```
#|

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 FOR ANY CLAIM BY ANY OTHER PARTY.

|#
```

EVENT: Start with the library "gcd" using the compiled version.

```
#|

The following C program computes the greatest common divisor of
three nonnegative integers a, b and c. We investigate the machine
code of this program generated by a widely used C compiler gcc,
and verify the correctness of the code.  The aim here is to see
how to handle subroutine calls.

gcd3(a, b, c)
long int a, b, c;
{
  gcd(gcd(a, b), c);
}

Here is the MC68020 assembly code of the above GCD program.  The code is
generated by gcc.
```

```
0x2324 <gcd3>:          linkw a6,#0
0x2328 <gcd3+4>:        movel a2,sp@-
0x232a <gcd3+6>:        movel a6@(16),sp@-
0x232e <gcd3+10>:       movel a6@(12),sp@-
0x2332 <gcd3+14>:       movel a6@(8),sp@-
0x2336 <gcd3+18>:       lea @#0x2350 <gcd>,a2
0x233c <gcd3+24>:       jsr a2@
0x233e <gcd3+26>:       addqw #8,sp
0x2340 <gcd3+28>:       movel d0,sp@-
0x2342 <gcd3+30>:       jsr a2@
0x2344 <gcd3+32>:       moveal a6@(-4),a2
0x2348 <gcd3+36>:       unlk a6
0x234a <gcd3+38>:       rts

The machine code of the above program is:

<gcd3>:       0x4e56  0x0000  0x2f0a  0x2f2e  0x0010  0x2f2e  0x000c  0x2f2e
<gcd3+16>:    0x0008  0x45f9  0x0000  0x2350  0x4e92  0x504f  0x2f00  0x4e92
<gcd3+32>:    0x246e  0xfffc  0x4e5e  0x4e75

'(78     86     0      0      47     10     47     46
  0      16     47     46     0      12     47     46
  0      8      69     249    0      0      35     80
  78     146    80     79     47     0      78     146
  36     110    255    252    78     94     78     117)
|#

; now we start to verify this GCD3 program,  defined by (gcd3-code).
```

Definition:
gcd3-code
= '(78 86 0 0 47 10 47 46 0 16 47 46 0 12 47 46 0 8 69
  249 -1 -1 -1 -1 78 146 80 79 47 0 78 146 36 110 255
  252 78 94 78 117)

Conservative Axiom: gcd3-load
gcd3-loadp (s)
= (evenp (gcd3-addr)
  ∧ (gcd3-addr $\in \mathbf{N}$)
  ∧ nat-rangep (gcd3-addr, 32)
  ∧ rom-addrp (gcd3-addr, mc-mem (s), 40)
  ∧ mcode-addrp (gcd3-addr, mc-mem (s), gcd3-code)
  ∧ gcd-loadp (s)
  ∧ (pc-read-mem (add (32, gcd3-addr, 20), mc-mem (s), 4) = gcd-addr))

Simultaneously, we introduce the new function symbols *gcd3-loadp* and *gcd3-addr*.

THEOREM: stepn-gcd3-loadp
$\text{gcd3-loadp}(\text{stepn}(s,\ n)) = \text{gcd3-loadp}(s)$

DEFINITION: $\text{gcd3}(a,\ b,\ c) = \text{gcd}(\text{gcd}(a,\ b),\ c)$

DEFINITION: $\text{gcd3-t0}(a,\ b,\ c) = \mathtt{7}$

DEFINITION: $\text{gcd3-t1}(a,\ b,\ c) = \text{gcd-t}(a,\ b)$

DEFINITION: $\text{gcd3-t2}(a,\ b,\ c) = \mathtt{3}$

DEFINITION: $\text{gcd3-t3}(a,\ b,\ c) = \text{gcd-t}(\text{gcd}(a,\ b),\ c)$

DEFINITION: $\text{gcd3-t4}(a,\ b,\ c) = \mathtt{3}$

DEFINITION:
$\text{gcd3-t}(a,\ b,\ c)$
$= \text{splus}(\text{gcd3-t0}(a,\ b,\ c),$
$\quad \text{splus}(\text{gcd3-t1}(a,\ b,\ c),$
$\quad\quad \text{splus}(\text{gcd3-t2}(a,\ b,\ c),\ \text{splus}(\text{gcd3-t3}(a,\ b,\ c),\ \text{gcd3-t4}(a,\ b,\ c)))))$

```
; the initial state.
```

