

EVENT: Start with the initial **nqthm** theory.

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;; Sets; Matt Kaufmann, Dec. 1989, revised March 1990.  The first few events
;; are some basic events about lists.  I'll take the approach that all these
;; basic functions will be disabled once enough algebraic properties have
;; been proved.
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;; Theories:
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;; (deftheory set-defns
;;   (length properp fix-properp member append subsetp delete
;;     disjoint intersection set-diff setp))
```

DEFINITION:

```
length(x)
=  if listp(x) then 1 + length(cdr(x))
    else 0 endif
```

THEOREM: length-nlistp
 $(x \simeq \mathbf{nil}) \rightarrow (\text{length}(x) = 0)$

THEOREM: length-cons
 $\text{length}(\text{cons}(a, x)) = (1 + \text{length}(x))$

THEOREM: length-append
 $\text{length}(\text{append}(x, y)) = (\text{length}(x) + \text{length}(y))$

EVENT: Disable length.

THEOREM: append-assoc
 $\text{append}(\text{append}(x, y), z) = \text{append}(x, \text{append}(y, z))$

THEOREM: member-cons
 $(a \in \text{cons}(x, l)) = ((a = x) \vee (a \in l))$

THEOREM: member-nlistp
 $(l \simeq \mathbf{nil}) \rightarrow (a \notin l)$

EVENT: Disable member.

DEFINITION:

```
subsetp(x, y)
=  if x  $\simeq$  nil then t
    else (car(x)  $\in$  y)  $\wedge$  subsetp(cdr(x), y) endif
```

DEFINITION:

$\text{subsetp-wit}(x, y)$
= **if** $x \simeq \text{nil}$ **then** **t**
 elseif $\text{car}(x) \in y$ **then** $\text{subsetp-wit}(\text{cdr}(x), y)$
 else $\text{car}(x)$ **endif**

THEOREM: subsetp-wit-witnesses

$\text{subsetp}(x, y)$
= $\neg ((\text{subsetp-wit}(x, y) \in x) \wedge (\text{subsetp-wit}(x, y) \notin y))$

THEOREM: subsetp-wit-witnesses-general-1

$((\text{subsetp-wit}(x, y) \notin x) \wedge (a \in x)) \rightarrow (a \in y)$

THEOREM: subsetp-wit-witnesses-general-2

$((\text{subsetp-wit}(x, y) \in y) \wedge (a \in x)) \rightarrow (a \in y)$

EVENT: Disable subsetp-wit-witnesses.

EVENT: Disable subsetp-wit-witnesses-general-1.

EVENT: Disable subsetp-wit-witnesses-general-2.

THEOREM: subsetp-cons-1

$\text{subsetp}(\text{cons}(a, x), y) = ((a \in y) \wedge \text{subsetp}(x, y))$

THEOREM: subsetp-cons-2

$\text{subsetp}(l, m) \rightarrow \text{subsetp}(l, \text{cons}(a, m))$

THEOREM: subsetp-reflexivity

$\text{subsetp}(x, x)$

THEOREM: cdr-subsetp

$\text{subsetp}(\text{cdr}(x), x)$

THEOREM: member-subsetp

$((x \in y) \wedge \text{subsetp}(y, z)) \rightarrow (x \in z)$

THEOREM: subsetp-is-transitive

$(\text{subsetp}(x, y) \wedge \text{subsetp}(y, z)) \rightarrow \text{subsetp}(x, z)$

THEOREM: member-append

$(a \in \text{append}(x, y)) = ((a \in x) \vee (a \in y))$

THEOREM: subsetp-append

$\text{subsetp}(\text{append}(x, y), z) = (\text{subsetp}(x, z) \wedge \text{subsetp}(y, z))$

THEOREM: subsetp-of-append-sufficiency
 $(\text{subsetp}(a, b) \vee \text{subsetp}(a, c)) \rightarrow \text{subsetp}(a, \text{append}(b, c))$

THEOREM: subsetp-nlistp
 $(x \simeq \mathbf{nil}) \rightarrow (\text{subsetp}(x, y) \wedge (\text{subsetp}(y, x) = (y \simeq \mathbf{nil})))$

THEOREM: subsetp-cons-not-member
 $(z \notin x) \rightarrow (\text{subsetp}(x, \text{cons}(z, v)) = \text{subsetp}(x, v))$

EVENT: Disable subsetp.

