

Coloring an Irregular Grid

Problem: You are given an $m \times n$ array A of zeros and ones. You are to construct an $m \times n$ array B such that $|A| = |B|$ (thus if $a_{i,j} = 0$ then $b_{i,j} = 0$ and if $a_{i,j} = 1$ then $b_{i,j} = 1$ or $b_{i,j} = -1$) so that all row sums and columns sums of B are either $-1, 0,$ or 1 .

Example: Given $A = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$, we could have $B = \begin{bmatrix} -1 & 0 & 1 & -1 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$.

We could refer to the components in A that are one as a grid and then the problem is to assign a color – black or red – to each element of the grid in such a fashion that the rows and columns are as close to being color-balanced as possible. Each row and column either has an equal number of blacks and reds or has exactly one extra black or red.

1. Prove that given any such A , a legal B exists.
2. Write a function in any programming language that takes A as input and produces B as output.

(Unnecessary Comment: You now know what “proof” means and having an algorithm that works on every example you’ve tried does not constitute a proof.)