CS388C: Combinatorics and Graph Theory — Problem Set No. 2

David Zuckerman — due: Sep. 25, 2007

1. A *line* in the vector space $\mathbb{F}_3^n$ is a 3-set $\{x, y, z\}$ such that $x + y + z = 0$. (Think about why this corresponds to the definition you might have expected. For players of the game SET, note that a "set" in that game corresponds to a line in $\mathbb{F}_3^4$.) Define a *no-line set* as a set which doesn't contain any line.

   a) (3 points) Show that there is a no-line set in $\mathbb{F}_3^n$ of size 2^n.

   b) (7 points) Show that the largest no-line set in $\mathbb{F}_3^2$ has size 4.

2. There are rs couples at a dance. The men are divided into r groups of size s, and similarly the women are divided into r groups of size s. The groups are not necessarily related to who is a couple. Show that there is a set of r couples such that the corresponding $2r$ people belong to distinct groups.

3. Show that any bipartite graph with n vertices on both sides of the bipartition and strictly more than $(k-1)n$ edges contains a matching of size k.

4. Show that for any positive integer n, there is a nonzero multiple of n containing only the digits 3 and 0.

5. Show that any graph on $2n$ vertices with all degrees at least n has a perfect matching. (Hint: consider short augmenting paths.)