

EVENT: Start with the library "interpreter".

```
;;;this files defines a token passing solution to the n processor  
;;;mutual exclusion problem.
```

DEFINITION:

```
status (state, index) = cdr (assoc (cons ('me, index), state))
```

DEFINITION:

```
wait (state, index) = (status (state, index) = 'wait)
```

DEFINITION:

```
non-critical (state, index) = (status (state, index) = 'non-critical)
```

DEFINITION:

```
critical (state, index)  
= ((¬ non-critical (state, index)) ∧ (¬ wait (state, index)))
```

DEFINITION: ticks (state, index) = fix (status (state, index))

DEFINITION:

```
critical-ticks (state, index, ticks)  
= (critical (state, index) ∧ (ticks (state, index) = fix (ticks)))
```

DEFINITION:

```
channel (state, index) = cdr (assoc (cons ('c, index), state))
```

DEFINITION: token (state, index) = listp (channel (state, index))

DEFINITION:

```
me (old, new, index, size)  
= if non-critical (old, index)  
  then if non-critical (new, index)  
    then if token (old, index)  
      then if add1-mod (size, index) = index  
        then (length (channel (new, index))  
              = length (channel (old, index)))  
              ∧ changed (old, new, list (cons ('c, index)))  
        else (channel (new, index)  
              = cdr (channel (old, index)))  
              ∧ (length (channel (new,  
                                add1-mod (size, index)))  
                  = (1 + length (channel (old,  
                                add1-mod (size,
```

```

index))))))

    ∧ changed (old,
        new,
        list (cons ('c, index),
            cons ('c,
                add1-mod (size,
                    index)))) endif

    else changed (old, new, nil) endif
else wait (new, index)
    ∧ changed (old, new, list (cons ('me, index))) endif
elseif wait (old, index)
then if token (old, index)
    then (channel (new, index) = cdr (channel (old, index)))
        ∧ critical (new, index)
        ∧ changed (old,
            new,
            list (cons ('me, index), cons ('c, index)))
    else changed (old, new, nil) endif
else ((ticks (new, index) < ticks (old, index))
    ∧ critical (new, index)
    ∧ changed (old, new, list (cons ('me, index))))
    ∨ (non-critical (new, index)
        ∧ (length (channel (new, add1-mod (size, index)))
            = (1 + length (channel (old,
                add1-mod (size,
                    index)))))

    ∧ changed (old,
        new,
        list (cons ('c, add1-mod (size, index)),
            cons ('me, index)))) endif

```

DEFINITION:

```

me-function (index, size, state)
= if non-critical (state, index)
    then if token (state, index)
        then update-assoc (cons ('c, add1-mod (size, index)),
            cons ('token,
                channel (update-assoc (cons ('c, index),
                    cdr (channel (state,
                        index)), state),
                    add1-mod (size, index))),
                update-assoc (cons ('c, index),
                    cdr (channel (state, index))),
```

```

state))
else state endif
elseif wait (state, index)
then if token (state, index)
    then update-assoc (cons ('c, index),
                           cdr (channel (state, index)),
                           update-assoc (cons ('me, index), 0, state)))
else state endif
elseif ticks (state, index)  $\simeq$  0
then update-assoc (cons ('c, add1-mod (size, index)),
                     cons ('token, channel (state, add1-mod (size, index))),
                     update-assoc (cons ('me, index),
                                   'non-critical,
                                   state)))
else update-assoc (cons ('me, index), ticks (state, index) - 1, state) endif

```

THEOREM: total-me

$$((index \in \mathbf{N}) \wedge (index < size)) \\ \rightarrow \text{me}(\text{old}, \text{me-function}(index, size, \text{old}), index, size)$$

DEFINITION:

$$\text{program}(index, size) \\ = \text{if } index \simeq 0 \text{ then nil} \\ \text{else cons}(\text{list}('me, index - 1, size), \text{program}(index - 1, size)) \text{endif}$$

DEFINITION: me-prg (size) = program (size, size)

