CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [20] Using only Definition 2', prove that the set of finitely long strings using characters from {A, B, C, …, Z, a, b, c, …., z} is infinite**.**

2. [20] Consider this theorem (that relies upon the Axiom of Choice):

$$\textit{If } f : A \xrightarrow[onto]{} B\textit{, then there exists a subset } \hat{A} \textit{ of } A \textit{ such that } f : \hat{A} \xrightarrow[onto]{1-1} B.$$

Use this theorem to prove: If $f : A \xrightarrow[onto]{} B$, and B is infinite then A is infinite.

3. [20] Is the set of infinitely long strings using characters from {A, B, C, …, Z, a, b, c, …., z} finite, countably infinite, or uncountably infinite**?** Prove your claim.

4. [20] Prove that $\sqrt{n^3+1} = o(n^2)$. (Hint: $1 \le n^3$ for $n \ge 1$.)

5. **[20]** Employing induction and Theorem 4, prove that for $k \ge 1$, if for $i = 1, 2, \ldots, k$, $f_i = \mathrm{O}(g)$, then $\prod_{i=1}^{k} f_i = \mathrm{O}(g^k)$.

6. [20] Prove that polynomials are asymptotically dominated by their largest power: That is, for $k \ge 0, \sum_{i=0}^{k} a_i n^i = \mathrm{O}(n^k)$.