`; Pigeon hole principle, as suggested by Randy Pollack.`

EVENT: Start with the initial **nqthm** theory.

DEFINITION:
subsetp $(x,\ y)$
$=$ **if** $x \simeq$ **nil then t**
elseif car $(x) \in y$ **then** subsetp (cdr $(x),\ y)$
else f endif

DEFINITION:
dups (l)
$=$ **if** $\neg$ listp (l) **then f**
elseif car $(l) \in$ cdr (l) **then t**
else dups (cdr (l)) **endif**

DEFINITION:
remove $(x,\ z)$
$=$ **if** $\neg$ listp (z) **then** z
elseif $x =$ car (z) **then** remove $(x,$ cdr $(z))$
else cons (car (z), remove $(x,$ cdr $(z))$) **endif**

THEOREM: remove-preserves-subsetp
(listp $(a) \land$ subsetp $(a,\ b)) \rightarrow$ subsetp (remove $(x,\ a)$, remove $(x,\ b)$)

DEFINITION:
double-list-ind $(l,\ m)$
$=$ **if** listp $(l) \land$ listp (m)
then double-list-ind (remove (car $(m),\ l)$, cdr (m))
else t endif

THEOREM: remove-car-non-dups
(listp $(m) \land (\neg$ dups $(m))) \rightarrow$ (remove (car $(m),\ m) =$ cdr (m))

DEFINITION:
length (lst)
$=$ **if** listp (lst) **then** $1 +$ length (cdr (lst))
else `0` **endif**

THEOREM: length-remove-leq
(length $(l) <$ length (remove $(x,\ l)$)) $=$ **f**

THEOREM: length-remove-lessp
$(x \in l) \rightarrow$ ((length (remove $(x,\ l)) <$ length $(l)) =$ **t**)

THEOREM: subsetp-dups
((length $(l) <$ length $(m)) \land$ subsetp $(m,\ l)) \rightarrow$ dups (m)

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