THEOREM: member-program

$$(statement \in \text{program}(index, size)) \\ = \text{if } index \simeq 0 \text{ then f} \\ \text{else (car(statement)} = 'me) \\ \wedge (\text{cadr(statement)} \in \mathbf{N}) \\ \wedge (\text{cadr(statement)} < index) \\ \wedge (\text{caddr(statement)} = size) \\ \wedge (\text{cdaddr(statement)} = \text{nil}) \text{endif}$$

THEOREM: member-me-prg

$$(statement \in \text{me-prg}(size)) \\ = \text{if } size \simeq 0 \text{ then f} \\ \text{else (car(statement)} = 'me) \\ \wedge (\text{cadr(statement)} \in \mathbf{N}) \\ \wedge (\text{cadr(statement)} < size) \\ \wedge (\text{caddr(statement)} = size) \\ \wedge (\text{cdaddr(statement)} = \text{nil}) \text{endif}$$

EVENT: Disable me-prg.

THEOREM: total-prg  
 $\text{total}(\text{me-prg}(size))$

DEFINITION:

$\text{weight-of-triple}(state, index, size)$   
 $= \text{if critical}(state, index) \text{ then } 1$   
 $\quad \text{else } 0 \text{ endif}$   
 $\quad + \text{length}(\text{channel}(state, index))$   
 $\quad + \text{length}(\text{channel}(state, \text{add1-mod}(size, index))))$

THEOREM: weight-of-triple-preserved

$((index \in \mathbf{N})$   
 $\wedge (1 < size)$   
 $\wedge (index < size)$   
 $\wedge \text{me}(old, new, index, size))$   
 $\rightarrow (\text{weight-of-triple}(new, index, size)$   
 $\quad = \text{weight-of-triple}(old, index, size))$

DEFINITION:

$\text{weight-but-triple}(state, index, size, n)$   
 $= \text{if } n \simeq 0 \text{ then } 0$   
 $\quad \text{elseif } (n - 1) = index$   
 $\quad \text{then weight-but-triple}(state, index, size, n - 1)$   
 $\quad \text{else if critical}(state, n - 1) \text{ then } 1$   
 $\quad \text{else } 0 \text{ endif}$   
 $\quad + \text{if } (n - 1) = \text{add1-mod}(size, index) \text{ then } 0$   
 $\quad \quad \text{else length}(\text{channel}(state, n - 1)) \text{ endif}$   
 $\quad + \text{weight-but-triple}(state, index, size, n - 1) \text{ endif}$

THEOREM: weight-but-triple-preserved

$((index \in \mathbf{N}) \wedge (index < size) \wedge \text{me}(old, new, index, size))$   
 $\rightarrow (\text{weight-but-triple}(new, index, size, n)$   
 $\quad = \text{weight-but-triple}(old, index, size, n))$

DEFINITION:

$\text{weight}(state, size)$   
 $= \text{if } size \simeq 0 \text{ then } 0$   
 $\quad \text{else if critical}(state, size - 1) \text{ then } 1$   
 $\quad \text{else } 0 \text{ endif}$   
 $\quad + \text{length}(\text{channel}(state, size - 1))$   
 $\quad + \text{weight}(state, size - 1) \text{ endif}$

THEOREM: weight-is-sum

$((index \in \mathbf{N}) \wedge (index < size) \wedge (1 < size))$   
 $\rightarrow (\text{weight}(state, n))$

```

=  (if index < n
    then if critical(state, index) then 1
        else 0 endif
        + length(channel(state, index))
    else 0 endif
    + if add1-mod(size, index) < n
        then length(channel(state, add1-mod(size, index)))
        else 0 endif
    + weight-but-triple(state, index, size, n)))

```

THEOREM: weight-preserved-1

$$\begin{aligned}
& ((\text{index} \in \mathbf{N}) \\
& \wedge (\text{index} < \text{size}) \\
& \wedge (1 < \text{size}) \\
& \wedge \text{me}(\text{old}, \text{new}, \text{index}, \text{size})) \\
\rightarrow & (\text{weight}(\text{new}, \text{size}) = \text{weight}(\text{old}, \text{size}))
\end{aligned}$$

