Homework 12 Solutions
CS 313H

The important issue is the logic you used to arrive at your answer.

1. Prove that for all sets A: $A \cap A = A$.

F or all sets A:
$x \in A \cap A$
$\Leftrightarrow x \in A \wedge x \in A$
$\Leftrightarrow x \in A$.

2. Prove that for all sets A: $A \cap \varnothing = \varnothing$.

F or all sets A:
$x \in A \cap \varnothing$
$\Leftrightarrow x \in A \wedge x \in \varnothing$
$\Leftrightarrow$ *false*
$\Leftrightarrow x \in \varnothing$.

3. Prove that for all sets A: $A \sim \varnothing = A$.

F or all sets A:
$x \in A \sim \varnothing = A$.
$\Leftrightarrow x \in A \wedge x \notin \varnothing$
$\Leftrightarrow x \in A \wedge true$
$\Leftrightarrow x \in A$.

4. Prove that for all sets A: $A \cup \varnothing = A$.

F or all sets A:
$x \in A \cup \varnothing = A$.
$\Leftrightarrow x \in A \vee x \in \varnothing$
$\Leftrightarrow x \in A \vee false$
$\Leftrightarrow x \in A$.

5. Prove that for all sets A, B, and C: $(A \subseteq B \wedge B \subseteq C) \Rightarrow A \subseteq C$.

For all sets A, B, and C: if$(A \subseteq B \wedge B \subseteq C)$then
$x \in A$
$\Rightarrow x \in B$
$\Rightarrow x \in C$
thus $A \subseteq C$.

6. Use **5.** to prove that for all sets B and C : $(B \subseteq C) \Rightarrow P(B) \subseteq P(C)$.

F or all sets B and C :
$A \in P(B)$
$\Rightarrow A \subseteq B$
$\Rightarrow A \subseteq C$
$\Rightarrow A \in P(C)$.

7. Prove that for all sets A , B, and C : $(A \subseteq B \cap C) \Rightarrow (A \subseteq B \wedge A \subseteq C)$.

For all sets A , B, and C : if($A \subseteq B \cap C$)then
$x \in A$
$\Rightarrow x \in B \wedge x \in C$
thus($A \subseteq B \wedge A \subseteq C$).

8. Prove or disprove with a simple counterexample, for all sets B and C :

$$P(B \cap C) = P(B) \cap P(C).$$

F or all sets B and C :
$A \in P(B \cap C)$
$\Leftrightarrow A \subseteq B \cap C$
$\Leftrightarrow A \subseteq B \wedge A \subseteq C$
$\Leftrightarrow A \in P(B) \wedge A \in P(C)$
$\Leftrightarrow A \in P(B) \cap P(C)$.

9. Prove or disprove with a simple counterexample, for all sets B and C :

$$P(B \cup C) = P(B) \cup P(C).$$

This is false. Let $B = \{0\}$ and $C = \{1\}$, then $P(B) = \{\emptyset, \{0\}\}$, $P(C) = \{\emptyset, \{1\}\}$, so $P(B) \cup P(C) = \{\emptyset, \{0\}, \{1\}\}$ but $B \cup C = \{0,1\}$ so $P(B \cup C) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$. We have $\{0,1\} \in P(B \cup C)$ but $\{0,1\} \notin P(B) \cup P(C)$.