;;;;; Other set-theoretic and list-theoretic definitions, and properp observations.

DEFINITION:
 $\text{properp}(x)$
 $=$ **if** listp(x) **then** properp(cdr(x))
 else $x = \mathbf{nil}$ **endif**

DEFINITION:
 $\text{fix-properp}(x)$
 $=$ **if** listp(x) **then** cons(car(x), fix-properp(cdr(x)))
 else $\mathbf{nil}$ **endif**

THEOREM: properp-fix-properp
 $\text{properp}(\text{fix-properp}(x))$

THEOREM: fix-properp-properp
 $\text{properp}(x) \rightarrow (\text{fix-properp}(x) = x)$

THEOREM: properp-cons
 $\text{properp}(\text{cons}(x, y)) = \text{properp}(y)$

THEOREM: properp-nlistp
 $(x \simeq \mathbf{nil}) \rightarrow (\text{properp}(x) = (x = \mathbf{nil}))$

THEOREM: fix-properp-cons
 $\text{fix-properp}(\text{cons}(x, y)) = \text{cons}(x, \text{fix-properp}(y))$

THEOREM: fix-properp-nlistp
 $(x \simeq \mathbf{nil}) \rightarrow (\text{fix-properp}(x) = \mathbf{nil})$

THEOREM: properp-append
 $\text{properp}(\text{append}(x, y)) = \text{properp}(y)$

THEOREM: fix-properp-append
 $\text{fix-properp}(\text{append}(x, y)) = \text{append}(x, \text{fix-properp}(y))$

THEOREM: append-nil
 $\text{append}(x, \mathbf{nil}) = \text{fix-properp}(x)$

DEFINITION:
 $\text{delete}(x, l)$
 $=$ **if** $\text{listp}(l)$
 then if $x = \text{car}(l)$ **then** $\text{cdr}(l)$
 else $\text{cons}(\text{car}(l), \text{delete}(x, \text{cdr}(l)))$ **endif**
 else l **endif**

THEOREM: properp-delete
 $\text{properp}(\text{delete}(x, l)) = \text{properp}(l)$

DEFINITION:
 $\text{disjoint}(x, y)$
 $=$ **if** $\text{listp}(x)$ **then** $(\text{car}(x) \notin y) \wedge \text{disjoint}(\text{cdr}(x), y)$
 else **t** **endif**

DEFINITION:
 $\text{disjoint-wit}(x, y)$
 $=$ **if** $\text{listp}(x)$
 then if $\text{car}(x) \in y$ **then** $\text{car}(x)$
 else $\text{disjoint-wit}(\text{cdr}(x), y)$ **endif**
 else **t** **endif**

THEOREM: disjoint-wit-witnesses
 $\text{disjoint}(x, y)$
 $=$ $(\neg ((\text{disjoint-wit}(x, y) \in x) \wedge (\text{disjoint-wit}(x, y) \in y)))$

EVENT: Disable disjoint-wit.

EVENT: Disable disjoint-wit-witnesses.

