Examination 1 Solutions

CS 313H

1. [10] Using a truth table prove that $((p \wedge q) \Rightarrow r) \Leftrightarrow (p \Rightarrow (q \Rightarrow r))$

2. [20] Using sentential calculus (with a four column format), prove that the conclusion $E \Rightarrow J$ follows from premises: $E \Rightarrow (F \wedge G), G \Rightarrow (H \vee J), \text{and} \sim H$.

3. [20] Using sentential calculus (with a four column format), prove that the conclusion "Peter is a student" follows from these premises:

Either Peter is a student or Karen is a student.
If Karen is a student then Tom and Sam are both students.
Either Robert is not a student or Sam is not a student.
If both Sam and Tom are students then Robert is a student.

Begin by giving them symbolic values to certain sentences and expressing the premises and conclusion in terms of those symbols.

Let p = "Peter is a student"
k = "Karen is a student"
t = "Tom is a student"
s = "Sam is a student"
r = "Robert is a student"

4. [20] Prove that the conclusion $(A \Rightarrow C) \Rightarrow F$ follows from the premises $\sim C \text{ and } (B \Rightarrow \sim C) \Rightarrow A$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

5. [15] Using the predicates defined on the set of real numbers:

Gxy x is greater than y ,
Px x is positive,
Nxy x is not equal to y ,
$Dxyz$ $x - y$ is an integral multiple of z ,

Express in the syntax of Predicate Calculus (you may use integers as constants):

a. "There are unequal numbers so that either the larger one is positive or that the two have an even difference."

b. "All integral multiples of 6 are also integral multiples of 3."