EVENT: Start with the initial **nqthm** theory.

DEFINITION:
subsetp $(x,\ y)$
$=$ **if** $x \simeq$ **nil then t**
elseif car $(x) \in y$ **then** subsetp (cdr $(x),\ y)$
else f endif

THEOREM: member-cons
$(a \in l) \rightarrow (a \in \text{cons}\,(x,\ l))$

THEOREM: subsetp-cons
$\text{subsetp}\,(l,\ m) \rightarrow \text{subsetp}\,(l,\ \text{cons}\,(a,\ m))$

THEOREM: subsetp-reflexivity
$\text{subsetp}\,(x,\ x)$

THEOREM: cdr-subsetp
$\text{subsetp}\,(\text{cdr}\,(x),\ x)$

THEOREM: member-subsetp
$((x \in y) \land \text{subsetp}\,(y,\ z)) \rightarrow (x \in z)$

THEOREM: subsetp-is-transitive
$(\text{subsetp}\,(x,\ y) \land \text{subsetp}\,(y,\ z)) \rightarrow \text{subsetp}\,(x,\ z)$

THEOREM: append-is-subsetp
$\text{subsetp}\,(x,\ y) \rightarrow \text{subsetp}\,(\text{append}\,(x,\ y),\ y)$

THEOREM: member-append-2
$(x \in z) \rightarrow (x \in \text{append}\,(y,\ z))$

THEOREM: member-append-1
$(x \in y) \rightarrow (x \in \text{append}\,(y,\ z))$

THEOREM: non-member-eliminates-append-1
$(x \notin y) \rightarrow ((x \in \text{append}\,(y,\ z)) = (x \in z))$

THEOREM: non-member-eliminates-append-2
$(x \notin z) \rightarrow ((x \in \text{append}\,(y,\ z)) = (x \in y))$

THEOREM: member-append-3
$(x \in \text{append}\,(y,\ z)) = ((x \in y) \lor (x \in z))$

THEOREM: subsetp-of-append-1
$\text{subsetp}\,(x,\ \text{append}\,(x,\ y))$

Theorem: subsetp-of-append-2
$\text{subsetp}(y,\ \text{append}(x,\ y))$

Theorem: append-preserves-subsetp-second-1
$\text{subsetp}(x,\ y) \rightarrow \text{subsetp}(x,\ \text{append}(y,\ z))$

Theorem: append-preserves-subsetp-second-2
$\text{subsetp}(x,\ z) \rightarrow \text{subsetp}(x,\ \text{append}(y,\ z))$

Theorem: append-preserves-subsetp-first
$(\text{subsetp}(x,\ z) \wedge \text{subsetp}(y,\ z)) \rightarrow \text{subsetp}(\text{append}(x,\ y),\ z)$

Theorem: subsetp-append-append
$(\text{subsetp}(a,\ b) \wedge \text{subsetp}(c,\ d)) \rightarrow \text{subsetp}(\text{append}(a,\ c),\ \text{append}(b,\ d))$

Theorem: union-append-subsetp
$\text{subsetp}(x \cup y,\ \text{append}(x,\ y))$

Theorem: append-union-subsetp
$\text{subsetp}(\text{append}(x,\ y),\ x \cup y)$

Theorem: union-is-subsetp
$\text{subsetp}(x,\ y) \rightarrow \text{subsetp}(x \cup y,\ y)$

Theorem: member-union-2
$(x \in z) \rightarrow (x \in (y \cup z))$

Theorem: member-union-1
$(x \in y) \rightarrow (x \in (y \cup z))$

Theorem: non-member-eliminates-union-1
$(x \notin y) \rightarrow ((x \in (y \cup z)) = (x \in z))$

Theorem: non-member-eliminates-union-2
$(x \notin z) \rightarrow ((x \in (y \cup z)) = (x \in y))$

Theorem: member-union-3
$(x \in (y \cup z)) = ((x \in y) \vee (x \in z))$

Theorem: subsetp-of-union-1
$\text{subsetp}(x,\ x \cup y)$

Theorem: subsetp-of-union-2
$\text{subsetp}(y,\ x \cup y)$

Theorem: union-preserves-subsetp-second-1
$\text{subsetp}(x,\ y) \rightarrow \text{subsetp}(x,\ y \cup z)$

THEOREM: union-preserves-subsetp-second-2
$\text{subsetp}(x, z) \rightarrow \text{subsetp}(x, y \cup z)$

THEOREM: union-preserves-subsetp-first
$(\text{subsetp}(x, z) \wedge \text{subsetp}(y, z)) \rightarrow \text{subsetp}(x \cup y, z)$

THEOREM: subsetp-union-union
$(\text{subsetp}(a, b) \wedge \text{subsetp}(c, d)) \rightarrow \text{subsetp}(a \cup c, b \cup d)$

THEOREM: union-subsetp
$\text{subsetp}(x, y) \rightarrow ((x \cup y) = y)$

DEFINITION:
make-set (l)
= **if** $\neg$ listp (l) **then nil**
elseif car $(l) \in$ cdr (l) **then** make-set (cdr (l))
else cons (car (l), make-set (cdr (l))) **endif**

THEOREM: make-set-preserves-member
$(x \in \text{make-set}(l)) = (x \in l)$

THEOREM: make-set-preserves-subsetp-1
$\text{subsetp}(\text{make-set}(x), \text{make-set}(y)) = \text{subsetp}(x, y)$

THEOREM: make-set-preserves-subsetp-2
$\text{subsetp}(x, \text{make-set}(y)) = \text{subsetp}(x, y)$

THEOREM: make-set-preserves-subsetp-3
$\text{subsetp}(\text{make-set}(x), y) = \text{subsetp}(x, y)$

DEFINITION:
setp (x)
= **if** $\neg$ listp (x) **then t**
else (car $(x) \notin$ cdr (x)) $\wedge$ setp (cdr (x)) **endif**

THEOREM: make-set-gives-setp
$\text{setp}(\text{make-set}(x))$

THEOREM: union-of-sets-is-setp
$(\text{setp}(x) \wedge \text{setp}(y)) \rightarrow \text{setp}(x \cup y)$

EVENT: Make the library `"subset"`.

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