Cardinality Theory

Definition 1: A set A is *finite* with cardinality n if it is empty or if there exists a one-to-one function mapping $\{1,2,\ldots,n\}$ onto A. A set is *infinite* if it is not finite.

Definition 2: A set A is *infinite* if there exists a one-to-one function mapping A onto a proper subset of A. A set is *finite* if it is not infinite.

Lemma 1: If $f: A \xrightarrow{1-1} B$ then f maps A one-to-one onto B $f(A)$ and thus is invertable.

Definition 2': A set A is *infinite* if there exists a one-to-one function mapping A into a proper subset of A. A set is *finite* if it is not infinite.

Theorem 1: The set N of natural numbers is infinite.

Theorem 2: The real interval $[0,1]$ is infinite.

Theorem 3: A superset of an infinite set is infinite.

Corollary: A subset of a finite set is finite.

Theorem 4: Let A be infinite and $f: A \xrightarrow{1-1} B$, then B is infinite.

Definition 3: A set A is *countably infinite* if there exists a one-to-one function mapping N onto A. A set is *countable* if it is finite or countably infinite. A set is *uncountably infinite* if it is not countable.

Theorem 5: The real interval $[0,1]$ is uncountably infinite.

Theorem 6: If there exists a function $f: \mathrm{N} \xrightarrow[onto]{} A$ then A is countable.

Theorem 7: A subset of a countable set is countable.

Corollary: A superset of an uncountably infinite set is uncountably infinite.

Theorem 8: The union of a finite collection of finite sets is finite.

Theorem 9: The union of a countably infinite collection of finite sets is countable.

Theorem 10: The union of a countably infinite collection of countably infinite sets is countably infinite..

Theorem 11: Let A be uncountably infinite and $f: A \xrightarrow{1-1} B$, then B is uncountably infinite.