Here is a mechanically-checked proof of the finite version of Ramsey's Theorem for exponent 2. This proof was shown to me (with ordinary mathematical notation rather than in the Boyer-Moore logic) by Jim Schmerl, who claims that it's not a new proof.

Theorem (RAMSEY-THM-2):

(IMPLIES (LEQ (RAMSEY P Q) (LENGTH DOMAIN))
   (AND
    (SUBSETP (HOM-SET PAIRS DOMAIN P Q) DOMAIN)
    (OR
     (AND (HOMOGENEOUS (HOM-SET PAIRS DOMAIN P Q) PAIRS 1)
      (LEQ P (LENGTH (HOM-SET PAIRS DOMAIN P Q))))
     (AND (HOMOGENEOUS (HOM-SET PAIRS DOMAIN P Q) PAIRS 2)
      (LEQ Q (LENGTH (HOM-SET PAIRS DOMAIN P Q)))))))

Here, HOM-SET can be thought of as a Skolem function which picks out the desired homogeneous subset of the given DOMAIN, i.e., homogeneous with respect to the relation PAIRS (presented as a list of ordered pairs). Notice that if DOMAIN has no repetitions (which is a harmless assumption for the statement of Ramsey's Theorem), then LENGTH is just cardinality. The function LEQ is just "less than or equal", at least on the natural numbers. (HOMOGENEOUS S PAIRS 1) is true, i.e. equals T, if the list S is homogeneous with respect to PAIRS in the positive sense, i.e., in that each pair from S is related by PAIRS (i.e. either the pair or its reverse belongs to PAIRS). Similarly, (HOMOGENEOUS S PAIRS 2) is true if no pair from S belongs to PAIRS (in the same sense). RAMSEY is a function which returns an integer which (according to the theorem above) is "large enough". However, at the end of this note we prove that (RAMSEY P Q) = (CHOOSE (PLUS P Q) P), where (CHOOSE M N) is the binomial coefficient (defined though with integer division).
; The description above completes the discussion of the theorem, 
; but to read on it helps to realize that we don’t define HOM-SET 
; until near the end of this note. Instead, we define a function 
; WIT which returns a pair, so that HOM-SET merely returns the CAR 
; of this pair. The CDR of the value of WIT is the "color", which 
; is 1 or 2 (as is proved in a lemma below). It’s really best to 
; think of WIT as a Skolem function, though in fact it is defined 
; to make the theorem true.

**Event:** Start with the initial `nqthm` theory.

**Definition:**

```lisp
ramsey (p, q) = if p ≃ 0 then 1 
elseif q ≃ 0 then 1 
else ramsey (p - 1, q) + ramsey (p, q - 1) endif
```

; this is a hint to get the
; definition accepted

**Definition:**

```lisp
related (i, j, pairs) = ((cons (i, j) ∈ pairs) ∨ (cons (j, i) ∈ pairs))
```

**Definition:**

```lisp
partition (n, rest, pairs) = 
if listp (rest) 
then if related (n, car (rest), pairs) 
then cons (cons (car (rest), car (partition (n, cdr (rest), pairs))), 
cdr (partition (n, cdr (rest), pairs)))
else cons (car (partition (n, cdr (rest), pairs)), 
cons (car (rest), cdr (partition (n, cdr (rest), pairs)))) endif 
else cons (nil, nil) endif
```

**Definition:**

```lisp
length (lst) = if listp (lst) then 1 + length (cdr (lst)) 
else 0 endif
```

**Definition:**

```lisp
wit (pairs, domain, p, q) = if p ≃ 0 then cons (nil, 1) 
elseif q ≃ 0 then cons (nil, 2) 
elseif length (car (partition (car (domain), cdr (domain), pairs)))
```
ramsey (p - 1, q) 

then if cdr (wit (pairs, 
    cdr (partition (car (domain), cdr (domain), pairs)), 
    p, 
    q - 1)) 

= 1 

then wit (pairs, 
    cdr (partition (car (domain), cdr (domain), pairs)), 
    p, 
    q - 1) 

else cons (cons (car (domain), 
    car (wit (pairs, 
        cdr (partition (car (domain), 
            cdr (domain), 
            pairs)), 
        p, 
        q - 1))), 

 endif 

elseif cdr (wit (pairs, 
    car (partition (car (domain), cdr (domain), pairs)), 
    p - 1, 
    q)) 