DEFINITION:
$\text{gcd3-statep}(s,\ a,\ b,\ c)$
$= ((\text{mc-status}(s) = \mathtt{'running})$
$\land\ \text{gcd3-loadp}(s)$
$\land\ (\text{mc-pc}(s) = \text{GCD3-ADDR})$
$\land\ \text{ram-addrp}(\text{sub}(\mathtt{32},\ \mathtt{36},\ \text{read-sp}(s)),\ \text{mc-mem}(s),\ \mathtt{52})$
$\land\ (a = \text{iread-mem}(\text{add}(\mathtt{32},\ \text{read-sp}(s),\ \mathtt{4}),\ \text{mc-mem}(s),\ \mathtt{4}))$
$\land\ (b = \text{iread-mem}(\text{add}(\mathtt{32},\ \text{read-sp}(s),\ \mathtt{8}),\ \text{mc-mem}(s),\ \mathtt{4}))$
$\land\ (c = \text{iread-mem}(\text{add}(\mathtt{32},\ \text{read-sp}(s),\ \mathtt{12}),\ \text{mc-mem}(s),\ \mathtt{4}))$
$\land\ (\mathtt{0} < a)$
$\land\ (\mathtt{0} < b)$
$\land\ (\mathtt{0} < c))$

```
; the state after the execution of the first JSR instruction, but before
; the execution of the subroutine GCD.
```

DEFINITION:
$\text{gcd3-s0p}(s,\ a,\ b,\ c)$
$= ((\text{mc-status}(s) = \mathtt{'running})$
$\land\ \text{gcd3-loadp}(s)$

$\wedge$ (mc-pc (s) = GCD-ADDR)
$\wedge$ (read-an (32, 2, s) = GCD-ADDR)
$\wedge$ (rts-addr (s) = add (32, GCD3-ADDR, 26))
$\wedge$ ram-addrp (sub (32, 12, read-sp (s)), mc-mem (s), 52)
$\wedge$ equal* (read-an (32, 6, s), add (32, read-sp (s), 20))
$\wedge$ (a = iread-mem (add (32, read-sp (s), 4), mc-mem (s), 4))
$\wedge$ (b = iread-mem (add (32, read-sp (s), 8), mc-mem (s), 4))
$\wedge$ (c = iread-mem (add (32, read-sp (s), 12), mc-mem (s), 4))
$\wedge$ ($0 < a$)
$\wedge$ ($0 < b$)
$\wedge$ ($0 < c$))

```
; the state right after return from the first call to subroutine GCD.
```

DEFINITION:
gcd3-s1p (s, a, b, c)
= ((mc-status (s) = 'running)
$\wedge$ gcd3-loadp (s)
$\wedge$ (read-an (32, 2, s) = GCD-ADDR)
$\wedge$ (mc-pc (s) = add (32, GCD3-ADDR, 26))
$\wedge$ ram-addrp (sub (32, 16, read-sp (s)), mc-mem (s), 52)
$\wedge$ equal* (read-an (32, 6, s), add (32, read-sp (s), 16))
$\wedge$ (iread-dn (32, 0, s) = gcd (a, b))
$\wedge$ (c = iread-mem (add (32, read-sp (s), 8), mc-mem (s), 4))
$\wedge$ ($0 < a$)
$\wedge$ ($0 < b$)
$\wedge$ ($0 < c$))

```
; the state after the execution of the second JSR, but before the
; execution of the subroutine GCD.
```

DEFINITION:
gcd3-s2p (s, a, b, c)
= ((mc-status (s) = 'running)
$\wedge$ gcd3-loadp (s)
$\wedge$ (mc-pc (s) = GCD-ADDR)
$\wedge$ (rts-addr (s) = add (32, GCD3-ADDR, 32))
$\wedge$ ram-addrp (sub (32, 16, read-sp (s)), mc-mem (s), 52)
$\wedge$ equal* (read-an (32, 6, s), add (32, read-sp (s), 16))
$\wedge$ (gcd (a, b) = iread-mem (add (32, read-sp (s), 4), mc-mem (s), 4))
$\wedge$ (c = iread-mem (add (32, read-sp (s), 8), mc-mem (s), 4))
$\wedge$ ($0 < a$)
$\wedge$ ($0 < b$)
$\wedge$ ($0 < c$))

```
; the state returned from the second call to the subroutine GCD.
```

