CS388C: Combinatorics and Graph Theory — October 25, 2005
David Zuckerman

Midterm

Instructions. There are five problems of equal weight. You may assume any theorems stated in class or in the book, unless the question is to prove such a theorem. Explain all answers. You are allowed one sheet of paper, with writing on both sides. You have 75 minutes.

Name: ______________________________

1. Show that any family of more than 2^{n-1} subsets of $[n]$ contains distinct sets A, B with $A \subset B$.

2. Consider 6 people sitting around a circular table. How many different ways can they change seats so that each person has a different neighbor to the right?

3. Show that any graph on $2n$ vertices with all degrees at least n has a perfect matching. (Hint: consider short augmenting paths.)

4. Show that there exists a constant c with the following property. Let M be an $n \times n$ matrix with ± 1 entries. Then there exists a vector v with ± 1 entries such that at least 99% of the entries in Mv are at most $c\sqrt{n}$ in absolute value.

5. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be any symmetric function (f(x,y)=f(y,x)), and S be any set of 17 integers. Show that there exist distinct $s, t, u \in S$ such that

$$f(s,t) \equiv f(t,u) \equiv f(s,u) \pmod 3.$$