Event: Start with the initial nqthm theory.

Definition:
delete (x, l)
= if listp (l)
    then if x = car (l) then cdr (l)
        else cons (car (l), delete (x, cdr (l))) endif
    else l endif

Definition:
bagdiff (x, y)
= if listp (y)
    then if car (y) ∈ x then bagdiff (delete (car (y), x), cdr (y))
        else bagdiff (x, cdr (y)) endif
    else x endif

Definition:
bagint (x, y)
= if listp (x)
    then if car (x) ∈ y
        then cons (car (x), bagint (cdr (x), delete (car (x), y)))
        else bagint (cdr (x), y) endif
    else nil endif

Definition:
ocurrences (x, l)
= if listp (l)
    then if x = car (l) then 1 + occurrences (x, cdr (l))
        else occurrences (x, cdr (l)) endif
    else 0 endif

Definition:
subbagp (x, y)
= if listp (x)
    then if car (x) ∈ y then subbagp (cdr (x), delete (car (x), y))
        else f endif
    else t endif

Theorem: listp-delete
listp (delete (x, l))
= if listp (l) then (x ≠ car (l)) ∨ listp (cdr (l))
    else f endif

Event: Disable listp-delete.
Theorem: delete-non-member
\((x \not\in y) \rightarrow (\text{delete}(x, y) = y)\)

Theorem: delete-delete
\(\text{delete}(y, \text{delete}(x, z)) = \text{delete}(x, \text{delete}(y, z))\)

Theorem: equal-occurrences-zero
\((\text{occurrences}(x, l) = 0) = (x \not\in l)\)

Theorem: member-non-list
\((\neg \text{listp}(l)) \rightarrow (x \not\in l)\)

Theorem: member-delete
\((x \in \text{delete}(y, l))\)
\(=\)
\(\begin{align*}
    &\text{if } x \in l \\
    &\text{then if } x = y \text{ then } 1 < \text{occurrences}(x, l) \\
    &\text{else } t \text{ endif} \\
    &\text{else } f \text{ endif}
\end{align*}\)

Theorem: member-delete-implies-membership
\((x \in \text{delete}(y, l)) \rightarrow (x \in l)\)

Theorem: occurrences-delete
\(\text{occurrences}(x, \text{delete}(y, l))\)
\(=\)
\(\begin{align*}
    &\text{if } x = y \\
    &\text{then if } x \in l \text{ then } \text{occurrences}(x, l) - 1 \\
    &\text{else } 0 \text{ endif} \\
    &\text{else } \text{occurrences}(x, l) \text{ endif}
\end{align*}\)

Theorem: member-bagdiff
\((x \in \text{bagdiff}(a, b)) = (\text{occurrences}(x, b) < \text{occurrences}(x, a))\)

Theorem: bagdiff-delete
\(\text{bagdiff}(\text{delete}(e, x), y) = \text{delete}(e, \text{bagdiff}(x, y))\)

Theorem: subbagp-delete
\(\text{subbagp}(x, \text{delete}(u, y)) \rightarrow \text{subbagp}(x, y)\)

Theorem: subbagp-cdr1
\(\text{subbagp}(x, y) \rightarrow \text{subbagp}(\text{cdr}(x), y)\)

Theorem: subbagp-cdr2
\(\text{subbagp}(x, \text{cdr}(y)) \rightarrow \text{subbagp}(x, y)\)

Theorem: subbagp-bagint1
\(\text{subbagp}(\text{bagint}(x, y), x)\)
THEOREM: subbagp-bagint2
\[ \text{subbagp} (\text{bagint} (x, y), y) \]

THEOREM: occurrences-bagint
\[ \text{occurrences} (x, \text{bagint} (a, b)) = \begin{cases} \text{occurrences} (x, a) & \text{if } \text{occurrences} (x, a) < \text{occurrences} (x, b) \\ \text{occurrences} (x, b) & \text{else} \end{cases} \]

THEOREM: occurrences-bagdiff
\[ \text{occurrences} (x, \text{bagdiff} (a, b)) = (\text{occurrences} (x, a) - \text{occurrences} (x, b)) \]

THEOREM: member-bagint
\[ x \in \text{bagint} (a, b) = ((x \in a) \land (x \in b)) \]

EVENT: Let us define the theory bags to consist of the following events: occurrences-bagint, bagdiff-delete, occurrences-bagdiff, member-bagint, member-bagdiff, subbagp-bagint2, subbagp-bagint1, subbagp-cdr2, subbagp-cdr1, subbagp-delete.

EVENT: Make the library "bags".
Index

bagdiff, 1–3
bagdiff-delete, 2
bagint, 1–3
bags, 3

delete, 1, 2
delete-delete, 2
delete-non-member, 2

equal-occurrences-zero, 2

listp-delete, 1

member-bagdiff, 2
member-bagint, 3
member-delete, 2
member-delete-implies-membership, 2
member-non-list, 2

occurrences, 1–3
occurrences-bagdiff, 3
occurrences-bagint, 3
occurrences-delete, 2

subbagp, 1–3
subbagp-bagint1, 2
subbagp-bagint2, 3
subbagp-cdr1, 2
subbagp-cdr2, 2
subbagp-delete, 2