

1. Let $c > 0$ be an arbitrary constant. Suppose that n balls are thrown randomly into n bins. Show that with probability at least $1 - n^{-c}$, every bin has $O((\log n)/\log \log n)$ balls.
2. Choose a and b randomly from \mathbb{F}_p , p prime. For $i \in \mathbb{F}_p$, set $X_i = ai + b \pmod{p}$. Show that the X_i are pairwise independent.
3. Jukna, Problem 14.23.
4. Show that the rank of the adjacency matrix A of a tournament on n vertices is at least $n - 1$.
Hint: express the definition of tournament algebraically. Show that if $Ax = 0$ and $Jx = 0$, then $x = 0$.
5. Suppose that the complete graph K_n can be expressed as the edge-disjoint union of m complete bipartite graphs. Show that $m \geq n - 1$.
Hint: $m \geq (n - 1)/2$ is easier. To double this lower bound, make the matrices sparser and apply the previous problem.