= 2 

then wit (pairs, car (partition (car (domain), cdr (domain), pairs)), p - 1, q) 

else cons (cons (car (domain), 
    car (wit (pairs, 
        car (partition (car (domain), 
            cdr (domain), 
            pairs)), 
        p - 1, 
        q))), 

 1) endif 

Definition: 

homogeneous1 (n, domain, pairs, flg) 
= if listp (domain) 

  then if flg = 1 then related (n, car (domain), pairs) 
       else ¬ related (n, car (domain), pairs) endif 

       ∧ homogeneous1 (n, cdr (domain), pairs, flg) 

  else t endif 

Definition: 

homogeneous (domain, pairs, flg) 
= if listp (domain)
then homogeneous1 (car (domain), cdr (domain), pairs, flg)
    ∧ homogeneous (cdr (domain), pairs, flg)
else t endif

THEOREM: length-of-partitions
length (cons (n, domain))
= (1 + (length (car (partition (n, domain, pairs))))
   + length (cdr (partition (n, domain, pairs))))

THEOREM: lessp-length-ramsey-2
(listp (domain)
 ∧ (i ≠ 0)
 ∧ (j ≠ 0)
 ∧ ((i + j) ≤ length (domain))
 ∧ (length (car (partition (car (domain), cdr (domain), pairs))) < i))
→ ((length (cdr (partition (car (domain), cdr (domain), pairs))) < j) = f)

THEOREM: ramsey-not-zerop
ramsey (p, q) ≠ 0

EVENT: Disable lessp-length-ramsey-2.

THEOREM: lessp-length-ramsey
((p ≠ 0)
 ∧ (q ≠ 0)
 ∧ (ramsey (p, q) ≤ length (domain))
 ∧ (length (car (partition (car (domain), cdr (domain), pairs)))
     < ramsey (p - 1, q)))
→ ((length (cdr (partition (car (domain), cdr (domain), pairs)))
    < ramsey (p, q - 1))
    = f)

THEOREM: wit-flag
(cdr (wit (pairs, domain, p, q)) ≠ 1) → (cdr (wit (pairs, domain, p, q)) = 2)

THEOREM: homogeneous1-partition-cdr
homogeneous1 (a, cdr (partition (a, domain, pairs)), pairs, 2)

THEOREM: homogeneous1-partition-car
homogeneous1 (a, car (partition (a, domain, pairs)), pairs, 1)

DEFINITION:
subsetp (x, y)
= if x ≈ nil then t
   elseif car (x) ∈ y then subsetp (cdr (x), y)
   else f endif
Theorem: subsetp-preserves-homogeneous1
(\text{subsetp}(l, m) \land \text{homogeneous1}(a, m, \text{pairs, flg}))
\rightarrow \text{homogeneous1}(a, l, \text{pairs, flg})

; The following lemmas, till further notice, are simply; lemmas about sets, some of which may be needed below.

Theorem: member-cons
(a \in l) \rightarrow (a \in \text{cons}(x, l))

Theorem: subsetp-cons
\text{subsetp}(l, m) \rightarrow \text{subsetp}(l, \text{cons}(a, m))

Theorem: subsetp-reflexivity
\text{subsetp}(x, x)

Theorem: cdr-subsetp
\text{subsetp}(\text{cdr}(x), x)

Theorem: member-subsetp
((x \in y) \land \text{subsetp}(y, z)) \rightarrow (x \in z)

Theorem: subsetp-is-transitive
(\text{subsetp}(x, y) \land \text{subsetp}(y, z)) \rightarrow \text{subsetp}(x, z)

; end of silly subset lemmas

Theorem: partition-subsetp
\text{subsetp} (\text{car} (\text{partition} (a, \text{dom}, \text{pairs})), \text{dom})
\land \text{subsetp} (\text{cdr} (\text{partition} (a, \text{dom}, \text{pairs})), \text{dom})

Theorem: ramsey-not-zero
\text{ramsey}(p, q) \neq 0

Theorem: witness-subsetp
(\text{ramsey}(p, q) \leq \text{length}(\text{domain}))
\rightarrow \text{subsetp}(\text{car}(\text{wit}(\text{pairs, domain, p, q})), \text{domain})

; Here are the commands to generate proofs of four of; the six induction steps in the theorem.