DEFINITION:
 $\text{intersection}(x, y)$
 $=$ **if** $\text{listp}(x)$
 then if $\text{car}(x) \in y$ **then** $\text{cons}(\text{car}(x), \text{intersection}(\text{cdr}(x), y))$
 else $\text{intersection}(\text{cdr}(x), y)$ **endif**
 else **nil** **endif**

THEOREM: properp-intersection
 $\text{properp}(\text{intersection}(x, y))$

DEFINITION:

$\text{set-diff}(x, y)$
= **if** $\text{listp}(x)$
 then if $\text{car}(x) \in y$ **then** $\text{set-diff}(\text{cdr}(x), y)$
 else $\text{cons}(\text{car}(x), \text{set-diff}(\text{cdr}(x), y))$ **endif**
 else nil endif

THEOREM: properp-set-diff
 $\text{properp}(\text{set-diff}(x, y))$

DEFINITION:

$\text{setp}(x)$
= **if** $\neg \text{listp}(x)$ **then** $x = \text{nil}$
 else $(\text{car}(x) \notin \text{cdr}(x)) \wedge \text{setp}(\text{cdr}(x))$ **endif**

THEOREM: setp-implies-properp
 $\text{setp}(x) \rightarrow \text{properp}(x)$

EVENT: Disable properp.

EVENT: Let us define the theory *set-defns* to consist of the following events:
length, properp, fix-properp, member, append, subsetp, delete, disjoint, intersection, set-diff, setp, properp.

:: Set theory lemmas

THEOREM: delete-cons
 $\text{delete}(a, \text{cons}(b, x))$
= **if** $a = b$ **then** x
 else $\text{cons}(b, \text{delete}(a, x))$ **endif**

THEOREM: delete-nlistp
 $(x \simeq \text{nil}) \rightarrow (\text{delete}(a, x) = x)$

THEOREM: listp-delete
 $\text{listp}(\text{delete}(x, l))$
= **if** $\text{listp}(l)$ **then** $(x \neq \text{car}(l)) \vee \text{listp}(\text{cdr}(l))$
 else f endif

THEOREM: delete-non-member
 $(x \notin y) \rightarrow (\text{delete}(x, y) = y)$

THEOREM: delete-delete
 $\text{delete}(y, \text{delete}(x, z)) = \text{delete}(x, \text{delete}(y, z))$

THEOREM: member-delete
 $\text{setp}(x) \rightarrow ((a \in \text{delete}(b, x)) = ((a \neq b) \wedge (a \in x)))$

THEOREM: setp-delete
 $\text{setp}(x) \rightarrow \text{setp}(\text{delete}(a, x))$

EVENT: Disable delete.

THEOREM: disjoint-cons-1
 $\text{disjoint}(\text{cons}(a, x), y) = ((a \notin y) \wedge \text{disjoint}(x, y))$

THEOREM: disjoint-cons-2
 $\text{disjoint}(x, \text{cons}(a, y)) = ((a \notin x) \wedge \text{disjoint}(x, y))$

THEOREM: disjoint-nlistp
 $((x \simeq \mathbf{nil}) \vee (y \simeq \mathbf{nil})) \rightarrow \text{disjoint}(x, y)$

THEOREM: disjoint-symmetry
 $\text{disjoint}(x, y) = \text{disjoint}(y, x)$

THEOREM: disjoint-append-right
 $\text{disjoint}(u, \text{append}(y, z)) = (\text{disjoint}(u, y) \wedge \text{disjoint}(u, z))$

THEOREM: disjoint-append-left
 $\text{disjoint}(\text{append}(y, z), u) = (\text{disjoint}(y, u) \wedge \text{disjoint}(z, u))$

THEOREM: disjoint-non-member
 $((a \in x) \wedge (a \in y)) \rightarrow (\neg \text{disjoint}(x, y))$

THEOREM: disjoint-subsetp-monotone-second
 $(\text{subsetp}(y, z) \wedge \text{disjoint}(x, z)) \rightarrow \text{disjoint}(x, y)$

THEOREM: subsetp-disjoint-2
 $(\text{subsetp}(x, y) \wedge \text{disjoint}(y, z)) \rightarrow \text{disjoint}(z, x)$

THEOREM: subsetp-disjoint-1
 $(\text{subsetp}(x, y) \wedge \text{disjoint}(y, z)) \rightarrow \text{disjoint}(x, z)$

THEOREM: subsetp-disjoint-3
 $(\text{subsetp}(x, y) \wedge \text{disjoint}(z, y)) \rightarrow \text{disjoint}(x, z)$

EVENT: Disable disjoint.

