EVENT: Start with the initial **nqthm** theory.

DEFINITION:
delete $(x,\ l)$
$=$ **if** listp (l)
then if $x = \text{car}\,(l)$ **then** cdr (l)
else cons (car (l), delete $(x$, cdr $(l)))$ **endif**
else l **endif**

DEFINITION:
bagdiff $(x,\ y)$
$=$ **if** listp (y)
then if car $(y) \in x$ **then** bagdiff (delete (car (y), x), cdr (y))
else bagdiff $(x$, cdr $(y))$ **endif**
else x **endif**

DEFINITION:
bagint $(x,\ y)$
$=$ **if** listp (x)
then if car $(x) \in y$
then cons (car (x), bagint (cdr (x), delete (car (x), y)))
else bagint (cdr (x), y) **endif**
else nil endif

DEFINITION:
occurrences $(x,\ l)$
$=$ **if** listp (l)
then if $x = \text{car}\,(l)$ **then** $1 +$ occurrences $(x$, cdr $(l))$
else occurrences $(x$, cdr $(l))$ **endif**
else 0 **endif**

DEFINITION:
subbagp $(x,\ y)$
$=$ **if** listp (x)
then if car $(x) \in y$ **then** subbagp (cdr (x), delete (car (x), y))
else f endif
else t endif

THEOREM: listp-delete
listp (delete $(x,\ l)$)
$=$ **if** listp (l) **then** $(x \neq \text{car}\,(l)) \lor \text{listp}\,(\text{cdr}\,(l))$
else f endif

EVENT: Disable listp-delete.

THEOREM: delete-non-member
$(x \notin y) \rightarrow (\text{delete}(x,\ y) = y)$

THEOREM: delete-delete
$\text{delete}(y,\ \text{delete}(x,\ z)) = \text{delete}(x,\ \text{delete}(y,\ z))$

THEOREM: equal-occurrences-zero
$(\text{occurrences}(x,\ l) = \mathtt{0}) = (x \notin l)$

THEOREM: member-non-list
$(\neg\ \text{listp}(l)) \rightarrow (x \notin l)$

THEOREM: member-delete
$$
\begin{array}{ll}
\multicolumn{2}{l}{(x \in \text{delete}(y,\ l))} \\
= & \textbf{if } x \in l \\
 & \textbf{then if } x = y \ \textbf{ then } \mathtt{1} < \text{occurrences}(x,\ l) \\
 & \qquad\quad \textbf{else t endif} \\
 & \textbf{else f endif}
\end{array}
$$

THEOREM: member-delete-implies-membership
$(x \in \text{delete}(y,\ l)) \rightarrow (x \in l)$

THEOREM: occurrences-delete
$$
\begin{array}{ll}
\multicolumn{2}{l}{\text{occurrences}(x,\ \text{delete}(y,\ l))} \\
= & \textbf{if } x = y \\
 & \textbf{then if } x \in l \ \textbf{ then } \text{occurrences}(x,\ l) - 1 \\
 & \qquad\quad \textbf{else } \mathtt{0} \textbf{ endif} \\
 & \textbf{else } \text{occurrences}(x,\ l) \textbf{ endif}
\end{array}
$$

THEOREM: member-bagdiff
$(x \in \text{bagdiff}(a,\ b)) = (\text{occurrences}(x,\ b) < \text{occurrences}(x,\ a))$

THEOREM: bagdiff-delete
$\text{bagdiff}(\text{delete}(e,\ x),\ y) = \text{delete}(e,\ \text{bagdiff}(x,\ y))$

THEOREM: subbagp-delete
$\text{subbagp}(x,\ \text{delete}(u,\ y)) \rightarrow \text{subbagp}(x,\ y)$

THEOREM: subbagp-cdr1
$\text{subbagp}(x,\ y) \rightarrow \text{subbagp}(\text{cdr}(x),\ y)$

THEOREM: subbagp-cdr2
$\text{subbagp}(x,\ \text{cdr}(y)) \rightarrow \text{subbagp}(x,\ y)$

THEOREM: subbagp-bagint1
$\text{subbagp}(\text{bagint}(x,\ y),\ x)$

THEOREM: subbagp-bagint2
subbagp (bagint $(x,\ y),\ y)$

THEOREM: occurrences-bagint
occurrences $(x,$ bagint $(a,\ b))$
$=$ **if** occurrences $(x,\ a) <$ occurrences $(x,\ b)$ **then** occurrences $(x,\ a)$
else occurrences $(x,\ b)$ **endif**

THEOREM: occurrences-bagdiff
occurrences $(x,$ bagdiff $(a,\ b)) =$ (occurrences $(x,\ a)$ $-$ occurrences $(x,\ b))$

THEOREM: member-bagint
$(x \in$ bagint $(a,\ b)) = ((x \in a) \land (x \in b))$

EVENT: Let us define the theory *bags* to consist of the following events: occurrences-bagint, bagdiff-delete, occurrences-bagdiff, member-bagint, member-bagdiff, subbagp-bagint2, subbagp-bagint1, subbagp-cdr2, subbagp-cdr1, subbagp-delete.

EVENT: Make the library `"bags"`.

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