

Viewing and Projections

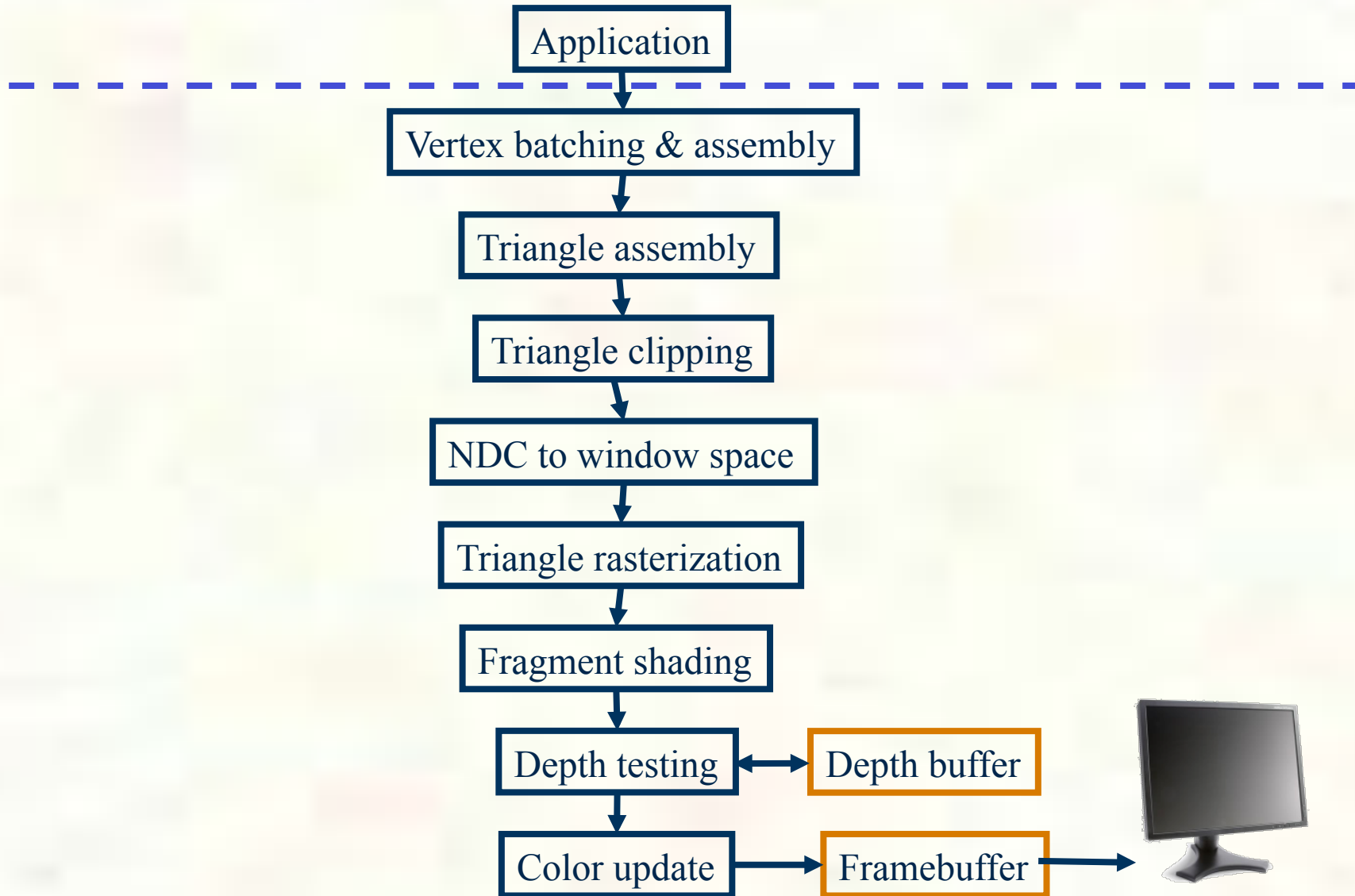