THEOREM: intersection-disjoint
 $(\text{intersection}(x, y) = \mathbf{nil}) = \text{disjoint}(x, y)$

THEOREM: intersection-nlistp

$$((x \simeq \mathbf{nil}) \vee (y \simeq \mathbf{nil})) \rightarrow (\text{intersection}(x, y) = \mathbf{nil})$$

THEOREM: member-intersection

$$(a \in \text{intersection}(x, y)) = ((a \in x) \wedge (a \in y))$$

THEOREM: subsetp-intersection

$$\text{subsetp}(x, \text{intersection}(y, z)) = (\text{subsetp}(x, y) \wedge \text{subsetp}(x, z))$$

THEOREM: intersection-symmetry

$$\text{subsetp}(\text{intersection}(x, y), \text{intersection}(y, x))$$

THEOREM: intersection-cons-1

$$\begin{aligned} & \text{intersection}(\text{cons}(a, x), y) \\ = & \quad \mathbf{if} \ a \in y \ \mathbf{then} \ \text{cons}(a, \text{intersection}(x, y)) \\ & \quad \mathbf{else} \ \text{intersection}(x, y) \ \mathbf{endif} \end{aligned}$$

THEOREM: intersection-cons-2

$$(a \notin y) \rightarrow (\text{intersection}(y, \text{cons}(a, x)) = \text{intersection}(y, x))$$

;; The following is needed because DISJOINT-INTERSECTION-COMMUTER,
;; added during polishing, caused the proof of
;; DISJOINT-DOMAIN-CO-RESTRICT (in "alists.events") to fail.

THEOREM: intersection-cons-3

$$\begin{aligned} & (w \in x) \\ \rightarrow & \quad (\text{subsetp}(\text{intersection}(y, \text{cons}(w, z)), x) \\ & \quad = \text{subsetp}(\text{intersection}(y, z), x)) \end{aligned}$$

THEOREM: intersection-cons-subsetp

$$\text{subsetp}(\text{intersection}(x, y), \text{intersection}(x, \text{cons}(a, y)))$$

THEOREM: subsetp-intersection-left-1

$$\text{subsetp}(\text{intersection}(x, y), x)$$

THEOREM: subsetp-intersection-left-2

$$\text{subsetp}(\text{intersection}(x, y), y)$$

THEOREM: subsetp-intersection-sufficiency-1

$$\text{subsetp}(y, z) \rightarrow \text{subsetp}(\text{intersection}(x, y), z)$$

THEOREM: subsetp-intersection-sufficiency-2

$$\text{subsetp}(y, z) \rightarrow \text{subsetp}(\text{intersection}(y, x), z)$$

THEOREM: intersection-associative

$$\text{intersection}(\text{intersection}(x, y), z) = \text{intersection}(x, \text{intersection}(y, z))$$

THEOREM: intersection-elimination

$$\text{subsetp}(x, y) \rightarrow (\text{intersection}(x, y) = \text{fix-properp}(x))$$

THEOREM: length-intersection

$$\text{length}(x) \not\prec \text{length}(\text{intersection}(x, y))$$

THEOREM: subsetp-intersection-member

$$\begin{aligned} & (\text{subsetp}(\text{intersection}(x, y), z) \wedge (a \notin z)) \\ \rightarrow & (((a \in x) \rightarrow (a \notin y)) \wedge ((a \in y) \rightarrow (a \notin x))) \end{aligned}$$

;; The following wasn't needed in the proof about generalization, but it's a nice rule.

THEOREM: intersection-append

$$\text{intersection}(\text{append}(x, y), z) = \text{append}(\text{intersection}(x, z), \text{intersection}(y, z))$$

;; I'd rather just prove that intersection distributes over append on
;; the right but that isn't true. Congruence relations would probably
;; help a lot with that problem. In the meantime, I content myself
;; with the following.

