1. How many bit strings contain exactly eight 0s and ten 1s if every 0 must be immediately followed by a 1?

2. a. Let A be a set of cardinality *m* and A^k be its *k*-fold Cartesian product with itself. Let B be a set of cardinality *n*. How many different functions map from A^k to B?

b. How many Boolean (i.e. truth- or false - valued) functions are defined for pairs of Boolean variables? (Hint: What are *m*, *n*, and *k* here?)

3. Present a combinatorial argument that for all positive values of *m*, *n*, and *r*:

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k}\binom{n}{k}$$

4. Consider a three-dimensional array with dimensions $m \times n \times p$. Now consider a path from the (1, 1, 1) element to the (m, n, p) element such that at each step exactly one of the indices increases by exactly one (i.e., from any (i, j, k), a step is taken to either (i+1, j, k), (i, j+1, k) or (i, j, k+1)). How many such paths are there?

5. Prove by induction that for $n \geq 1$ and any set of events $E_1, E_2, \ldots, E_n$ for which probabilities are defined:

$$\Pr(\bigcup_{i=1}^{n} E_i) \leq \sum_{i=1}^{n} \Pr(E_i)$$

6. Let *A* be the set of finite length strings of 0s and 1s and let ***s*** = <010101...> (i.e., the infinite string alternating 0s and 1s). Let B be the set of all infinite strings of the form $\boldsymbol{a} \,||\, \boldsymbol{s}$ where $\boldsymbol{a} \in A$ and $||$ indicates concatenation. Is B finite, countably infinite, or uncountably infinite? Prove your assertion.

7. Using only Definition 2', show that the unit circle $A = \{(x,y) | x \text{ and } y \text{ are real and } x^2 + y^2 \leq 1\}$ is infinite**.**

8. Consider arrays of positive integers whose sum is 17 (e.g., <17>, <9, 8>, and <1, 5, 1, 6, 4>). Is the set of all such arrays finite, countably infinite, or uncountably infinite? Prove your assertion.

9. Since there are *n*! permutations of *n* elements, a binary decision tree whose leaves are the *n*! permutations must have height $\lceil \log_2 n! \rceil$. This is thus the number of comparisons necessary to determine which permutation one has - thus to sort the elements. Yet this number is often quoted as $n \log_2 n$. Prove that $\log_2 n! = O(n \log_2 n)$.

10. Show with a simple counter-example that even if $f = O(g)$ and *a* > 1, it does not follow that $a^f = O(a^g)$. (Hint: you may want to use that if $0 < c < b$, then $b^n \neq O(c^n)$.)

11. Suppose $f_1 = O(g_1)$ and $f_2 = o(g_2)$, show $f_1 + f_2 = O(|g_1| + |g_2|)$.

12. Prove that the following code is partial correct and terminates for preconditions $p > 0 \wedge q > 0$ and postcondition $k = p \, \mathbf{div} \, q$:

```
m := p
k := 0
while m ≥ q
    m := m-q
    k := k+1
```

13. Determine *wp*(**if** *(x div 2)*2=x* **then** x := x div 2 **else** x := x*x, *x is odd*). (Remember for even *x*, x **div** 2 = x/2. For odd, positive *x*, x **div** 2 = (x-1)/2, and for odd, negative *x*, x **div** 2 = (x+1)/2.)

14. With respect to the precondition $x > y > 0$ and postcondition $z \geq 1$ prove that the following code is correct or provide a counterexample to its correctness:

```
z := (x+1)div(y-1).
```

15. Some people believe goto statements, especially those that point back up into the code, make verification quite difficult. Rewrite this code avoiding both gotos then prove partial correctness and termination with respect to the precondition $n \geq 1$ and postcondition $A[k] = x \vee k = n + 1$:

```
k := 1;
loop: if A[k] = y then goto exit;
k := k+1
if k ≤ n then goto loop;
exit
```