THEOREM: weight-preserved-2

$$\text{me}(\text{old}, \text{new}, 0, 1) \rightarrow (\text{weight}(\text{new}, 1) = \text{weight}(\text{old}, 1))$$

THEOREM: about-size-index

$$\begin{aligned}
& ((\text{index} \in \mathbf{N}) \wedge (\text{index} < \text{size})) \\
\rightarrow & (((\text{index} = 0) \wedge (\text{size} = 1)) \vee (1 < \text{size}))
\end{aligned}$$

THEOREM: weight-preserved

$$\begin{aligned}
& ((\text{index} \in \mathbf{N}) \wedge (\text{index} < \text{size}) \wedge \text{me}(\text{old}, \text{new}, \text{index}, \text{size})) \\
\rightarrow & (\text{weight}(\text{new}, \text{size}) = \text{weight}(\text{old}, \text{size}))
\end{aligned}$$

EVENT: Disable weight-is-sum.

EVENT: Disable weight-preserved-1.

EVENT: Disable weight-preserved-2.

EVENT: Disable weight-of-triple-preserved.

EVENT: Disable weight-but-triple-preserved.

EVENT: Disable total-me.

DEFINITION:

$$\text{mutual-exclusionp}(\text{state}, \text{size}) = (\text{weight}(\text{state}, \text{size}) = 1)$$

THEOREM: mutual-exclusionp-unless-sufficient  
 unless-sufficient (*statement*,  
     me-prg (*size*),  
     *old*,  
     *new*,  
     ‘(mutual-exclusionp state ’,size),  
     ’(false))

EVENT: Disable weight-preserved.

THEOREM: mutual-exclusionp-prg  
 unless (‘(mutual-exclusionp state ’,size), ’(false), me-prg (*size*))

THEOREM: wait-unless-critical-1  
 ((*index* ∈ **N**)  
   ∧ (*index* < *size*)  
   ∧ me (*old*, *new*, *index*, *size*)  
   ∧ wait (*old*, *index*))  
   → (wait (*new*, *index*) ∨ critical (*new*, *index*))

THEOREM: wait-unless-critical-2  
 ((*index* ∈ **N)  
   ∧ (*index* < *size*)  
   ∧ (*index* ∈ **N**)  
   ∧ (*index* < *size*)  
   ∧ (*index* ≠ *index*)  
   ∧ me (*old*, *new*, *index*, *size*)  
   ∧ wait (*old*, *index*))  
   → (wait (*new*, *index*) ∨ critical (*new*, *index*))**

THEOREM: wait-unless-critical-unless-sufficient  
 ((*index* ∈ **N**) ∧ (*index* < *size*))  
   → unless-sufficient (*statement*,  
     me-prg (*size*),  
     *old*,  
     *new*,  
     ‘(wait state ’,index),  
     ‘(critical state ’,index))

THEOREM: wait-unless-critical  
 ((*index* ∈ **N**) ∧ (*index* < *size*) ∧ (*size* ≠ 0))  
   → unless (‘(wait state ’,index),  
     ‘(critical state ’,index),  
     me-prg (*size*))

THEOREM: celcor-ensures-key

$$((index \in \mathbf{N}) \wedge (index < size)) \rightarrow \text{ensures-key}(\text{list}('me, index, size),$$

$$\text{me-prg}(size),$$

$$old,$$

$$new,$$

$$'(\text{and}$$

$$(\text{critical state } ', index)$$

$$(\text{lessp}$$

$$(\text{ticks state } ', index)$$
 $'(\text{or}$ 

$$(\text{and}$$

$$(\text{critical state } ', index)$$

$$(\text{lessp} (\text{ticks state } ', index) ', ticks))$$

$$(\text{token state } ', (add1-mod size index))))$$

THEOREM: celcor-ensures-rest

$$((index \in \mathbf{N}) \wedge (index < size)) \rightarrow \text{ensures-rest}(\text{statement},$$

$$\text{list}('me, index, size),$$

$$\text{me-prg}(size),$$

$$old,$$

$$new,$$

$$'(\text{and}$$

$$(\text{critical state } ', index)$$

$$(\text{lessp}$$

$$(\text{ticks state } ', index)$$
 $q)$ 

THEOREM: critical-ensures-less-critical-or-right

$$((index \in \mathbf{N}) \wedge (index < size)) \rightarrow \text{ensures}('(\text{and}$$