DEFINITION:
gcd3-s3p (s, a, b, c)
= ((mc-status (s) = 'running)
  ∧ gcd3-loadp (s)
  ∧ (mc-pc (s) = add (32, GCD3-ADDR, 32))
  ∧ ram-addrp (sub (32, 12, read-sp (s)), mc-mem (s), 44)
  ∧ equal* (read-an (32, 6, s), add (32, read-sp (s), 12))
  ∧ (gcd (gcd (a, b), c) = iread-dn (32, 0, s))
  ∧ ($0 < a$)
  ∧ ($0 < b$)
  ∧ ($0 < c$))

```
; from the initial state to s0.
```

THEOREM: gcd3-s-s0
let *s0* **be** stepn (s, gcd3-t0 (a, b, c))
in
gcd3-statep (s, a, b, c)
→ (gcd3-s0p (*s0*, a, b, c)
  ∧ (linked-rts-addr (*s0*) = rts-addr (s))
  ∧ (linked-a6 (*s0*) = read-an (32, 6, s))
  ∧ (read-rn (32, 14, mc-rfile (*s0*))
    = sub (32, 4, read-sp (s)))
  ∧ (rn-saved (*s0*) = read-an (32, 2, s))) **endlet**

THEOREM: gcd3-s-s0-rfile
(gcd3-statep (s, a, b, c) ∧ d2-7a3-5p (*rn*))
→ (read-rn (*oplen*, *rn*, mc-rfile (stepn (s, gcd3-t0 (a, b, c))))
  = read-rn (*oplen*, *rn*, mc-rfile (s)))

THEOREM: gcd3-s-s0-mem
(gcd3-statep (s, a, b, c) ∧ disjoint (x, k, sub (32, 36, read-sp (s)), 52))
→ (read-mem (x, mc-mem (stepn (s, gcd3-t0 (a, b, c))), k)
  = read-mem (x, mc-mem (s), k))

```
; from s0 to s1.
```

THEOREM: gcd3-s0-s1
let *s1* **be** stepn (s, gcd3-t1 (a, b, c))
in
gcd3-s0p (s, a, b, c)
→ (gcd3-s1p (*s1*, a, b, c)
  ∧ (linked-rts-addr (*s1*) = linked-rts-addr (s))

$\wedge \quad (\text{read-rn}\,(32,\ 14,\ \text{mc-rfile}\,(s1))$
$\quad = \quad \text{read-rn}\,(32,\ 14,\ \text{mc-rfile}\,(s)))$
$\wedge \quad (\text{linked-a6}\,(s1) = \text{linked-a6}\,(s))$
$\wedge \quad (\text{rn-saved}\,(s1) = \text{rn-saved}\,(s)))$ **endlet**

THEOREM: gcd3-s0-s1-rfile
$(\text{gcd3-s0p}\,(s,\ a,\ b,\ c) \wedge \text{d2-7a2-5p}\,(rn) \wedge (oplen \leq 32))$
$\rightarrow \quad (\text{read-rn}\,(oplen,\ rn,\ \text{mc-rfile}\,(\text{stepn}\,(s,\ \text{gcd3-t1}\,(a,\ b,\ c))))$
$\quad = \quad \text{read-rn}\,(oplen,\ rn,\ \text{mc-rfile}\,(s)))$

THEOREM: gcd3-s0-s1-mem
$(\text{gcd3-s0p}\,(s,\ a,\ b,\ c) \wedge \text{disjoint}\,(x,\ k,\ \text{sub}\,(32,\ 32,\ \text{read-an}\,(32,\ 6,\ s)),\ 52))$
$\rightarrow \quad (\text{read-mem}\,(x,\ \text{mc-mem}\,(\text{stepn}\,(s,\ \text{gcd3-t1}\,(a,\ b,\ c))),\ k)$
$\quad = \quad \text{read-mem}\,(x,\ \text{mc-mem}\,(s),\ k))$

```
; from s1 to s2.
```

