Name:

1 a	15	
b	15	
c	15	
d	15	
2	15	
3 a	8	
b	8	
c	8	
4 a	5	
b	15	
5 a	8	
b	8	
6	20	
Total	155	

CS 341
Second Midterm Exam
Practice

Use extra paper to determine your solutions then neatly transcribe them onto these sheets.

You may use any claim we proved in class as a theorem. But make sure that you only use such a claim when it is exactly what you need. If we have proved something "close" in class, then you must do a complete proof here, but you can use the proof we did in class as a model for your proof.

(1) For each of the following languages L, state whether it is regular, context-free but not regular, or neither. Prove your answer. Make sure, if you say that a language is context free, that you show that it is not also regular.

(a) $\{w \in \{0, 1\}^* : \exists k \geq 0$ and w is a binary encoding (leading zeros allowed) of $2^k+1\}$.

(b) $\{a^*b^*c^* - \{a^n b^n c^n : n \geq 0\}\}$.

(c) $\{(ab)^n a^n b^n : n > 0\}$.

(d) $\{x \in \{a, b\}^* : |x|$ is even and the first half of x has one more a than does the second half$\}$.

(2) Give a decision procedure to answer the question, "Given a context-free grammar G, does G generate at least three strings?"

(3) Let $L = \{w \in \{a, b\}^* :$ the first, middle, and last characters of w are identical$\}$.
- **(a)** Show a context-free grammar that generates L.
- **(b)** Show a natural PDA that accepts L.
- **(c)** Prove that L is not regular.

(4) Let *middle* be a function that maps from any language L over some alphabet Σ to a new language L' as follows:

$$middle(L) = \{x: \exists y, z \in \Sigma^* (yxz \in L)\}.$$

(a) Let $L = \{w \in \{a, b\}^* : \#_a(w) = \#_b(w)\}$. What is $middle(L)$?

(b) Prove that, for any language L, if L is context free then $M(L)$ is context free.

(5) Provide short answers to each of the following questions:

(a) Let $L_1 = L_2 \cap L_3$.

(i) Show values for L_1, L_2, and L_3, such that L_1 is context-free but neither L_2 nor L_3 is.

(ii) Show values for L_1, L_2, and L_3, such that L_2 is context-free but neither L_1 nor L_3 is.

(b) Give an example of a regular language L_1 that has a superset L_2 that is context-free but not regular. (Specify both L_1 and L_2 in your example.)

(6) Consider the following grammar G:

$$S \rightarrow \mathtt{1}\ S\ \mathtt{1} \mid T$$
$$T \rightarrow \mathtt{1}\ X\ \mathtt{1} \mid X$$
$$X \rightarrow \mathtt{0}\ X\ \mathtt{0} \mid \mathtt{1}$$

(a) What are the first four strings in the lexicographic enumeration of $L(G)$?

(b) Give an example of a string $w \in \{\mathtt{0}, \mathtt{1}\}^+$ such that $|w| > 7$ and $w \notin L(G)$.

(c) Show that G is ambiguous.

(7) Give a short English description of what the following machine M does. We are looking for a high-level description of the result of running M, not a play-by-play description of how it works. Assume that the input to M is in $\mathtt{1}\ (\mathtt{0} \cup \mathtt{1})^*$. M =