$$(\text{critical state } ', index)$$

$$(\text{lessp} (\text{ticks state } ', index) ', (add1 ticks))),$$

$$'(\text{or}$$

$$(\text{and}$$

$$(\text{critical state } ', index)$$

$$(\text{lessp} (\text{ticks state } ', index) ', ticks))$$

$$(\text{token state } ', (add1-mod size index))),$$

$$\text{me-prg}(size))$$

THEOREM: critical-unless-less-critical-or-right

$$((index \in \mathbf{N}) \wedge (index < size))$$

```

→ unless('(
  (and
    (critical state ',index)
    (lessp (ticks state ',index) ',(add1 ticks))),
  '(
    (or
      (and
        (critical state ',index)
        (lessp (ticks state ',index) ',ticks))
        (token state ',(add1-mod size index)),
      me-prg(size)))

```

THEOREM: wlcec-ensures-key  
 $((index \in \mathbf{N}) \wedge (index < size))$   
→ ensures-key(list('me, index, size),  
 me-prg(size),  
 old,  
 new,  
 '(
 (and
 (wait state ',index)
 (token state ',index)),
 '(critical state ',index))

THEOREM: wlcec-ensures-rest  
 $((index \in \mathbf{N}) \wedge (index < size))$   
→ ensures-rest(statement,  
 list('me, index, size),  
 me-prg(size),  
 old,  
 new,  
 '(
 (and
 (wait state ',index)
 (token state ',index)),
 q)

THEOREM: wait-and-left-channel-ensures-critical  
 $((index \in \mathbf{N}) \wedge (index < size))$   
→ ensures('(
 (and
 (wait state ',index)
 (token state ',index)),
 '(critical state ',index),
 me-prg(size)))

THEOREM: wait-and-left-channel-unless-critical  
 $((index \in \mathbf{N}) \wedge (index < size))$   
→ unless('(
 (and
 (wait state ',index)

```

(token state ',index)),
‘(critical state ',index),
me-prg(size))

```

THEOREM: nclcewlcorc-ensures-key

```

((index ∈ N) ∧ (index < size))
→ ensures-key(list('me, index, size),
               me-prg(size),
               old,
               new,
               ‘(and
                  (non-critical state ',index)
                  (token state ',index)),
               ‘(or
                  (and
                     (wait state ',index)
                     (token state ',index))
                     (token state ',(add1-mod size index))))

```

THEOREM: nclcewlcorc-ensures-rest

```

((index ∈ N) ∧ (index < size))
→ ensures-rest(statement,
               list('me, index, size),
               me-prg(size),
               old,
               new,
               ‘(and
                  (non-critical state ',index)
                  (token state ',index)),
               q)

```

THEOREM: non-critical-left-ensures-wait-left-or-right

```

((index ∈ N) ∧ (index < size))
→ ensures(‘(and
               (non-critical state ',index)
               (token state ',index)),
               ‘(or
                  (and
                     (wait state ',index)
                     (token state ',index))
                     (token state ',(add1-mod size index))),
               me-prg(size)))

```

THEOREM: non-critical-left-unless-wait-left-or-right

```

((index ∈ N) ∧ (index < size))

```

```

→ unless('(
  (and
    (non-critical state ',index)
    (token state ',index)),
  '(
    (or
      (and
        (wait state ',index)
        (token state ',index))
      (token state ',(add1-mod size index))),
    me-prg(size)))

```

THEOREM: listp-me-prg  
 $(size \not\leq 0) \rightarrow \text{listp(me-prg}(size))$

THEOREM: mutual-exclusionp-is-invariant  
 $(\text{initial-condition}('(\text{mutual-exclusionp state ',size}), \text{me-prg}(size))$   
 $\wedge (size \not\leq 0))$   
 $\rightarrow \text{invariant}('(\text{mutual-exclusionp state ',size}), \text{me-prg}(size))$