THEOREM: gcd3-s1-s2
let $s2$ **be** $\text{stepn}\,(s,\ \text{gcd3-t2}\,(a,\ b,\ c))$
in
$\text{gcd3-s1p}\,(s,\ a,\ b,\ c)$
$\rightarrow \quad (\text{gcd3-s2p}\,(s2,\ a,\ b,\ c)$
$\wedge \quad (\text{linked-rts-addr}\,(s2) = \text{linked-rts-addr}\,(s))$
$\wedge \quad (\text{read-rn}\,(32,\ 14,\ \text{mc-rfile}\,(s2))$
$\quad = \quad \text{read-rn}\,(32,\ 14,\ \text{mc-rfile}\,(s)))$
$\wedge \quad (\text{linked-a6}\,(s2) = \text{linked-a6}\,(s))$
$\wedge \quad (\text{rn-saved}\,(s2) = \text{rn-saved}\,(s)))$ **endlet**

THEOREM: gcd3-s1-s2-rfile
$(\text{gcd3-s1p}\,(s,\ a,\ b,\ c) \wedge \text{d2-7a3-5p}\,(rn))$
$\rightarrow \quad (\text{read-rn}\,(oplen,\ rn,\ \text{mc-rfile}\,(\text{stepn}\,(s,\ \text{gcd3-t2}\,(a,\ b,\ c))))$
$\quad = \quad \text{read-rn}\,(oplen,\ rn,\ \text{mc-rfile}\,(s)))$

THEOREM: gcd3-s1-s2-mem
$(\text{gcd3-s1p}\,(s,\ a,\ b,\ c) \wedge \text{disjoint}\,(x,\ k,\ \text{sub}\,(32,\ 32,\ \text{read-an}\,(32,\ 6,\ s)),\ 52))$
$\rightarrow \quad (\text{read-mem}\,(x,\ \text{mc-mem}\,(\text{stepn}\,(s,\ \text{gcd3-t2}\,(a,\ b,\ c))),\ k)$
$\quad = \quad \text{read-mem}\,(x,\ \text{mc-mem}\,(s),\ k))$

```
; from s2 to s3.
```

THEOREM: gcd-nonzero
$((a \not\simeq 0) \wedge (b \not\simeq 0)) \rightarrow (\text{gcd}\,(a,\ b) \neq 0)$

THEOREM: gcd3-s2-s3
let $s3$ **be** $\text{stepn}\,(s,\ \text{gcd3-t3}\,(a,\ b,\ c))$
in

gcd3-s2p $(s,\ a,\ b,\ c)$
$\rightarrow$ (gcd3-s3p $(s3,\ a,\ b,\ c)$
$\wedge$ (linked-rts-addr $(s3)$ = linked-rts-addr (s))
$\wedge$ (read-rn (32, 14, mc-rfile $(s3)$)
= read-rn (32, 14, mc-rfile (s)))
$\wedge$ (linked-a6 $(s3)$ = linked-a6 (s))
$\wedge$ (rn-saved $(s3)$ = rn-saved (s))) **endlet**

THEOREM: gcd3-s2-s3-rfile
(gcd3-s2p $(s,\ a,\ b,\ c)$ $\wedge$ d2-7a2-5p (rn) $\wedge$ $(oplen \leq 32)$)
$\rightarrow$ (read-rn $(oplen,\ rn,$ mc-rfile (stepn $(s,$ gcd3-t3 $(a,\ b,\ c)$)))
= read-rn $(oplen,\ rn,$ mc-rfile (s)))

THEOREM: gcd3-s2-s3-mem
(gcd3-s2p $(s,\ a,\ b,\ c)$ $\wedge$ disjoint $(x,\ k,$ sub (32, 32, read-an (32, 6, s)), 52))
$\rightarrow$ (read-mem $(x,$ mc-mem (stepn $(s,$ gcd3-t3 $(a,\ b,\ c)$)), k)
= read-mem $(x,$ mc-mem $(s),\ k$))

```
; from s3 to exit.
```

THEOREM: gcd3-s3-sn
let sn **be** stepn $(s,$ gcd3-t4 $(a,\ b,\ c)$)
in
gcd3-s3p $(s,\ a,\ b,\ c)$
$\rightarrow$ ((mc-status (sn) = 'running)
$\wedge$ (mc-pc (sn) = linked-rts-addr (s))
$\wedge$ (iread-dn (32, 0, sn) = gcd (gcd $(a,\ b),\ c$))
$\wedge$ (read-rn (32, 14, mc-rfile (sn)) = linked-a6 (s))
$\wedge$ (read-rn (32, 15, mc-rfile (sn))
= add (32, read-an (32, 6, s), 8))) **endlet**

