Name ______________________________

Examination 1

CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [5] Suppose all rolls of a six-side die are equally likely. What is the probability the roll is a six given that it is not one?

2. a. [10] Present a combinatorial argument that for all $n \geq 1$:

$$\sum_{k=1}^{n} \binom{n}{k} = 2^n - 1$$

(**Note**: The summation begins with $k = 1$.)

b. [10] Present a combinatorial argument that for all integers *k* and *n* satisfying $3 \leq k \leq n$

$$\binom{n}{k} = \binom{n-3}{k} + 3\binom{n-3}{k-1} + 3\binom{n-3}{k-2} + \binom{n-3}{k-3}$$

(**Hint:** Consider three special elements.)

3. [10] How many distinct permutations are there of the letters in "mississippi"?

4. [10] A bin has 100 blue balls, 100 red balls, and 100 green. How many different collections can I obtain using 100 of these balls? (Balls of the same color are indistinguishable from one another but are distinguishable from balls of another color. A "collection" has no order to it.)

5. [10] Assume all strings of length five using characters from $\{a, b, c, d\}$ are equally likely. What is the probability that there is a substring $\langle abc \rangle$ in the string?

6. [10] Suppose a number *k* from {1, 2, …, 100} is to be drawn and that all numbers are equally likely. Let A be the event *k* is a power of two. Let B be the event *k* is an integer multiple of four. Prove either that the events A and B are statistically independent or that they are statistically dependent.

7. [10] For $n \geq 1$, how many strings of length *n* employing the characters $\{a,b,c\}$ have at least one *a*?