

**Examination 1****CS 336**

- 1. The important issue is the logic you used to arrive at your answer.**
- 2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.**
- 3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.**
- 4. Comment on all logical flaws and omissions and enclose the**  

**comments in boxes**

**1.** For  $n \geq m \geq 1$ , let  $A = \{1, 2, \dots, 2m\}$ ,  $B = \{1, 2, \dots, 2n\}$ , and consider functions mapping from  $A$  into  $B$ .

- a. [5]** How many of these functions map even numbers to even numbers and odd number to odd numbers (i.e., how many functions have **both** properties)?
- b. [5]** Of these functions that map even numbers to even numbers and odd number to odd numbers, how many are one-to-one?

**2.a [10]** Present a combinatorial argument that for all positive integers  $n$ :

$$\binom{2n}{2} = 2 \binom{n}{2} + n^2.$$

**b. [10]** Present a combinatorial argument that for all positive integers  $n$

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}.$$

(Hint: Consider distinct sets  $A$  and  $B$  each of cardinalities  $n$ .)

- 3. [10]** For  $n \geq m \geq 1$ , in how many ways can  $n$  **identical** coins be distributed among  $m$  **non-identical** people such that every person has at least one coin?
- 4. [10]** For  $n \geq n_1, n_2, n_3, n_4 \geq 0$ , you are given  $n$  non-identical books and five non-identical boxes. How many ways are there to distribute books into the boxes so that box 1 has exactly  $n_1$  books, box 2 has exactly  $n_2$  books, ..., and box 5 has the remaining books (if any)?
- 5. [10]** For  $n \geq 3$ , how many strings of length  $n$  consisting of  $a$ s,  $b$ s, and  $c$ s are there that have exactly one  $a$  and at least two  $b$ s?
- 6.** Consider strings of length  $n \geq 5$  containing exactly  $k$  1s and  $n - k$  0s, where  $k \geq 5$ . Consider that all such strings are equally likely.
  - a. [5]** What is the probability that such a string begins with five 1s?

**b. [5]** What is the probability that such a string begins with five 1s given that it begins with three 1s?