CS388C: Combinatorics and Graph Theory — Problem Set No. 3
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1. Show that every 2-coloring of the edges of K_n contains a monochromatic spanning tree.

2. Show that if G is a regular graph on n vertices, then $\omega(G) = n$ or $\omega(G) \leq n/2$. (Recall that $\omega(G)$ is the size of the maximum clique.)

3. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be any function, and S be any set of 17 integers. Show that there exist distinct $s, t, u \in S$ such that

$$f(s,t) \equiv f(t,u) \equiv f(s,u) \pmod 3.$$

Hint: first assume f is symmetric: $(\forall x, y) f(x,y) = f(y,x)$.

4. A *tournament* is a directed graph $T = (V, E)$ with no self loops such that for all $v \neq w \in V$, exactly one of (v,w) and (w,v) is in E. T is *transitive* if there is a permutation π of V such that $(v,w) \in E \iff \pi(v) < \pi(w)$. Show that there exists a tournament on n vertices which has no transitive subtournament on $\lceil 2 \log_2 n \rceil + 1$ vertices.

5. An (N, M, D, K, ϵ)-*disperser* is a bipartite graph $([N], [M], E)$ such that every node in $[N]$ has degree at most D, and for every subset $S \subseteq [N]$ of K nodes, $|\Gamma(S)| \geq (1-\epsilon)M$. Show that $(N, M, D = \lceil \log_2 N \rceil, K = M, 1/2)$-dispersers exist for all positive integers $N \geq M$.