THEOREM: disjoint-intersection-append

$$\begin{aligned} & \text{disjoint}(x, \text{intersection}(y, \text{append}(z1, z2))) \\ = & (\text{disjoint}(x, \text{intersection}(y, z1)) \wedge \text{disjoint}(x, \text{intersection}(y, z2))) \end{aligned}$$

;; See comment just above DISJOINT-INTERSECTION-APPEND

THEOREM: subsetp-intersection-append

$$\begin{aligned} & \text{subsetp}(\text{intersection}(u, \text{append}(x, y)), z) \\ = & (\text{subsetp}(\text{intersection}(u, x), z) \wedge \text{subsetp}(\text{intersection}(u, y), z)) \end{aligned}$$

THEOREM: subsetp-intersection-elimination-lemma

$$\begin{aligned} & (\text{subsetp}(y, x) \wedge (\neg \text{subsetp}(y, z))) \\ \rightarrow & (\neg \text{subsetp}(\text{intersection}(x, y), z)) \end{aligned}$$

THEOREM: subsetp-intersection-elimination

$$\text{subsetp}(y, x) \rightarrow (\text{subsetp}(\text{intersection}(x, y), z) \leftrightarrow \text{subsetp}(y, z))$$

THEOREM: disjoint-intersection

$$\text{disjoint}(\text{intersection}(x, y), z) = \text{disjoint}(x, \text{intersection}(y, z))$$

THEOREM: subsetp-intersection-monotone-1

$$\begin{aligned} & (\text{subsetp}(\text{intersection}(x, y), z) \wedge \text{subsetp}(x1, x)) \\ \rightarrow & \text{subsetp}(\text{intersection}(x1, y), z) \end{aligned}$$

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;; The lemma SUBSETP-INTERSECTION-MONOTONE-2 below was added during
;; polishing of the final proof in "generalize.events", since the
;; lemma immediately above wasn't enough at that point. Actually
;; I realized at this point that intersection commutes from the point
;; of view of subsest:
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THEOREM: subsest-intersest-commuter
 $\text{subsest}(\text{intersest}(x, y), z) = \text{subsest}(\text{intersest}(y, x), z)$

THEOREM: subsest-intersest-monotone-2
 $(\text{subsest}(\text{intersest}(y, x), z) \wedge \text{subsest}(x1, x))$
 $\rightarrow \text{subsest}(\text{intersest}(x1, y), z)$

THEOREM: disjoint-intersest-commuter
 $\text{disjoint}(x, \text{intersest}(y, z)) = \text{disjoint}(x, \text{intersest}(z, y))$

THEOREM: disjoint-intersest3
 $\text{disjoint}(\text{free}, \text{intersest}(\text{vars}, x))$
 $\rightarrow (\text{intersest}(x, \text{intersest}(\text{vars}, \text{free})) = \mathbf{nil})$

EVENT: Disable intersest.

THEOREM: member-set-diff
 $(a \in \text{set-diff}(y, z)) = ((a \in y) \wedge (a \notin z))$

THEOREM: subsest-set-diff-1
 $\text{subsest}(\text{set-diff}(x, y), x)$

THEOREM: disjointp-set-diff
 $\text{disjoint}(\text{set-diff}(x, y), y)$

THEOREM: subsest-set-diff-2
 $\text{subsest}(x, \text{set-diff}(y, z)) = (\text{subsest}(x, y) \wedge \text{disjoint}(x, z))$

THEOREM: set-diff-cons
 $\text{set-diff}(\text{cons}(a, x), y)$
 $= \text{if } a \in y \text{ then set-diff}(x, y)$
 $\text{else cons}(a, \text{set-diff}(x, y)) \text{ endif}$

THEOREM: set-diff-nlistp
 $(x \simeq \mathbf{nil}) \rightarrow (\text{set-diff}(x, y) = \mathbf{nil})$

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;; The following was discovered during final polishing, for the
;; proof of MAIN-HYPS-RELIEVED-6-FIRST.
