Four Interpolation Problems
Due Tuesday Feb 3, 2009

1. Find a function $f:\mathbb{R}\to\mathbb{R}$ that is a linear combination of $1, e^x$, and $\sin x$ so that $f(0)=1$, $f(2)=1$, and $f(-1)=0$.

2. Determine a Lagrangian basis for the space of functions spanned by 1 and x^2.

3. Determine a Lagrangian basis for the space of functions spanned by $1, x,$ and e^x.

4. Consider the Planar Interpolation Problem:

Given $\{((x_1^1, x_1^2), y_1), ((x_2^1, x_2^2), y_2), ((x_3^1, x_3^2), y_3)\}$, find a function $p:\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ of the form $p(x^1, x^2) = a_1 + a_2 x^1 + a_3 x^2$ so that for $i = 1, 2, 3$

$$p(x_i^1, x_i^2) = y_i .$$

Build a Lagrangian basis for the space of functions spanned by $1, x^1,$ and x^2 .