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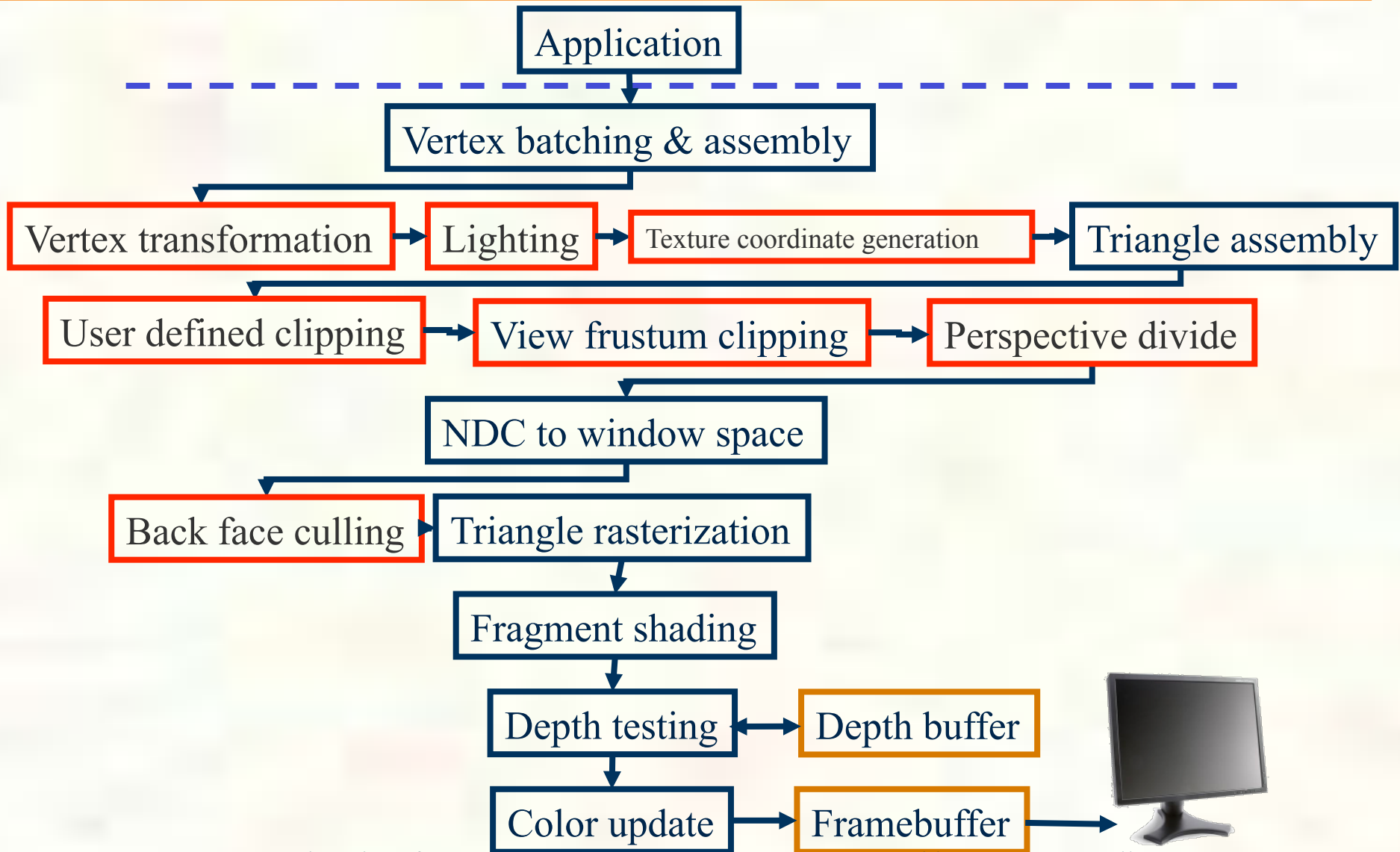