THEOREM: critical-leads-to-less-critical-or-right  
 $((index \in \mathbf{N}) \wedge (index < size))$   
 $\rightarrow \text{leads-to}('(
 (and
 (critical state ',index)
 (lessp
 (ticks state ',index)
 ',(add1 ticks))),$   
 $'(
 (or
 (and
 (critical state ',index)
 (lessp
 (ticks state ',index) ',ticks))
 (token state ',(add1-mod size index))),
 me-prg(size)))$

THEOREM: critical-ticks-leads-to-right  
 $((index \in \mathbf{N}) \wedge (index < size))$   
 $\rightarrow \text{leads-to}('(
 (and
 (critical state ',index)
 (lessp
 (ticks state ',index)
 ',(add1 ticks))),$   
 $'(
 (token state ',(add1-mod size index)),
 me-prg(size)))$

THEOREM: critical-leads-to-right  
 $((index \in \mathbf{N}) \wedge (index < size))$   
 $\rightarrow \text{leads-to}('(\text{critical state ',index}),$

```

‘(token state ’,(add1-mod size index)),
me-prg (size))

```

THEOREM: wait-left-leads-to-critical

```

((index ∈ N) ∧ (index < size))
→ leads-to (‘(and
            (wait state ’,index)
            (token state ’,index)),
            ‘(critical state ’,index),
            me-prg (size)))

```

THEOREM: non-critical-left-leads-to-wait-left-or-right

```

((index ∈ N) ∧ (index < size))
→ leads-to (‘(and
            (non-critical state ’,index)
            (token state ’,index)),
            ‘(or
                (and
                    (wait state ’,index)
                    (token state ’,index))
                    (token state ’,(add1-mod size index))),
                me-prg (size)))

```

THEOREM: wait-left-leads-to-right

```

((index ∈ N) ∧ (index < size))
→ leads-to (‘(and
            (wait state ’,index)
            (token state ’,index)),
            ‘(token state ’,(add1-mod size index)),
            me-prg (size)))

```

THEOREM: non-critical-left-leads-to-right

```

((index ∈ N) ∧ (index < size))
→ leads-to (‘(and
            (non-critical state ’,index)
            (token state ’,index)),
            ‘(token state ’,(add1-mod size index)),
            me-prg (size)))

```

THEOREM: critical-wait-non-critical

```

critical (state, index) ∨ wait (state, index) ∨ non-critical (state, index)

```

DEFINITION:

```

tokens (state, size)
= if size ≈ 0 then nil

```

```

elseif token(state, size - 1)
then cons(size - 1, tokens(state, size - 1))
else tokens(state, size - 1) endif

```

DEFINITION:

```

criticals(state, size)
= if size  $\simeq 0$  then nil
   elseif critical(state, size - 1)
   then cons(size - 1, criticals(state, size - 1))
   else criticals(state, size - 1) endif

```

THEOREM: equal-plus-1

```

((a + b) = 1)
= (((a = 1)  $\wedge$  (b  $\simeq 0$ ))  $\vee$  ((b = 1)  $\wedge$  (a  $\simeq 0$ )))

```

THEOREM: about-mutual-exclusionp

```

if weight(state, size)  $\simeq 0$ 
then (tokens(state, size) = nil)  $\wedge$  (criticals(state, size) = nil)
elseif weight(state, size) = 1
then ((length(tokens(state, size)) = 1)
       $\wedge$  (criticals(state, size) = nil))
       $\vee$  ((length(criticals(state, size)) = 1)
       $\wedge$  (tokens(state, size) = nil))
else t endif

```

EVENT: Disable equal-plus-1.

