

Solution

The first posted set of axioms was in fact incorrect. + Identity was listed as $(A+0=0)$, which clearly is a problem. I will accept submissions with this axiom. However, the correct + identity axiom is listed below. A (small) challenge awaits you to demonstrate this (updated) set of axioms is still wrong.

Given axioms:

+ identity	$A + 0 = A$	# inert	$A\#0 = 0$
+ commute	$A + B = B + A$	# commute	$A\#B = B\#A$
+ associate	$A + (B + C) = (A + B) + C$	# associate	$A\#(B\#C) = (A\#B)\#C$
× product	$A \times B = A\#B + A + B$	distributivity	$A\#(B + C) = A\#B + A\#C$
+ idempotence	$A + A = A$	# involution	$A\#A = 0$

And the fact: $A \leq A + B$ for all A and B

1. Show there is an inconsistency using any number of the above axioms.

Proof.

$A \leq A + B$	given
$(A + B)\#A \leq (A + B)\#(A + B)$	Apply (A+B)# to both sides
$(A + B)\#A \leq 0$	By # involution
$A\#B + A\#A \leq 0$	By # distributing over +
$A\#B + 0 \leq 0$	By # involution
$A\#B \leq 0$	By + identity

This means that, basically, the interaction of any two features is = 0, an absurdity or inconsistency.

2. Show that $A \leq A + B$ for all A and B is inconsistent in universes where $A+A=0$

Proof.

$A \leq A + B$	given
$A \leq A + A$	Set A = B
$A \leq 0$	By $A+A=0$ axiom

This means that all features are 0, an absurdity or inconsistency.