A Simplified Graphics Pipeline



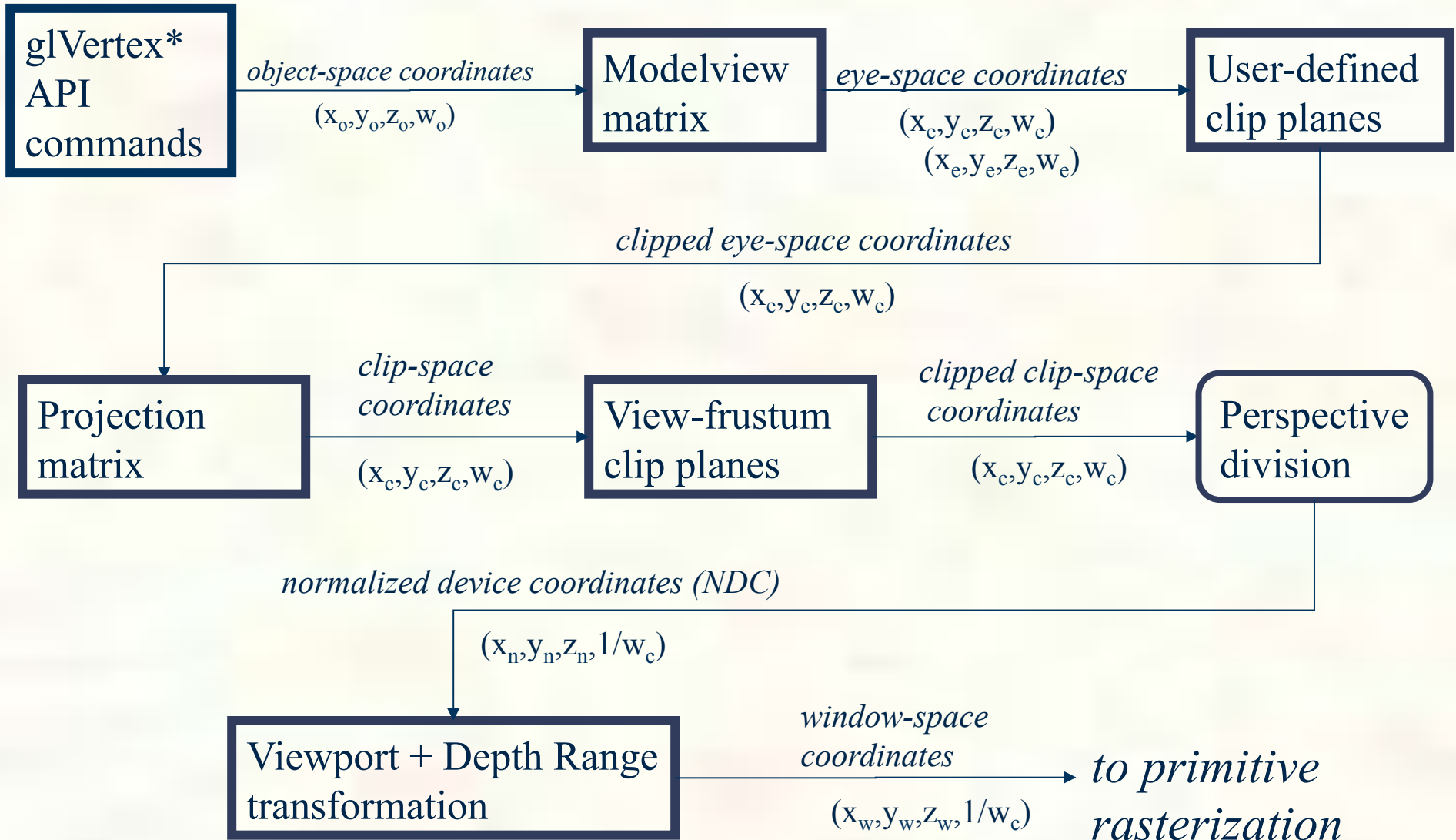


A few more steps expanded



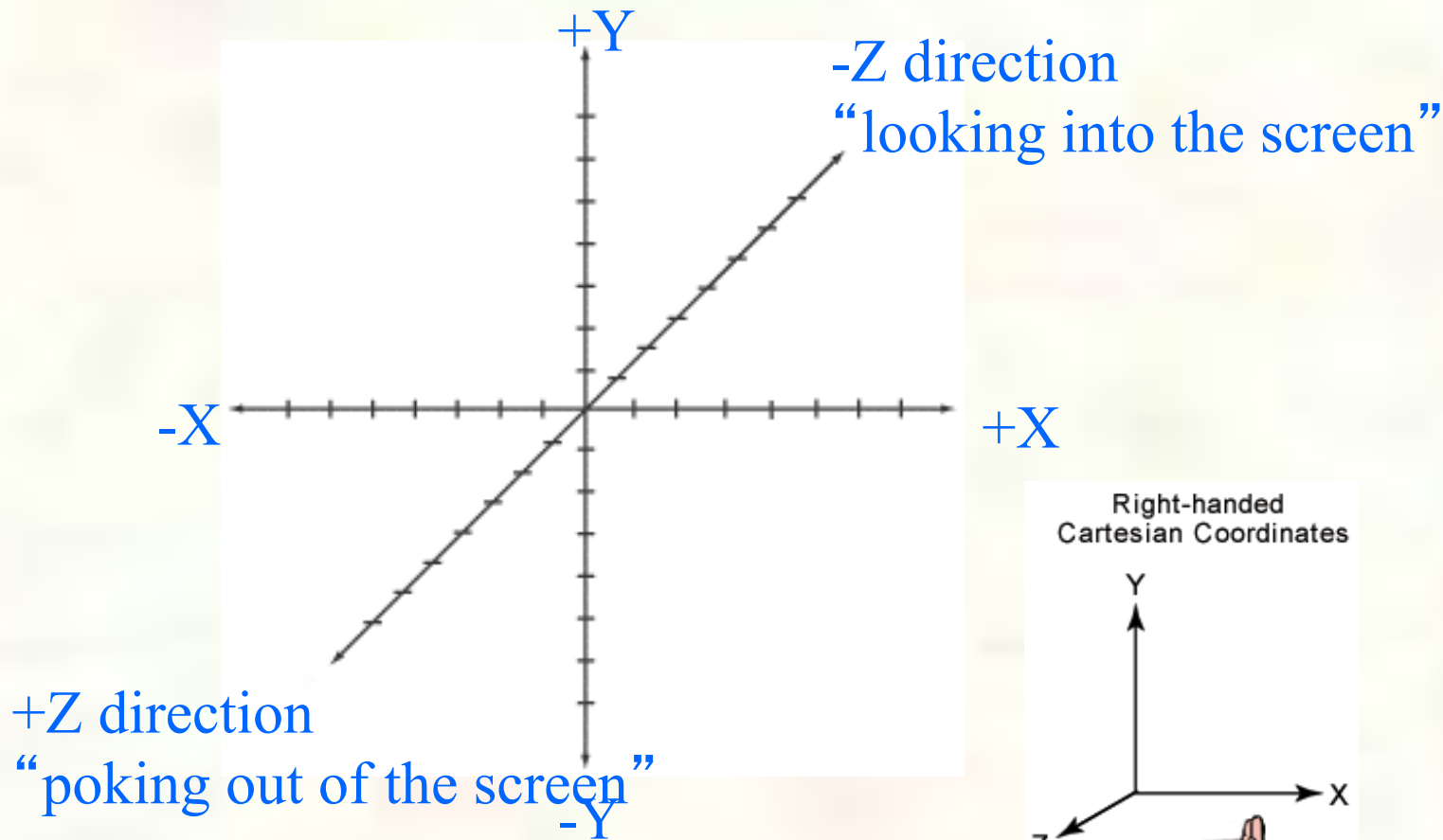


Conceptual Vertex Transformation

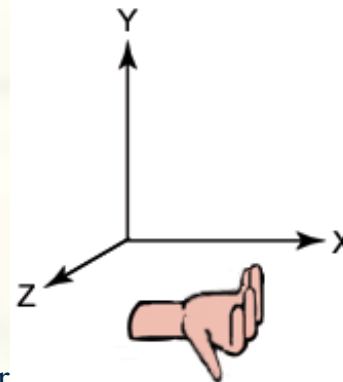




Eye Coordinates (not NDC)



Right-handed Cartesian Coordinates



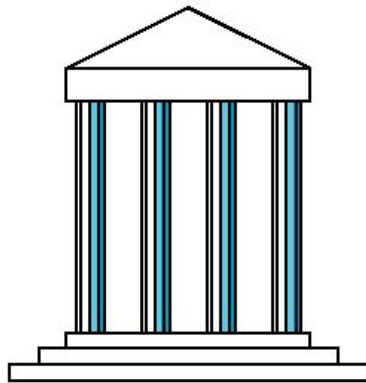


Planar Geometric Projections

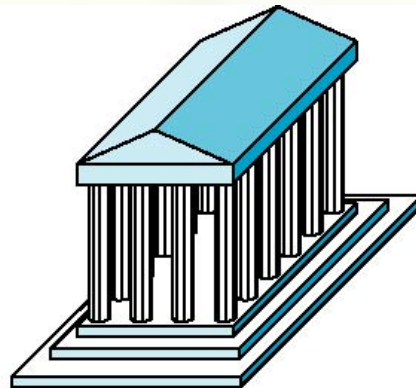
- Standard projections project onto a plane
- Projectors are lines that either
 - converge at a center of projection
 - are parallel
- Such projections preserve lines
 - but not necessarily angles
- Nonplanar projections are needed for applications such as map construction