THEOREM: gcd3-s3-sn-rfile
(gcd3-s3p $(s,\ a,\ b,\ c)$ $\wedge$ $(oplen \leq 32)$ $\wedge$ d2-7a2-5p (rn))
$\rightarrow$ (read-rn $(oplen,\ rn,$ mc-rfile (stepn $(s,$ gcd3-t4 $(a,\ b,\ c)$)))
= **if** $rn = 10$ **then** head (rn-saved $(s),\ oplen$)
else read-rn $(oplen,\ rn,$ mc-rfile (s)) **endif**)

THEOREM: gcd3-s3-sn-mem
(gcd3-s3p $(s,\ a,\ b,\ c)$ $\wedge$ disjoint $(x,\ k,$ sub (32, 32, read-an (32, 6, s)), 52))
$\rightarrow$ (read-mem $(x,$ mc-mem (stepn $(s,$ gcd3-t4 $(a,\ b,\ c)$)), k)
= read-mem $(x,$ mc-mem $(s),\ k$))

EVENT: Disable gcd3-t0.

EVENT: Disable gcd3-t1.

EVENT: Disable gcd3-t2.

EVENT: Disable gcd3-t3.

EVENT: Disable gcd3-t4.

```
; the correctness of the program GCD3.
```

THEOREM: gcd3-correctness

let sn **be** stepn $(s,$ gcd3-t $(a,\ b,\ c))$
in
gcd3-statep $(s,\ a,\ b,\ c)$
$\rightarrow$ ((mc-status (sn) = `'running`)
$\wedge$ (mc-pc (sn) = rts-addr (s))
$\wedge$ (read-rn (32, 14, mc-rfile (sn))
= read-rn (32, 14, mc-rfile (s))))
$\wedge$ (read-rn (32, 15, mc-rfile (sn))
= add (32, read-rn (32, 15, mc-rfile (s)), 4))
$\wedge$ ((($oplen \leq 32$) $\wedge$ d2-7a2-5p (rn))
$\rightarrow$ (read-rn $(oplen,\ rn,$ mc-rfile $(sn))$
= read-rn $(oplen,\ rn,$ mc-rfile $(s))))$
$\wedge$ (disjoint $(x,\ k,$ sub (32, 36, read-sp (s)), 52)
$\rightarrow$ (read-mem $(x,$ mc-mem $(sn),\ k)$
= read-mem $(x,$ mc-mem $(s),\ k)))$
$\wedge$ (iread-dn (32, 0, sn) = gcd (gcd $(a,\ b),\ c$))) **endlet**

EVENT: Disable gcd3-t.

```
; in the logic, the function gcd3 does computes the greatest common divisor
; of its three arguments.
```

THEOREM: remainder-trans

$(((a \textbf{ mod } b) = 0) \wedge ((b \textbf{ mod } c) = 0)) \rightarrow ((a \textbf{ mod } c) = 0)$

THEOREM: gcd3-is-cd

$((a \textbf{ mod } \text{gcd3}(a,\ b,\ c)) = 0)$
$\wedge\ ((b \textbf{ mod } \text{gcd3}(a,\ b,\ c)) = 0)$
$\wedge\ ((c \textbf{ mod } \text{gcd3}(a,\ b,\ c)) = 0)$

THEOREM: cd-divides-gcd

$(((a \textbf{ mod } x) = 0) \wedge ((b \textbf{ mod } x) = 0)) \rightarrow ((\text{gcd}(a,\ b) \textbf{ mod } x) = 0)$

THEOREM: gcd3-the-greatest

$$
\begin{array}{ll}
\multicolumn{2}{l}{((a \not\simeq \mathsf{0})} \\
\wedge & (b \not\simeq \mathsf{0}) \\
\wedge & (c \not\simeq \mathsf{0}) \\
\wedge & ((a \ \mathbf{mod}\ x) = \mathsf{0}) \\
\wedge & ((b \ \mathbf{mod}\ x) = \mathsf{0}) \\
\wedge & ((c \ \mathbf{mod}\ x) = \mathsf{0})) \\
\rightarrow & (\mathrm{gcd3}\,(a,\ b,\ c) \not< x)
\end{array}
$$

EVENT: Disable remainder-trans.

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