THEOREM: numberp-car-tokens  
 $\text{car}(\text{tokens}(\text{state}, \text{size})) \in \mathbb{N}$

THEOREM: numberp-car-criticals  
 $\text{car}(\text{criticals}(\text{state}, \text{size})) \in \mathbb{N}$

THEOREM: lessp-car-tokens  
 $(\text{car}(\text{tokens}(\text{state}, \text{size})) < \text{size}) = (\text{size} \not\simeq 0)$

THEOREM: lessp-car-criticals  
 $(\text{car}(\text{criticals}(\text{state}, \text{size})) < \text{size}) = (\text{size} \not\simeq 0)$

THEOREM: mutual-exclusionp-requires-state  
 $\text{mutual-exclusionp}(\text{state}, \text{size}) \rightarrow (\text{size} \not\simeq 0)$

THEOREM: lessp-car-tokens-and-criticals  
 $\text{mutual-exclusionp}(\text{state}, \text{size})$   
 $\rightarrow ((\text{car}(\text{tokens}(\text{state}, \text{size})) < \text{size})$   
 $\wedge (\text{car}(\text{criticals}(\text{state}, \text{size})) < \text{size}))$

THEOREM: about-member-list-length-one  
 $(\text{length}(\text{list}) = 1) \rightarrow ((x \in \text{list}) = (x = \text{car}(\text{list})))$

THEOREM: about-member-car-tokens-criticals  
 $\text{mutual-exclusionp}(\text{state}, \text{size})$   
 $\rightarrow (((\text{index} \in \text{tokens}(\text{state}, \text{size}))$   
 $= (\text{listp}(\text{tokens}(\text{state}, \text{size}))$   
 $\wedge (\text{index} = \text{car}(\text{tokens}(\text{state}, \text{size}))))))$   
 $\wedge ((\text{index} \in \text{criticals}(\text{state}, \text{size}))$   
 $= (\text{listp}(\text{criticals}(\text{state}, \text{size}))$   
 $\wedge (\text{index} = \text{car}(\text{criticals}(\text{state}, \text{size}))))))$

THEOREM: not-member-tokens  
 $(\text{index} \not\in \text{size}) \rightarrow (\text{index} \not\in \text{tokens}(\text{state}, \text{size}))$

THEOREM: not-member-criticals  
 $(\text{index} \not\in \text{size}) \rightarrow (\text{index} \not\in \text{criticals}(\text{state}, \text{size}))$

THEOREM: token-equals  
 $((\text{index} \in \mathbf{N}) \wedge (\text{index} < \text{size}))$   
 $\rightarrow (\text{token}(\text{state}, \text{index}) = (\text{index} \in \text{tokens}(\text{state}, \text{size})))$

THEOREM: critical-equals  
 $((\text{index} \in \mathbf{N}) \wedge (\text{index} < \text{size}))$   
 $\rightarrow (\text{critical}(\text{state}, \text{index}) = (\text{index} \in \text{criticals}(\text{state}, \text{size})))$

THEOREM: mtoken-equals  
 $(\text{mutual-exclusionp}(\text{state}, \text{size}) \wedge (\text{index} \in \mathbf{N}) \wedge (\text{index} < \text{size}))$   
 $\rightarrow (\text{token}(\text{state}, \text{index})$   
 $= (\text{listp}(\text{tokens}(\text{state}, \text{size}))$   
 $\wedge (\text{index} = \text{car}(\text{tokens}(\text{state}, \text{size}))))))$

THEOREM: mcritical-equals  
 $(\text{mutual-exclusionp}(\text{state}, \text{size}) \wedge (\text{index} \in \mathbf{N}) \wedge (\text{index} < \text{size}))$   
 $\rightarrow (\text{critical}(\text{state}, \text{index})$   
 $= (\text{listp}(\text{criticals}(\text{state}, \text{size}))$   
 $\wedge (\text{index} = \text{car}(\text{criticals}(\text{state}, \text{size}))))))$

THEOREM: mutual-exclusionp-implies  
 $\text{mutual-exclusionp}(\text{state}, \text{size})$   
 $\rightarrow (\text{listp}(\text{tokens}(\text{state}, \text{size})) \leftrightarrow (\neg \text{listp}(\text{criticals}(\text{state}, \text{size}))))$

EVENT: Disable lessp-car-criticals.

EVENT: Disable lessp-car-tokens.

EVENT: Disable about-member-list-length-one.

EVENT: Disable not-member-tokens.

EVENT: Disable not-member-criticals.

EVENT: Disable token.

EVENT: Disable critical.

EVENT: Disable non-critical.

EVENT: Disable wait.