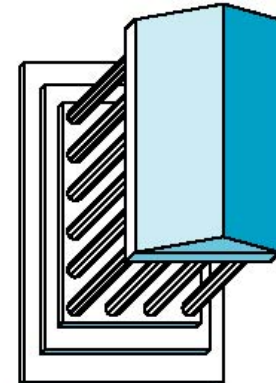
Classical Projections



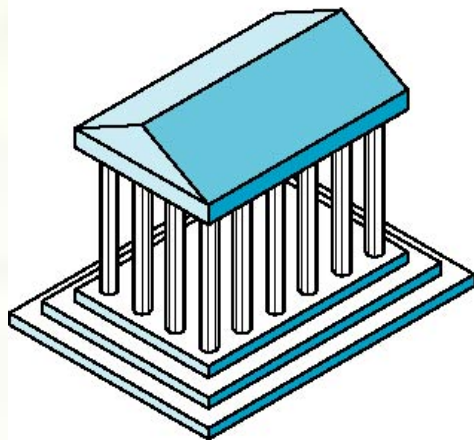
Front elevation



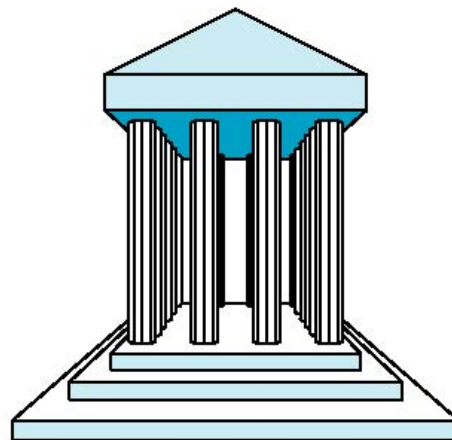
Elevation oblique



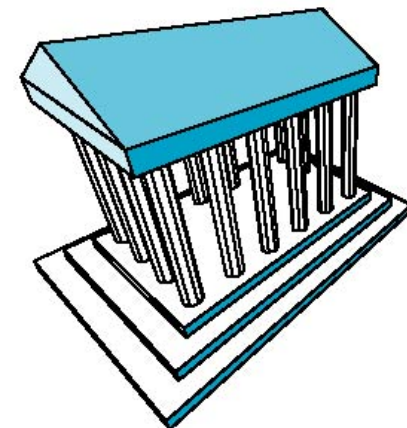
Plan oblique



Isometric



One-point perspective



Three-point perspective

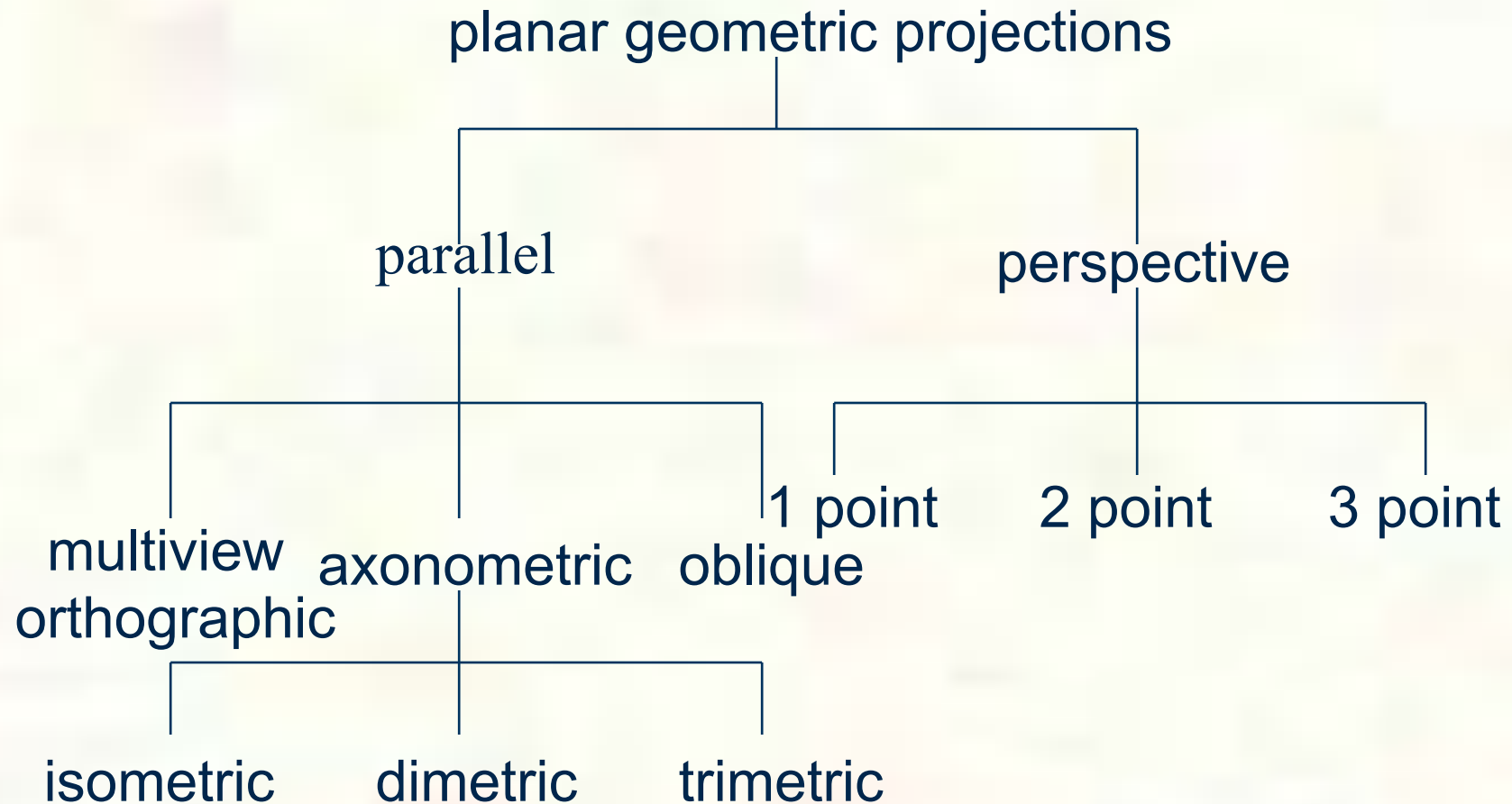


Perspective vs Parallel

- Computer graphics treats all projections the same and implements them with a single pipeline
- Classical viewing developed different techniques for drawing each type of projection
- Fundamental distinction is between parallel and perspective viewing even though mathematically parallel viewing is the limit of perspective viewing

