Examination 2 Solutions

CS 336

1. [20] Using only Definition 2', prove that the set of reciprocals of positive integers (i.e. $\{\frac{1}{1},\frac{1}{2},\frac{1}{3},\ldots\}=\{\frac{1}{k}\mid k\in\mathbb{Z}\wedge k\geq 1\}$ is infinite**.**

2. [20] Let $P=\{\text{infinitely long bit strings containing exactly ten zeros}\}$. Is the set P finite, countably infinite, or uncountably infinite? Prove your claim.

3. [20] Is the set of circles in the plane finite, countably infinite, or uncountably infinite**?** Prove your claim.

4. [20] Using no other asymptotic dominance theory than definitions, prove that $6n^{7/8}+5n^{3/2}=O(n^2)$.

5. **[20]** Employing induction prove that for $k\geq 1$, if for $i=1,2,\ldots,k$, $f_i=O(f_{i+1})$, then $f_1=O(f_{k+1})$.

6. [20] Prove that $2^n=o(n!)$. (Hint: $\prod_{i=1}^{n}\frac{2}{i}=\prod_{i=1}^{3}\frac{2}{i}\cdot\prod_{i=4}^{n}\frac{2}{i}=\frac{4}{3}\prod_{i=4}^{n}\frac{2}{i}$ and $\frac{2}{i}\leq\frac{1}{2}$ for $i\geq 4$.)