**Examination 1 Solutions**

**CS 313H**

**1. [10]** Using a truth table prove that $((p \wedge q) \Rightarrow r) \Leftrightarrow (p \Rightarrow (q \Rightarrow r))$

| $p$ | $q$ | $r$ | $p \wedge q$ | $(p \wedge q) \Rightarrow r$ | $q \Rightarrow r$ | $p \Rightarrow (q \Rightarrow r)$ | $((p \wedge q) \Rightarrow r) \Leftrightarrow (p \Rightarrow (q \Rightarrow r))$ |
|---|---|---|---|---|---|---|---|
| F | F | F | F | T | T | T | T |
| F | F | T | F | T | T | T | T |
| F | T | F | F | T | F | T | T |
| F | T | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | T | T | T | T | T | T | T |

**2. [20]** Using sentential calculus (with a four column format), prove that the conclusion $E \Rightarrow J$ follows from premises: $E \Rightarrow (F \wedge G), G \Rightarrow (H \vee J),$ and $\sim H$.

| | | |
|---|---|---|
| $\{Pr_1\}$ | (1.) $E \Rightarrow (F \wedge G)$ | P |
| $\{Pr_2\}$ | (2.) $G \Rightarrow (H \wedge J)$ | P |
| $\{Pr_3\}$ | (3.) $\sim H$ | P |
| $\{Pr_4\}$ | (4.) $E$ | P |
| $\{Pr_1, Pr_4\}$ | (5. ) $F \wedge G$ | MP (1),(4) |
| $\{Pr_1, Pr_4\}$ | (6.) $G$ | Simp (5) |
| $\{Pr_1, Pr_2, Pr_4\}$ | (7.) $H \vee J$ | MP (2), (6) |
| $\{Pr_1, Pr_2, Pr_3, Pr_4\}$ | (8.) $J$ | DS (3), (7) |
| $\{Pr_1, Pr_2, Pr_3\}$ | (9.) $E \Rightarrow J$ | C (4), (8) |

**3. [20]** Prove that the conclusion $(A \Rightarrow C) \Rightarrow F$ follows from the premises $\sim C$ and $(B \Rightarrow \sim C) \Rightarrow A$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

$\sim C$

$(B \Rightarrow \sim C) \Rightarrow A$
$\sim (B \Rightarrow \sim C) \vee A$
$\sim (\sim B \vee \sim C) \vee A$
$(B \wedge C) \vee A$
$(B \vee A) \wedge (C \vee A)$

$\sim ((A \Rightarrow C) \Rightarrow F)$

$\sim(\sim(A \Rightarrow C) \vee F)$
$\sim(\sim(\sim A \vee C) \vee F)$
$\sim((A \wedge \sim C) \vee F)$
$\sim(A \wedge \sim C) \wedge \sim F$
$(\sim A \vee C) \wedge \sim F$

| | | |
|---|---|---|
| 1. | $\sim C$ | P |
| 2. | $B \vee A$ | P |
| 3. | $C \vee A$ | P |
| 4. | $\sim A \vee C$ | P |
| 5. | $\sim F$ | P |
| 6. | $A$ | Res (1), (3) |
| 7. | $C$ | Res (4), (6) |
| 8. | *false* | Conj. (1), (7) |

**4. [10]** Using the predicates defined on the set of real numbers:

| | |
|---|---|
| $Gxy$ | $x$ is greater than $y$, |
| $Px$ | $x$ is positive, |
| $Nxy$ | $x$ is not equal to $y$, |
| $Dxyz$ | $x - y$ is an integral multiple of $z$, |

Express in the syntax of Predicate Calculus (you may use integers as constants):

*"There are unequal numbers so that either the larger one is positive or that the two have an even difference."*

$(\exists x)(\exists y)(Nxy \wedge (((Gxy \Rightarrow Px) \wedge (Gyx \Rightarrow Py)) \vee Dxy2))$

or equivalently

$(\exists x)(\exists y)((Gxy \wedge Px) \vee (Nxy \wedge Dxy2))$

**5. [25]** Prove that $((\exists x)Px \wedge A)$ follows from $(\exists x)(Px \wedge A)$ (Rather than using the TC rule be specific about the sentential calculus rule.)

| | | |
|---|---|---|
| $\{P_1\}$ | (1). $(\exists x)(Px \wedge A)$ | P |
| $\{P_1\}$ | (2). $Pa \wedge A$ | EI (1) |
| $\{P_1\}$ | (3). $Pa$ | Simp (2) |
| $\{P_1\}$ | (4). $(\exists x)Px$ | EG (3) |
| $\{P_1\}$ | (5). $A$ | Simp (2) |
| $\{P_1\}$ | (6). $((\exists x)Px \wedge A)$ | Conj (4), (5) |