

## Examination 1

## CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the 

comments in boxes
-------------------

1. [5] For  $n \geq 3$ , how many subsets of size 3 from  $\{a_1, a_2, \dots, a_n\}$  are there that either contain  $a_1$  or  $a_2$  (or both)?
2. [10] Given a set  $A$  of  $m$  characters, for  $n \geq 2$ , consider strings of length  $n$  using any of the characters of  $A$ . How many such strings begin and end with the same character?
3. [10] Present a combinatorial argument that for all positive integers  $m, n$ , and  $r$ , satisfying  $r \leq \min\{m, n\}$ :

$$\binom{m+n}{r} = \sum_{k=0}^n \binom{m}{k} \binom{n}{r-k}.$$

(Hint: Consider selecting from two sets.)

- b. [10] Present a combinatorial argument that for all positive integers  $n$ :

$$3^n = \sum_{i=0}^n \left( \sum_{j=0}^{n-i} \binom{n}{i} \binom{n-i}{j} \right)$$

(Note: Be very specific about the roles of  $i$  and  $j$ .)

4. [10] How many distinct permutations are there of the digits in 1121231234?
5. [10] Given  $n \geq r \geq 1$ , in how many ways can  $n$  identical balls be placed into  $r$  distinct bins such that each bin contains at least one ball? (Hint: Consider strings with balls and special dividers.)
6. [10] For  $n \geq 1$ , consider strings of length  $2n$  0's and 1's. Assuming all such strings are equally likely, what is the probability that such a string has an equal number of 0's and 1's?
7. a. [10] For  $n \geq 5$ , consider strings of length  $n$  using elements of  $\{a, b, c, d\}$ . Assume all such strings are equally likely. What is the probability that a string has exactly two  $a$ 's?  
  
b. [5] What is the probability that such a string has exactly three  $b$ 's given that it has exactly two  $a$ 's?