**Event**: Start with the initial `nqthm` theory.

**Definition**:  
\[
delete(x, l) = \begin{cases} 
\text{if } \text{listp}(l) \text{ then if } x = \text{car}(l) \text{ then } \text{cdr}(l) \\
& \text{else cons(car(l), delete(x, cdr(l))) endif} \\
\text{else l endif}
\end{cases}
\]

**Definition**:  
\[
\text{bagdiff}(x, y) = \begin{cases} 
\text{if } \text{listp}(y) \text{ then if } \text{car}(y) \in x \text{ then bagdiff(delete(car(y), x), cdr(y)) } \\
& \text{else bagdiff(x, cdr(y)) endif} \\
\text{else x endif}
\end{cases}
\]

**Definition**:  
\[
\text{bagint}(x, y) = \begin{cases} 
\text{if } \text{listp}(x) \text{ then if } \text{car}(x) \in y \\
& \text{then cons(car(x), bagint(cdr(x), delete(car(x), y)))) } \\
& \text{else bagint(cdr(x), y) endif} \\
\text{else nil endif}
\end{cases}
\]

**Definition**:  
\[
\text{occurrences}(x, l) = \begin{cases} 
\text{if } \text{listp}(l) \text{ then if } x = \text{car}(l) \text{ then } 1 + \text{occurrences}(x, \text{cdr}(l)) \\
& \text{else occurrences}(x, \text{cdr}(l)) endif} \\
\text{else 0 endif}
\end{cases}
\]

**Definition**:  
\[
\text{subbagp}(x, y) = \begin{cases} 
\text{if } \text{listp}(x) \text{ then if } \text{car}(x) \in y \\
& \text{then subbagp(cdr(x), delete(car(x), y)) } \\
& \text{else t endif} \\
\text{else f endif}
\end{cases}
\]

**Theorem**: `listp-delete`  
\[
\text{listp}(\text{delete}(x, l)) = \begin{cases} 
\text{if } \text{listp}(l) \text{ then } (x \neq \text{car}(l)) \lor \text{listp}(\text{cdr}(l)) \\
& \text{else f endif}
\end{cases}
\]

**Event**: Disable `listp-delete`. 
Theorem: delete-non-member
\((x \notin y) \rightarrow (\text{delete}(x, y) = y)\)

Theorem: delete-delete
\(\text{delete}(y, \text{delete}(x, z)) = \text{delete}(x, \text{delete}(y, z))\)

Theorem: equal-occurrences-zero
\((\text{occurrences}(x, l) = 0) = (x \notin l)\)

Theorem: member-non-list
\((\neg \text{listp}(l)) \rightarrow (x \notin l)\)

Theorem: member-delete
\((x \in \text{delete}(y, l)) =\)
\(\begin{aligned}
   \text{if } x \in l & \text{ then if } x = y \text{ then } 1 < \text{occurrences}(x, l) \\
   & \text{else } t \text{ endif} \\
   \text{else } f \text{ endif}
\end{aligned}\)

Theorem: member-delete-implies-membership
\((x \in \text{delete}(y, l)) \rightarrow (x \in l)\)

Theorem: occurrences-delete
\(\text{occurrences}(x, \text{delete}(y, l)) =\)
\(\begin{aligned}
   \text{if } x = y & \text{ then if } x \in l \text{ then } \text{occurrences}(x, l) - 1 \\
   & \text{else } 0 \text{ endif} \\
   \text{else } \text{occurrences}(x, l) \text{ endif}
\end{aligned}\)

Theorem: member-bagdiff
\((x \in \text{bagdiff}(a, b)) = (\text{occurrences}(x, b) < \text{occurrences}(x, a))\)

Theorem: bagdiff-delete
\(\text{bagdiff}(\text{delete}(e, x), y) = \text{delete}(e, \text{bagdiff}(x, y))\)

Theorem: subbagp-delete
\(\text{subbagp}(x, \text{delete}(u, y)) \rightarrow \text{subbagp}(x, y)\)

Theorem: subbagp-cdr1
\(\text{subbagp}(x, y) \rightarrow \text{subbagp}(\text{cdr}(x), y)\)

Theorem: subbagp-cdr2
\(\text{subbagp}(x, \text{cdr}(y)) \rightarrow \text{subbagp}(x, y)\)

Theorem: subbagp-bagint1
\(\text{subbagp}(\text{bagint}(x, y), x)\)
THEOREM: subbagp-bagint2
subbagp (bagint (x, y), y)

THEOREM: occurrences-bagint
occurrences (x, bagint (a, b))
= if occurrences (x, a) < occurrences (x, b) then occurrences (x, a)
   else occurrences (x, b) endif

THEOREM: occurrences-bagdiff
occurrences (x, bagdiff (a, b)) = (occurrences (x, a) — occurrences (x, b))

THEOREM: member-bagint
(x ∈ bagint (a, b)) = ((x ∈ a) ∧ (x ∈ b))

EVENT: Let us define the theory bags to consist of the following events: occurrences-
bagint, bagdiff-delete, occurrences-bagdiff, member-bagint, member-bagdiff, subbagp-
bagint2, subbagp-bagint1, subbagp-cdr2, subbagp-cdr1, subbagp-delete.

EVENT: Make the library "bags".
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