Theorem: ramsey-thm-2-goals-1-2-3-5
\begin{itemize}
\item \textbf{Theorem: all-but-goal-4}
\item \textbf{Theorem: ramsey-thm-2-except-length}
\begin{align*}
\text{ramsey}(p, q) &\leq \text{length}(\text{domain}) \\
\rightarrow &\quad \text{homogeneous}(\text{car}(\text{wit}([\text{pairs}, \text{domain}, p, q]), \\
&\text{pairs}, \\
&\text{cdr}(\text{wit}([\text{pairs}, \text{domain}, p, q])))
\end{align*}
\item \textbf{Theorem: ramsey-thm-2-length}
\begin{align*}
\text{ramsey}(p, q) &\leq \text{length}(\text{domain}) \\
\rightarrow &\quad ((\text{length}(\text{car}(\text{wit}([\text{pairs}, \text{domain}, p, q]))) \\
< \begin{cases} 
\text{if} &\text{cdr}(\text{wit}([\text{pairs}, \text{domain}, p, q])) = 1 \text{ then } p \\
\text{else} &q \text{ endif}
\end{cases} \\
= f)
\end{align*}
\item \textbf{Definition:}
\begin{align*}
\text{hom-set}([\text{pairs}, \text{domain}, p, q]) &= \text{car}(\text{wit}([\text{pairs}, \text{domain}, p, q]))
\end{align*}
\item \textbf{Theorem: ramsey-thm-2}
\begin{align*}
\text{ramsey}(p, q) &\leq \text{length}(\text{domain}) \\
\rightarrow &\quad (\text{subsetp}([\text{hom-set}([\text{pairs}, \text{domain}, p, q]), \text{domain}]) \\
&\quad \land ((\text{homogeneous}([\text{hom-set}([\text{pairs}, \text{domain}, p, q]), \text{pairs}, 1] \\
&\quad \land (p \leq \text{length}([\text{hom-set}([\text{pairs}, \text{domain}, p, q])])) \\
&\quad \lor (\text{homogeneous}([\text{hom-set}([\text{pairs}, \text{domain}, p, q]), \text{pairs}, 2] \\
&\quad \land (q \leq \text{length}([\text{hom-set}([\text{pairs}, \text{domain}, p, q])])))))
\end{align*}
\item \textbf{Definition:}
\begin{align*}
\text{setp}(x) &= \begin{cases} 
\text{if} &\text{listp}(x) \text{ then } (\text{car}(x) \not\in \text{cdr}(x)) \land \text{setp}(\text{cdr}(x)) \\
\text{else} &t \text{ endif}
\end{cases}
\end{align*}
\item \textbf{Theorem: set-partition}
\begin{align*}
\text{setp}(x) &\rightarrow (\text{setp}(\text{car}(\text{partition}(a, x, \text{pairs}))) \land \text{setp}(\text{cdr}(\text{partition}(a, x, \text{pairs}))))
\end{align*}
\end{itemize}
Theorem: member-partition
\[(a \notin x) \rightarrow ((a \notin \text{car}(\text{partition}(b, x, \text{pairs}))) \land (a \notin \text{cdr}(\text{partition}(b, x, \text{pairs}))))\]

Theorem: member-cons-expand
\[(a \in \text{cons}(b, y)) = \begin{cases} \text{t} & \text{if } a = b \\ a \in y & \text{else} \end{cases}\]

Theorem: setp-cons-expand
\[
\text{setp}(\text{cons}(b, y)) = \begin{cases} \text{f} & \text{if } b \in y \\ \text{setp}(y) & \text{else} \end{cases}
\]

Theorem: member-hom-set-implies-member-domain
\[
((\text{length}(\text{domain}) \not< \text{ramsey}(p, q)) \land (a \notin \text{domain})) \rightarrow (a \notin \text{car}(\text{wit}(\text{pairs}, \text{domain}, p, q)))
\]

Theorem: setp-hom-set
\[
((\text{length}(\text{domain}) \not< \text{ramsey}(p, q)) \land \text{setp}(\text{domain})) \rightarrow \text{setp}(\text{hom-set}(\text{pairs}, \text{domain}, p, q))
\]
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