Practice Examination 1 Solutions

CS 313H

1. [10] Using a truth table prove that $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \wedge (q \Rightarrow r))$

p	q	r	$p \vee q$	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \wedge (q \Rightarrow r)$	$((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \wedge (q \Rightarrow r))$
F	F	F	F	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	T	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
T	F	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	T	T	T	T	T	T	T	T

2. [20] Using the rules of Sentential Calculus conclude $(P \wedge \sim B) \Rightarrow C$ from the premises $A \vee (B \vee C)$ and $P \Rightarrow \sim A$.

$\{P_1\}$	(1.) $A \vee (B \vee C)$	P
$\{P_2\}$	(2.) $P \Rightarrow \sim A$	P
$\{P_3\}$	(3.) $P \wedge \sim B$	P
$\{P_3\}$	(4.) P	Simp (3)
$\{P_2, P_3\}$	(5.) $\sim A$	MP (2), (4)
$\{P_3\}$	(6.) $\sim B$	Simp (3)
$\{P_2, P_3\}$	(7.) $\sim A \wedge \sim B$	Conj (5), (6)
$\{P_2, P_3\}$	(8.) $\sim (A \vee B)$	DeM (7)
$\{P_1\}$	(9.) $(A \vee B) \vee C$	Assoc (1)
$\{P_1, P_2, P_3\}$	(10.) C	DS (9)
$\{P_1, P_2\}$	(11.) $(P \wedge \sim B) \Rightarrow C$	C (3), (10)

3. [20] Prove that the conclusion $(A \Rightarrow D) \vee (C \Rightarrow B)$ follows from the premise $(A \Rightarrow B) \vee (C \Rightarrow D)$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

$(A \Rightarrow B) \vee (C \Rightarrow D)$
$\sim A \vee B \vee \sim C \vee D$

$\sim ((A \Rightarrow D) \vee (C \Rightarrow B))$
$\sim (A \Rightarrow D) \wedge \sim (C \Rightarrow B)$
$\sim (\sim A \vee D) \wedge \sim (\sim C \vee B)$
$(A \wedge \sim D) \wedge (C \wedge \sim B)$

1.	$\sim A \vee B \vee \sim C \vee D$	P
2.	A	P
3.	$\sim D$	P
4.	C	P
5.	$\sim B$	P
6.	$B \vee \sim C \vee D$	Res (1), (2)
7.	$\sim C \vee D$	Res (5), (6)
8.	D	Res (4), (7)
9.	*false*	Conj. (3), (8)

4. [10] Using the predicates:

Bx	x is a loaf of bread,
Hx	x is a pound on hamburger,
Cxy	x costs more than y dollars,
Px	x is a person,
Exy	x eats y,
Tx	x is tofu.

Express in the syntax of Predicate Calculus:

"If some loaf of bread costs more than \$4 and some pound of hamburger costs more than \$8 then the only thing everyone eats is tofu."

$$((\exists x)(Bx \wedge Cx4) \wedge (\exists y)(Hy \wedge Cy8)) \Rightarrow (\forall w)(\forall z)((Pw \wedge Ewz) \Rightarrow Tz)$$

5. [25] Prove that $(\forall w)(\exists z)Rzw$ follows from $(\exists x)(\forall y)Rxy$ (Rather than using the TC rule be specific about the sentential calculus rule.)

$\{P_1\}$	(1). $(\exists x)(\forall y)Rxy$	P
$\{P_1\}$	(2). $(\forall y)Ray$	EI (1)
$\{P_1\}$	(3). Rab	UI (2)
$\{P_1\}$	(4). $(\exists z)Rzb$	EG (3)
$\{P_1\}$	(5). $(\forall w)(\exists z)Rzw$	UG (4)