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THEOREM: disjoint-set-diff-general

$\text{disjoint}(x, \text{set-diff}(y, z)) = \text{subsetp}(\text{intersection}(x, y), z)$

;; No longer relevant:

;(prove-lemma disjoint-set-diff-subsetp (rewrite)

; (implies (and (disjoint x (set-diff y z))

; (subsetp z z1))

; (disjoint x (set-diff y z1)))

; ((use (disjoint-wit-witnesses (y (set-diff y z1))))

; (disable member-set-diff set-diff)))

;; Instead of the following I'll prove the corresponding (in light of

;; DISJOINT-SET-DIFF-GENERAL) fact INTERSECTION-X-X about intersection.

;(prove-lemma disjoint-set-diff (rewrite)

; (disjoint x (set-diff y x)))

THEOREM: intersection-subsetp-identity

$(\text{properp}(x) \wedge \text{subsetp}(x, y)) \rightarrow (\text{intersection}(x, y) = x)$

THEOREM: intersection-x-x

$\text{properp}(x) \rightarrow (\text{intersection}(x, x) = x)$

THEOREM: subsetp-set-diff-mononone-2

$\text{subsetp}(\text{set-diff}(x, \text{append}(y, z)), \text{set-diff}(x, z))$

THEOREM: subsetp-set-diff-monotone-second

$\text{subsetp}(\text{set-diff}(x, y), \text{set-diff}(x, z)) = \text{subsetp}(\text{intersection}(x, z), y)$

THEOREM: set-diff-nil

$\text{set-diff}(x, \text{nil}) = \text{fix-properp}(x)$

THEOREM: set-diff-cons-non-member-1

$(a \notin x) \rightarrow (\text{set-diff}(x, \text{cons}(a, y)) = \text{set-diff}(x, y))$

THEOREM: length-intersection-set-diff

$\text{length}(x) = (\text{length}(\text{set-diff}(x, y)) + \text{length}(\text{intersection}(x, y)))$

THEOREM: length-set-diff-opener

$\text{length}(\text{set-diff}(x, y)) = (\text{length}(x) - \text{length}(\text{intersection}(x, y)))$

THEOREM: listp-set-diff

$\text{listp}(\text{set-diff}(x, y)) = (\neg \text{subsetp}(x, y))$

;; Here is a messy lemma about disjoint and such

THEOREM: disjoint-intersection-set-diff-intersection
 $\text{disjoint}(x, \text{intersection}(y, \text{set-diff}(z, \text{intersection}(y, x))))$

EVENT: Disable set-diff.

THEOREM: member-fix-properp
 $(a \in \text{fix-properp}(x)) = (a \in x)$

THEOREM: setp-append
 $\text{setp}(\text{append}(x, y)) = (\text{disjoint}(x, y) \wedge \text{setp}(\text{fix-properp}(x)) \wedge \text{setp}(y))$

THEOREM: setp-cons
 $\text{setp}(\text{cons}(a, x)) = ((a \notin x) \wedge \text{setp}(x))$

THEOREM: setp-nlistp
 $(x \simeq \mathbf{nil}) \rightarrow (\text{setp}(x) = (x = \mathbf{nil}))$

DEFINITION:
 $\text{make-set}(l)$
 $=$ **if** $\neg \text{listp}(l)$ **then** **nil**
 elseif $\text{car}(l) \in \text{cdr}(l)$ **then** $\text{make-set}(\text{cdr}(l))$
 else $\text{cons}(\text{car}(l), \text{make-set}(\text{cdr}(l)))$ **endif**

THEOREM: make-set-preserves-member
 $(x \in \text{make-set}(l)) = (x \in l)$

THEOREM: make-set-preserves-subsetp-1
 $\text{subsetp}(\text{make-set}(x), \text{make-set}(y)) = \text{subsetp}(x, y)$

THEOREM: make-set-preserves-subsetp-2
 $\text{subsetp}(x, \text{make-set}(y)) = \text{subsetp}(x, y)$

THEOREM: make-set-preserves-subsetp-3
 $\text{subsetp}(\text{make-set}(x), y) = \text{subsetp}(x, y)$

THEOREM: make-set-gives-setp
 $\text{setp}(\text{make-set}(x))$

THEOREM: make-set-set-diff
 $\text{make-set}(\text{set-diff}(x, y)) = \text{set-diff}(\text{make-set}(x), \text{make-set}(y))$

THEOREM: set-diff-make-set
 $\text{set-diff}(x, \text{make-set}(y)) = \text{set-diff}(x, y)$

THEOREM: listp-make-set
 $\text{listp}(\text{make-set}(x)) = \text{listp}(x)$

EVENT: Disable setp.

