

EVENT: Start with the initial **nqthm** theory.

DEFINITION:

$\text{subsetp}(x, y)$
= **if** $x \simeq \text{nil}$ **then t**
 elseif $\text{car}(x) \in y$ **then** $\text{subsetp}(\text{cdr}(x), y)$
 else f endif

THEOREM: member-cons
 $(a \in l) \rightarrow (a \in \text{cons}(x, l))$

THEOREM: subsetp-cons
 $\text{subsetp}(l, m) \rightarrow \text{subsetp}(l, \text{cons}(a, m))$

THEOREM: subsetp-reflexivity
 $\text{subsetp}(x, x)$

THEOREM: cdr-subsetp
 $\text{subsetp}(\text{cdr}(x), x)$

THEOREM: member-subsetp
 $((x \in y) \wedge \text{subsetp}(y, z)) \rightarrow (x \in z)$

THEOREM: subsetp-is-transitive
 $(\text{subsetp}(x, y) \wedge \text{subsetp}(y, z)) \rightarrow \text{subsetp}(x, z)$

THEOREM: append-is-subsetp
 $\text{subsetp}(x, y) \rightarrow \text{subsetp}(\text{append}(x, y), y)$

THEOREM: member-append-2
 $(x \in z) \rightarrow (x \in \text{append}(y, z))$

THEOREM: member-append-1
 $(x \in y) \rightarrow (x \in \text{append}(y, z))$

THEOREM: non-member-eliminates-append-1
 $(x \notin y) \rightarrow ((x \in \text{append}(y, z)) = (x \in z))$

THEOREM: non-member-eliminates-append-2
 $(x \notin z) \rightarrow ((x \in \text{append}(y, z)) = (x \in y))$

THEOREM: member-append-3
 $(x \in \text{append}(y, z)) = ((x \in y) \vee (x \in z))$

THEOREM: subsetp-of-append-1
 $\text{subsetp}(x, \text{append}(x, y))$

THEOREM: subsetp-of-append-2

$\text{subsetp}(y, \text{append}(x, y))$

THEOREM: append-preserves-subsetp-second-1

$\text{subsetp}(x, y) \rightarrow \text{subsetp}(x, \text{append}(y, z))$

THEOREM: append-preserves-subsetp-second-2

$\text{subsetp}(x, z) \rightarrow \text{subsetp}(x, \text{append}(y, z))$

THEOREM: append-preserves-subsetp-first

$(\text{subsetp}(x, z) \wedge \text{subsetp}(y, z)) \rightarrow \text{subsetp}(\text{append}(x, y), z)$

THEOREM: subsetp-append-append

$(\text{subsetp}(a, b) \wedge \text{subsetp}(c, d)) \rightarrow \text{subsetp}(\text{append}(a, c), \text{append}(b, d))$

THEOREM: union-append-subsetp

$\text{subsetp}(x \cup y, \text{append}(x, y))$

THEOREM: append-union-subsetp

$\text{subsetp}(\text{append}(x, y), x \cup y)$

THEOREM: union-is-subsetp

$\text{subsetp}(x, y) \rightarrow \text{subsetp}(x \cup y, y)$

THEOREM: member-union-2

$(x \in z) \rightarrow (x \in (y \cup z))$

THEOREM: member-union-1

$(x \in y) \rightarrow (x \in (y \cup z))$

THEOREM: non-member-eliminates-union-1

$(x \notin y) \rightarrow ((x \in (y \cup z)) = (x \in z))$

THEOREM: non-member-eliminates-union-2

$(x \notin z) \rightarrow ((x \in (y \cup z)) = (x \in y))$

THEOREM: member-union-3

$(x \in (y \cup z)) = ((x \in y) \vee (x \in z))$

THEOREM: subsetp-of-union-1

$\text{subsetp}(x, x \cup y)$

THEOREM: subsetp-of-union-2

$\text{subsetp}(y, x \cup y)$

THEOREM: union-preserves-subsetp-second-1

$\text{subsetp}(x, y) \rightarrow \text{subsetp}(x, y \cup z)$

THEOREM: union-preserves-subsetp-second-2

$\text{subsetp}(x, z) \rightarrow \text{subsetp}(x, y \cup z)$

THEOREM: union-preserves-subsetp-first

$(\text{subsetp}(x, z) \wedge \text{subsetp}(y, z)) \rightarrow \text{subsetp}(x \cup y, z)$

THEOREM: subsetp-union-union

$(\text{subsetp}(a, b) \wedge \text{subsetp}(c, d)) \rightarrow \text{subsetp}(a \cup c, b \cup d)$

THEOREM: union-subsetp

$\text{subsetp}(x, y) \rightarrow ((x \cup y) = y)$

DEFINITION:

$\text{make-set}(l)$

```
=  if  $\neg \text{listp}(l)$  then nil
    elseif  $\text{car}(l) \in \text{cdr}(l)$  then  $\text{make-set}(\text{cdr}(l))$ 
    else  $\text{cons}(\text{car}(l), \text{make-set}(\text{cdr}(l)))$  endif
```

THEOREM: make-set-preserves-member

$(x \in \text{make-set}(l)) = (x \in l)$

THEOREM: make-set-preserves-subsetp-1

$\text{subsetp}(\text{make-set}(x), \text{make-set}(y)) = \text{subsetp}(x, y)$

THEOREM: make-set-preserves-subsetp-2

$\text{subsetp}(x, \text{make-set}(y)) = \text{subsetp}(x, y)$

THEOREM: make-set-preserves-subsetp-3

$\text{subsetp}(\text{make-set}(x), y) = \text{subsetp}(x, y)$

DEFINITION:

$\text{setp}(x)$

```
=  if  $\neg \text{listp}(x)$  then t
    else  $(\text{car}(x) \notin \text{cdr}(x)) \wedge \text{setp}(\text{cdr}(x))$  endif
```

THEOREM: make-set-gives-setp

$\text{setp}(\text{make-set}(x))$

THEOREM: union-of-sets-is-setp

$(\text{setp}(x) \wedge \text{setp}(y)) \rightarrow \text{setp}(x \cup y)$

EVENT: Make the library "**subset**".

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