The Construction of Truth Tables

One technique for constructing proofs in prepositional logic is to employ truth tables. They always work. However, if the there is a large number of operands, then the tables with be huge (with n operands the table will have 2^n rows). Furthermore, if we see the primary function of theorem proving as gaining insight into why a theorem is true, then truth tables are pretty much worthless – they don't render much insight.

The process itself is fairly simple – so simple that a computer program could be written to do it. Actually, there are two standard formats: one uses about one extra column for every operator; the other has a single column with lots of truth values crowded together.

Both formats begin with the variables being listed across the top and all combinations of truth values placed on the rows beneath.

Thus with a single operand we have:

p
F
T

and with two it's:

p	*q*
F	F
F	T
T	F
T	T

(You can start with all T's or all F's. You should just proceed in an orderly fashion that ensures all combinations will be present.) As said above, in general with n operands the table will have 2^n rows. What happens next separates the two formats.

1. Super Safe "I-Always-Wear-My-Seat-Belt" Format:

Using this format the conclusion is built up one operand at a time and there will be one additional column for every operand. This is quite clean although possibly lengthy. It is safer than the other approach because it's generally pretty clear what is happening.

Consider the example of building a truth table for the expression $\sim((\sim p \wedge q) \vee p)$.

We begin with

p	q
F	F
F	T
T	F
T	T

and then build it up the columns in four steps (one for each of $\sim p$, $\sim p \wedge q$, $(\sim p \wedge q) \vee p$, and $\sim((\sim p \wedge q) \vee p)$):

p	q	$\sim p$
F	F	T
F	T	T
T	F	F
T	T	F

p	q	$\sim p$	$\sim p \wedge q$
F	F	T	F
F	T	T	T
T	F	F	F
T	T	F	F

p	q	$\sim p$	$\sim p \wedge q$	$(\sim p \wedge q) \vee p$
F	F	T	F	F
F	T	T	T	T
T	F	F	F	T
T	T	F	F	T

p	q	$\sim p$	$\sim p \wedge q$	$(\sim p \wedge q) \vee p$	$\sim((\sim p \wedge q) \vee p)$
F	F	T	F	F	T
F	T	T	T	T	F
T	F	F	F	T	F
T	T	F	F	T	F

The final column shows that the expression $\sim((\sim p \wedge q) \vee p)$ is neither a *tautology* (true for all truth values of the operands) nor a *contradiction* (false for all truth values of the operands).

2. Very Risky "I-Drive-With-My-Feet" Format:

With this format you have a single additional column with the entire expression. Underneath there are truth values associated with the operators AND operands. The final truth values (i.e., the ones that correspond to the last column in the first format) are the ones beneath the final operator applied. Yes, than can be hard to find. Yes, this can get extremely messy. If you use this, make sure that your truth values line up very nice vertically.

Here are the steps for our example.

Notice we begin by just copying the truth values over under the associated operands:

p	q	$\sim((\sim p \wedge q) \vee p)$
F	F	F F F
F	T	F T F
T	F	T F T
T	T	T T T

Then we add truth values under the negation of p under the ~:

p	q	$\sim((\sim p \wedge q) \vee p)$
F	F	T F F F
F	T	T F T F
T	F	F T F T
T	T	F T T T

Now under the $\wedge$ we place the truth values for $\sim p \wedge q$:

p	q	$\sim((\sim p \wedge q) \vee p)$
F	F	T F F F F
F	T	T F T T F
T	F	F T F F T
T	T	F T F T T

And now under the $\vee$ we place the truth values for $(\sim p \wedge q) \vee p$:

p	q	$\sim((\sim p \wedge q) \vee p)$
F	F	T F F F F F
F	T	T F T T T F
T	F	F T F F T T
T	T	F T F T T T

And finally under the leading ~ we place the truth values for $\sim((\sim p \wedge q) \vee p)$:

p	q	$\sim((\sim p \wedge q) \vee p)$
F	F	T T F F F F F
F	T	F T F T T T F
T	F	F F T F F T T
T	T	F F T F T T T

Again we see the expression is neither tautology nor contradiction. In this case, the final values showed up at the front of the column. Realize that they could occur deep in the middle of the list of Ts and Fs since they correspond to the final operation – wherever that is.