;;;;; The following were proved in the course of the final run
;;;;; through the generalization proof. There are a couple or
;;;;; so noted above here, too.

THEOREM: set-diff-append
 $\text{set-diff}(x, \text{append}(y, z)) = \text{set-diff}(\text{set-diff}(x, z), y)$

THEOREM: length-set-diff-leq
 $\text{length}(x) \not\leq \text{length}(\text{set-diff}(x, y))$

THEOREM: lessp-length
 $\text{listp}(x) \rightarrow (0 < \text{length}(x))$

THEOREM: listp-intersection
 $\text{listp}(\text{intersection}(x, y)) = (\neg \text{disjoint}(x, y))$

THEOREM: length-set-diff-lessp
 $(\neg \text{disjoint}(x, \text{new})) \rightarrow (\text{length}(\text{set-diff}(x, \text{new})) < \text{length}(x))$

THEOREM: disjoint-imply-empty-intersection
 $\text{disjoint}(x, y) \rightarrow (\text{intersection}(x, y) = \mathbf{nil})$

;; The following lemma DISJOINT-INTERSECTION3-MIDDLE is needed for the
;; proof of ALL-VARS-DISJOINT-OR-SUBSETP-GEN-CLOSURE in
;; generalize.events. I think I could avoid lemmas like this one
;; INTERSECTION were actually commutative-associative (in which case
;; I'd get rid of disjoint and rely on normalization).

THEOREM: disjoint-intersection3-middle
 $\text{disjoint}(y, \text{intersection}(x, z))$
 $\rightarrow (\text{intersection}(x, \text{intersection}(y, z)) = \mathbf{nil})$

;; Maybe I should redo the notion of disjoint sometime, perhaps using
;; the fact that intersection is commutative and associative when it's
;; equated with nil.

THEOREM: disjoint-subsetp-hack
 $(\text{disjoint}(x, \text{intersection}(u, v)) \wedge \text{subsetp}(w, x))$
 $\rightarrow \text{disjoint}(u, \text{intersection}(w, v))$

THEOREM: subsetp-set-diff-sufficiency
 $\text{subsetp}(x, y) \rightarrow \text{subsetp}(\text{set-diff}(x, z), y)$

;; The following lemma SETP-INTERSECTION-SUFFICIENCY is needed for
;; MAPPING-RESTRICT from "alists.events", because (I believe)
;; DOMAIN-RESTRICT, which was added during polishing, changed the
;; course of the previous proof. Similarly for
;; SETP-SET-DIFF-SUFFICIENCY and the lemma MAPPING-CO-RESTRICT.

THEOREM: setp-intersection-sufficiency
 $\text{setp}(x) \rightarrow \text{setp}(\text{intersection}(x, y))$

THEOREM: setp-set-diff-sufficiency
 $\text{setp}(x) \rightarrow \text{setp}(\text{set-diff}(x, y))$

;; The definition of FIX-PROPERP was also added in polishing because
;; of a problem with the proof of GEN-CLOSURE-ACCEPT in
;; "generalize.events". Here are a couple of lemmas about it that
;; might or might not be useful; all other lemmas about it above, and
;; the definition, were added during polishing.

EVENT: Disable fix-properp.

THEOREM: subsetp-fix-properp-1
 $\text{subsetp}(\text{fix-properp}(x), y) = \text{subsetp}(x, y)$

THEOREM: subsetp-fix-properp-2
 $\text{subsetp}(x, \text{fix-properp}(y)) = \text{subsetp}(x, y)$

EVENT: Make the library "sets".

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