1. Use extra paper to determine your solutions then neatly transcribe them (including intermediate steps) onto these sheets.
2. It’s possible that you won’t be able to finish. Read through the whole exam once and start working on the problems you’re sure you know how to do. Come back to the harder ones as you have time.

(1) Consider the following problem: Given a database $D$ and a query $Q$, what result is returned when $Q$ is executed against $D$?

(2) Let $L = \{w \in \{a, b\}^* : w$ does not end in $ab\}$
(a) Show a regular expression that generates $L$.

(b) Show an FSM that accepts $L$.

(3) Show a (possibly nondeterministic) FSM that accepts $\{w \in \{a, b\}^* : w$ contains at least one instance of $aaba, bbb$ or $ababa\}$.

(4) For each of the following languages $L$, state whether it is regular or not and prove your answer.
(a) $\{x#y : x, y \in \{0, 1\}^* \text{ when viewed as binary numbers, } x+y = 3y\}$. Example: $1000#100 \in L$.

(b) Let $\Sigma = \{a, b\}$. $L = \{w \in \Sigma^* : (w$ contains the substring $ab) \rightarrow (w$ contains the substring $ba)\}$

(c) $\{w = xyz : x, y, z \in \{0, 1\}^+\}$.

(d) $\{w = st : s \in \{a, b\}^* \text{ and } t \in \{b, c\}^* \text{ and } \#_a(s) = 2 \cdot \#_a(s) \text{ and } \#_c(t) = 3 \cdot \#_c(t)\}$. 

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<th>No.</th>
<th>Symbol Grade</th>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>a 5, b 5, c 5</td>
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(5) Recall that maxstring(L) = \{w: w \in L \text{ and } \forall z \in \Sigma^* (z \neq \varepsilon \rightarrow wz \notin L)\}.

(a) What is maxstring(L_1 L_2), where L_1 = \{w \in \{a, b\}^* : \text{contains exactly one } a\} \text{ and } L_2 = \{a\}? 

(b) Prove that the regular languages are closed under maxstring.

(c) If maxstring(L) is regular, must L also be regular? Prove your answer.

(6) Consider the following NDFSM M. Use ndfsmtodfsm to construct an equivalent DFSM. Begin by showing the value of \text{eps}(q) for each state q:

\[ \begin{array}{c}
1 \quad \varepsilon, a \quad \delta \\
2 \quad b \\
3 \quad b \\
4 \quad a \\
5 \\
\end{array} \]

(7) Define a decision procedure to answer the following question. You may use as subroutines all the procedures that we have discussed in class. Let \(\Sigma = \{a, b\}\) and let \(\alpha\) and \(\beta\) be regular expressions. Is the following sentence true:

\((L(\beta) = a^*) \lor (\forall w (w \in \{a, b\}^* \wedge |w| \text{ even}) \rightarrow w \in L(\alpha))\)

(8) Prove or disprove each of the following statements:

(a) It is possible that the intersection of an infinite number of regular languages is not regular.

(b) Every subset of a regular language is regular.

(c) Let \(L_4 = L_1 L_2 L_3\). If \(L_1\) and \(L_2\) are regular and \(L_3\) is not regular, it is possible that \(L_4\) is regular.