Solutions to Categorical Constructions

1. A natural transformation that is javac. That is, give me two categories C and D, and two functors F:C→D and G:C→D where a natural transformation (a D-object-to-D-object mapping nt:D→D) is javac. Hint: define D as the union of two distinct objects/domains.

**Ans:** Let C be the domain of programs and D be the union (sum) of the domains of Java programs and Java bytecodes. Functor F maps a product line of programs to a product line J of Java programs; functor G maps a product line of programs to a product line B of bytecode programs. A natural transformation that maps Java programs to their bytecodes is javac.

2. Consider the MDELite category M that computed Java programs that implemented finite state machines:

   Category N is isomorphic to M. Functor F:M→N maps M to N. Give a realistic example of F and M. That is, what is the semantics of functor F?

   **Ans:** there are quite a few possibilities, all of which are “features” (extensions). F introduces “code snippets” to each FSM transition. FSM’ is a domain of (FSM spec)×(code snippets). PrologDB’ is a domain of (PrologDB documents with code snippets). Arrow parse:FSM’→PrologDB’ produces the correct PrologDB statistics, in addition to what parse did. And so on.
3. Consider the product line $P$ below:

![Diagram of product line P]

Give an example of functors, $R:P \to X$, $S:P \to Y$, $T:P \to Z$, where $X$, $Y$, $Z$ are categories that are isomorphic to $P$ AND functors $U:X \to Y$ and $W:Y \to Z$, for a total of 5 distinct (but related) functors.

**Ans:**
- $R$ maps SPL $P$ to an SPL of FSMs.
- $S$ maps SPL $P$ to an SPL of Java programs.
- $T$ maps SPL $P$ to an SPL of bytecode programs.
- $U$ maps an SPL of FSMs to an SPL of Java programs.
- $W$ maps an SPL of Java programs to an SPL of bytecode programs.

4. Below is an abstraction $\alpha$ of a DxT derivation that maps PIM $J$ to PSM $K$. There is a corresponding graph $\beta$ that expresses the proof of correctness of this design. Recall the Pierce text. Clip out the part/section of the Pierce's text that says you can interpret $\beta$ as the proof of correctness for $\alpha$.

![Diagram of derivation]

15 Example  By a twist of perspective, we can call the objects in an arbitrary category *formulas* and the arrows *proofs*. An arrow $f : A \to B$ is viewed as a proof of the implication $A \to B$. In particular, the identity arrow $\text{id}_A : A \to A$ is an instance of the reflexivity axiom, and the composition of arrows

$$f : A \to B \quad g : B \to C$$

$$g \circ f : A \to C$$

is a rule of inference asserting the transitivity of implication.