EVENT: Disable mutual-exclusionp.

THEOREM: mutual-exclusionp-implies-prg  
mutual-exclusionp (*state*, *size*) → listp (me-prg (*size*))

THEOREM: left-leads-to-right

((*index* ∈ **N**)  
  ^  (*index* < *size*)  
  ^  initial-condition(' (mutual-exclusionp state ', *size*),  
                      me-prg (*size*)))  
→ leads-to(' (token state ', *index*),  
              ' (token state ', (add1-mod *size* *index*)),  
              me-prg (*size*))

DEFINITION:

walk-ring (*a*, *b*, *size*)  
=   **if** *a* ∈ **N  
    **then if** *b* ∈ **N  
        **then if** *a* < *size*  
          **then if** *b* < *size*  
            **then if** *a* = *b* **then t**  
              **else** walk-ring (add1-mod (*size*, *a*), *b*, *size*) **endif**  
            **else t endif**  
        **else t endif**  
    **else t endif**  
  **else t endif******

THEOREM: any-left-leads-to-right

$$\begin{aligned} & ((left \in \mathbf{N}) \\ & \quad \wedge (left < size) \\ & \quad \wedge (right \in \mathbf{N}) \\ & \quad \wedge (right < size) \\ & \quad \wedge \text{initial-condition}('(\text{mutual-exclusionp state } ', size), \\ & \quad \quad \text{me-prg}(size))) \\ \rightarrow & \text{leads-to}('(\text{token state } ', left), \\ & \quad '(\text{token state } ', right), \\ & \quad \text{me-prg}(size)) \end{aligned}$$

THEOREM: lessp-add1-mod-size

$$\text{mutual-exclusionp}(state, size) \rightarrow (\text{add1-mod}(size, n) < size)$$

THEOREM: any-critical-leads-to-right

$$\begin{aligned} & ((left \in \mathbf{N}) \\ & \quad \wedge (left < size) \\ & \quad \wedge (right \in \mathbf{N}) \\ & \quad \wedge (right < size) \\ & \quad \wedge \text{initial-condition}('(\text{mutual-exclusionp state } ', size), \\ & \quad \quad \text{me-prg}(size))) \\ \rightarrow & \text{leads-to}('(\text{critical state } ', left), \\ & \quad '(\text{token state } ', right), \\ & \quad \text{me-prg}(size)) \end{aligned}$$

THEOREM: any-leads-to-right

$$\begin{aligned} & ((index \in \mathbf{N}) \\ & \quad \wedge (index < size) \\ & \quad \wedge \text{initial-condition}('(\text{mutual-exclusionp state } ', size), \\ & \quad \quad \text{me-prg}(size))) \\ \rightarrow & \text{leads-to}('(\text{true}), '(\text{token state } ', index), \text{me-prg}(size)) \end{aligned}$$

THEOREM: me-prg-non-zero-size

$$\text{listp}(\text{me-prg}(size)) = (size \neq 0)$$

THEOREM: wait-leads-to-left-wait-or-critical

$$\begin{aligned} & ((index \in \mathbf{N}) \\ & \quad \wedge (index < size) \\ & \quad \wedge \text{initial-condition}('(\text{mutual-exclusionp state } ', size), \\ & \quad \quad \text{me-prg}(size))) \\ \rightarrow & \text{leads-to}('(\text{and} (\text{true}) (\text{wait state } ', index)), \\ & \quad '(\text{or} \\ & \quad \quad (\text{and} \\ & \quad \quad \quad (\text{token state } ', index) \\ & \quad \quad \quad (\text{wait state } ', index))) \end{aligned}$$

```
(critical state ',index)),  
me-prg(size))
```

THEOREM: wait-leads-to-critical  
(( $\text{index} \in \mathbf{N}$ )  
   $\wedge$  ( $\text{index} < \text{size}$ )  
   $\wedge$  initial-condition('mutual-exclusionp state ', $\text{size}$ ),  
    me-prg( $\text{size}$ )))  
→ leads-to('wait state ', $\text{index}$ ),  
    ('critical state ', $\text{index}$ ),  
    me-prg( $\text{size}$ ))

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