Name ______________________________

Final Examination

CS 336

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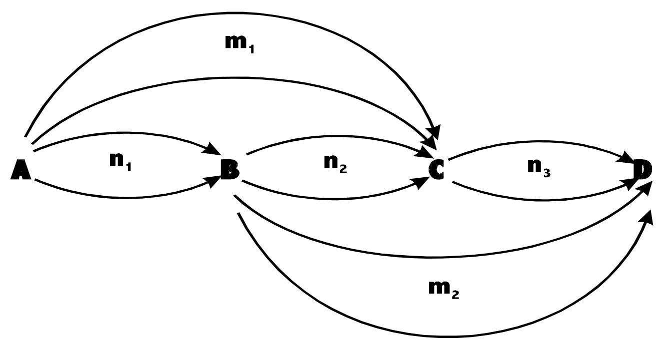
1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes

1. a [5] Consider this directed multigraph having n_1 edges from vertex A to vertex B, n_2 edges from vertex B to vertex C, etc. . How many different paths are there from vertex A to vertex D?



b [5] Given that the length of any shortest path along gridlines from *(0,0)* to *(m, n)* is

m+n, how many such shortest length paths are there. ?

2. [10] For $n \geq 1$, consider sequences on length *n* composed of *a*'s, *b*'s, *c*'s, *d*'s and *e*'s. How many of the sequences are in alphabetical order (i.e., all *a*'s precede all *b*'s, all *b*'s precede all *c*'s, ..., etc.)?

3. a. [10] Using a combinatorial argument, prove that for $n \geq 2$ and $m \geq 2$:

$$\binom{n+m}{2} = n \cdot m + \binom{n}{2} + \binom{m}{2}$$

b. [10] Using a combinatorial argument, prove that for integers $m, n, p \geq 1$:

$$(m+n)^p = \sum_{k=0}^{p} \binom{p}{k} m^k \, n^{p-k}$$

4. a. [10] For $n \geq 5$, consider strings of length n using elements of $\{a, b, c\}$. Assume all such strings are equally likely. What is the probability that a string has at least one a?

b. [5] What is the probability that such a string has at least one b given that it has at least one a?

5. **[10]** Using definition 2' (and no cardinality theorems) prove that $\mathrm{N} \times \mathrm{N}$, the set of ordered pairs of natural numbers, is infinite.

6. a. [10] Let A be a nonempty set. Prove that $\mathcal{P}(A)$, the power set of A, cannot be put into one-to-one correspondence with A (i.e., there exists no function $f: A \xrightarrow[onto]{1-1} \mathcal{P}(A)$). (Hint you may want to employ the set $C = \{a \mid a \subseteq A \text{ and } a \notin f(a)\}$.)

b. [5] Use the above result to conclude that for any nonempty set A, its power set cannot be countably infinite.

7. [10] Prove that $1 + 2n + 3n^2 + 4n^3 = \mathrm{O}(n^3)$.

8. [10] .Prove that if $f_1 = \mathrm{o}(g_1)$ and $f_2 = \mathrm{o}(g_2)$, then $f_1 + f_2 = \mathrm{o}(|\, g_1 \,| + |\, g_2 \,|)$.

9. **[10]** Prove the following code is correct with respect to precondition "***true***" and postcondition "$((z = w) \vee (z = x)) \wedge (z \geq w) \wedge (z \geq x)$":

```
z := w
if x > z then
        z := x
```

10. a. [10] Prove the following code is partially correct with respect to precondition "$m \geq 1$ and $n \geq 0$" and postcondition "$c = \binom{m}{n}$" (assume c, m, n, and k are integer variables.):

```
c := 1
k := 1
while k≤n do
        c := (c*(m-k+1))/k
        k := k+1
endwhile
```

Be explicit about your loop invariant. (You may use the following theorem:

$$\text{for } m \geq 1 \text{ and } k \geq 1: \binom{m}{k-1} \cdot \frac{m-k+1}{k} = \binom{m}{k},$$

and you may assume that integer division of $c*(m-k+1)$ by k is done exactly. That can be easily proved but do not waste the time.)

...b. [5] Prove that the loop terminates.

11. [10] Determine the weakest precondition with respect to the postcondition "$z \geq 6$" for the following code (assume *z*, *y*, and *x* are integer variables and that y is defined):

```
x := 5
z := x+y
if y>0 then
        z := 3+x
else
        z := 3*z
endif
```

12. [10] Prove that the weakest precondition with respect to the postcondition " $post(c)$ " for the following code

```
b : = 1
c := exp_1(a,b)
if test(a,b,c) then
        c := exp_2(a,b,c)
else
        c := exp_3(a,b,c)
endif
```

is:

$$(test(a,1,exp_1(a,1)) \wedge post(exp_2(a,1,exp_1(a,1)))) \vee (\neg test(a,1,exp_1(a,1)) \wedge post(exp_3(a,1,exp_1(a,1))))$$

(Hint: You may want to use the logical identity:
$((p \Rightarrow r) \wedge (\neg p \Rightarrow q)) \equiv ((p \wedge r) \vee (\neg p \wedge q))$