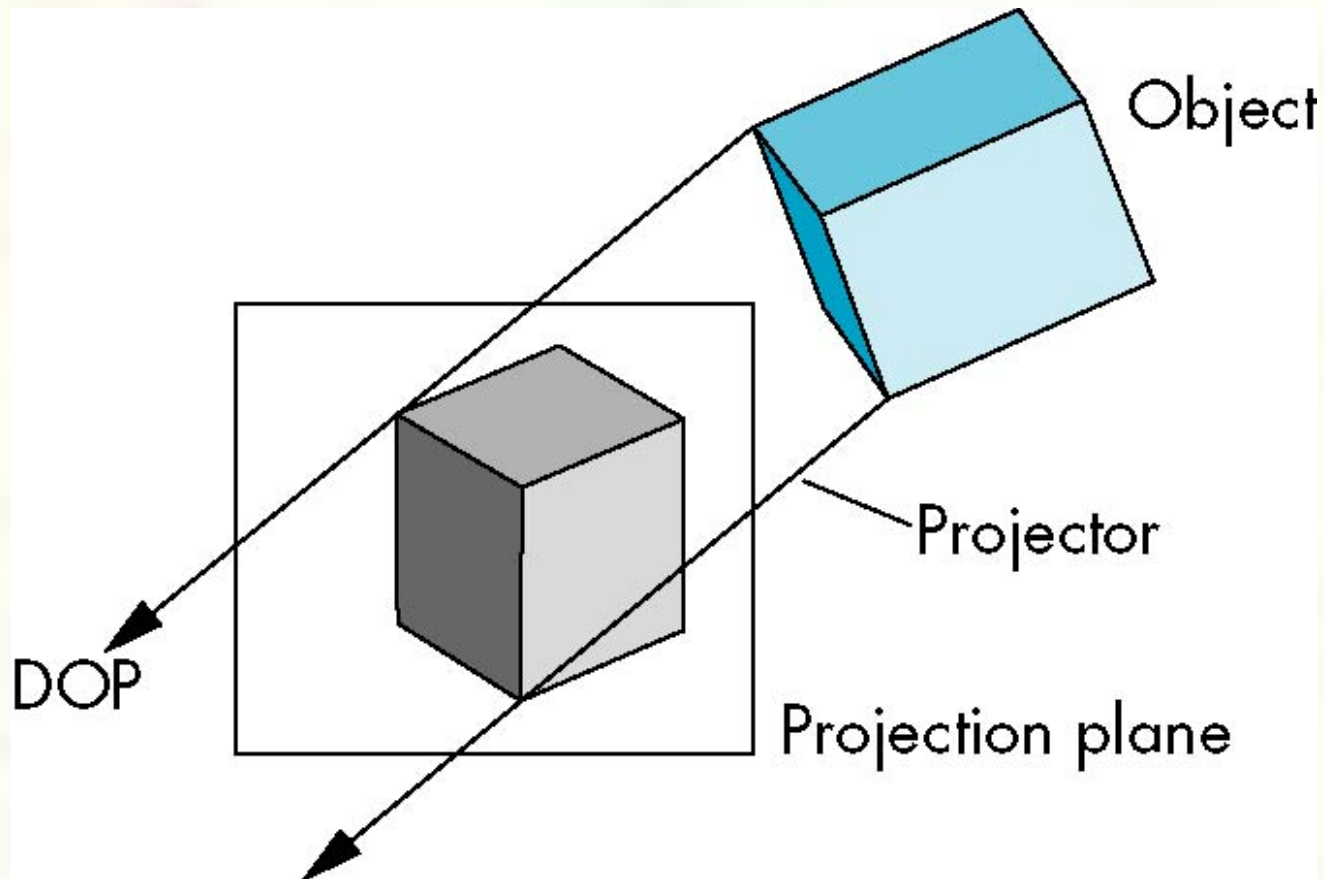


Taxonomy of Projections



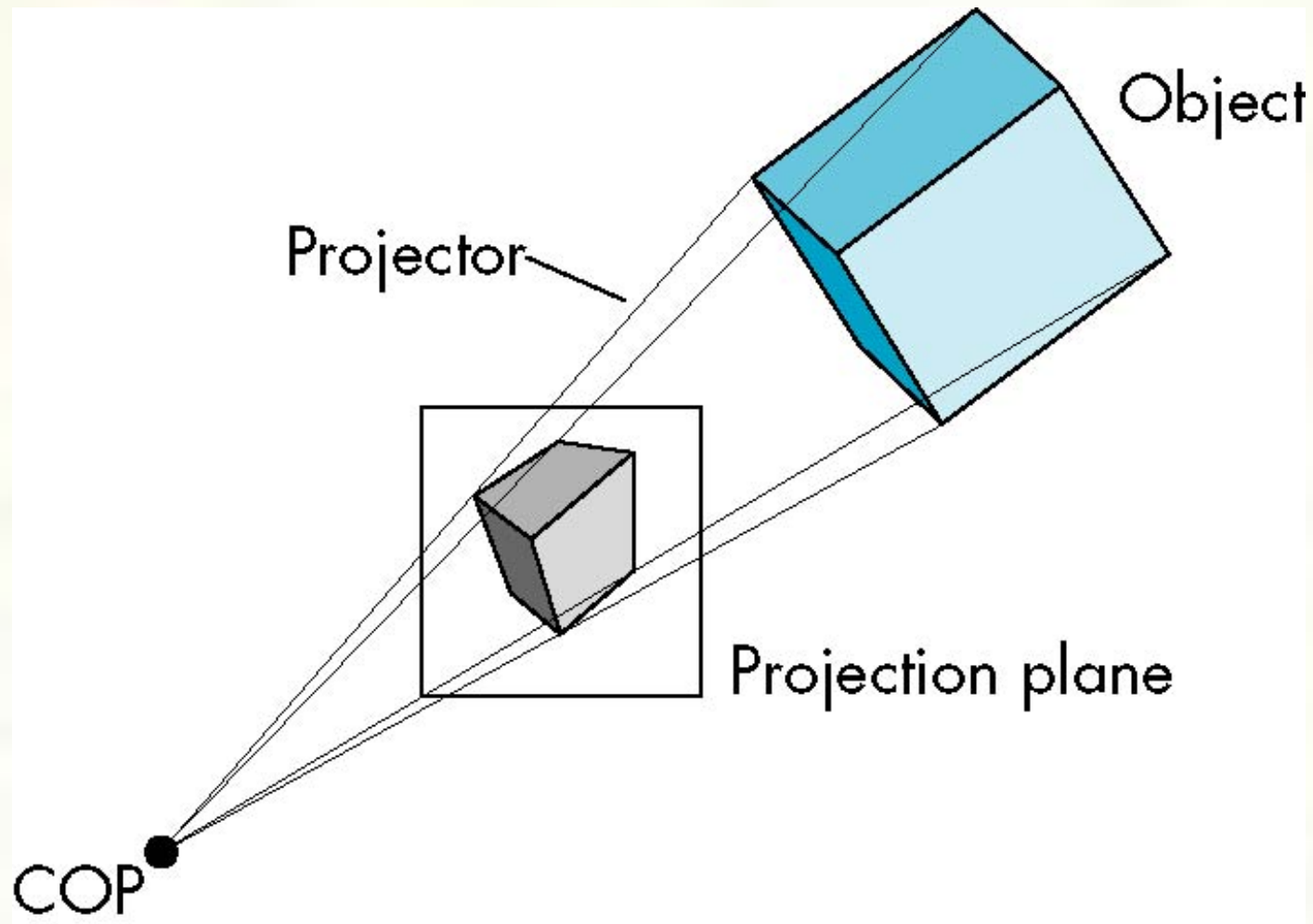


Parallel Projection





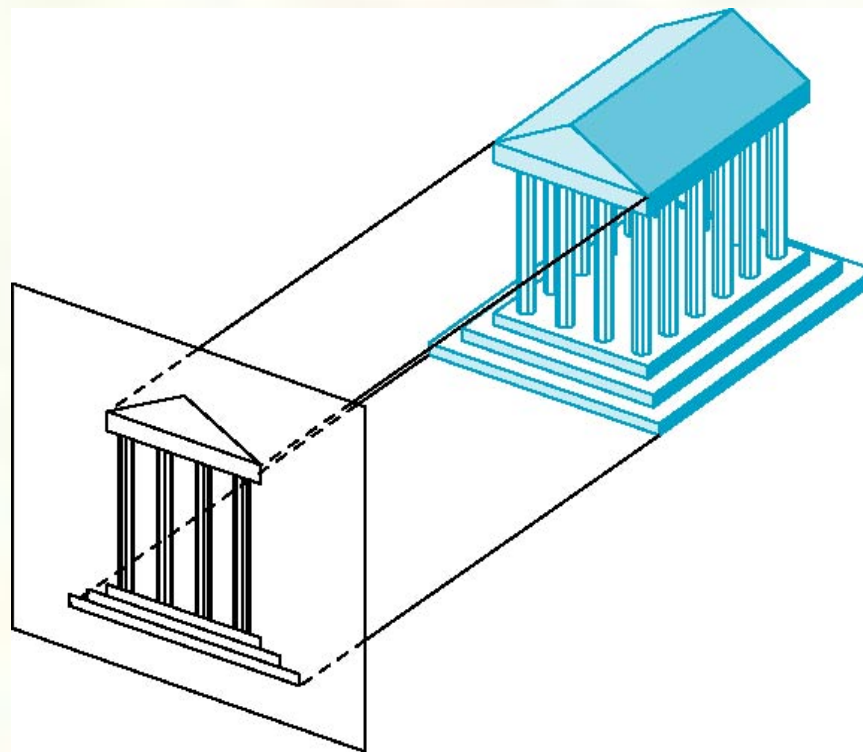
Perspective Projection





Orthographic Projection

Projectors are orthogonal to projection surface





Multiview Orthographic Projection

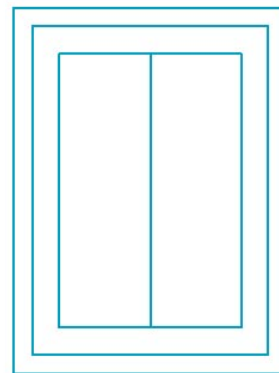
- Projection plane parallel to principal face
- Usually form front, top, side views

isometric (not multiview orthographic view)

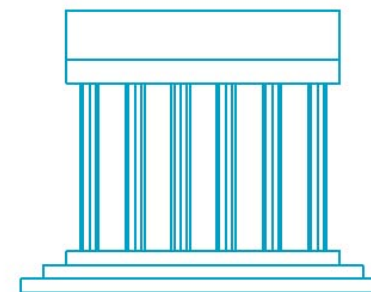


front

in CAD and architecture,
we often display three
multiviews plus isometric



top



side



Advantages and Disadvantages

- Preserves both distances and angles
 - Shapes preserved
 - Can be used for measurements
 - Building plans
 - Manuals
- Cannot see what object really looks like because many surfaces hidden from view
 - Often we add the isometric



Projections and Normalization

- The default projection in the eye (camera) frame is orthogonal
- For points within the default view volume

$$x_p = x$$

$$y_p = y$$

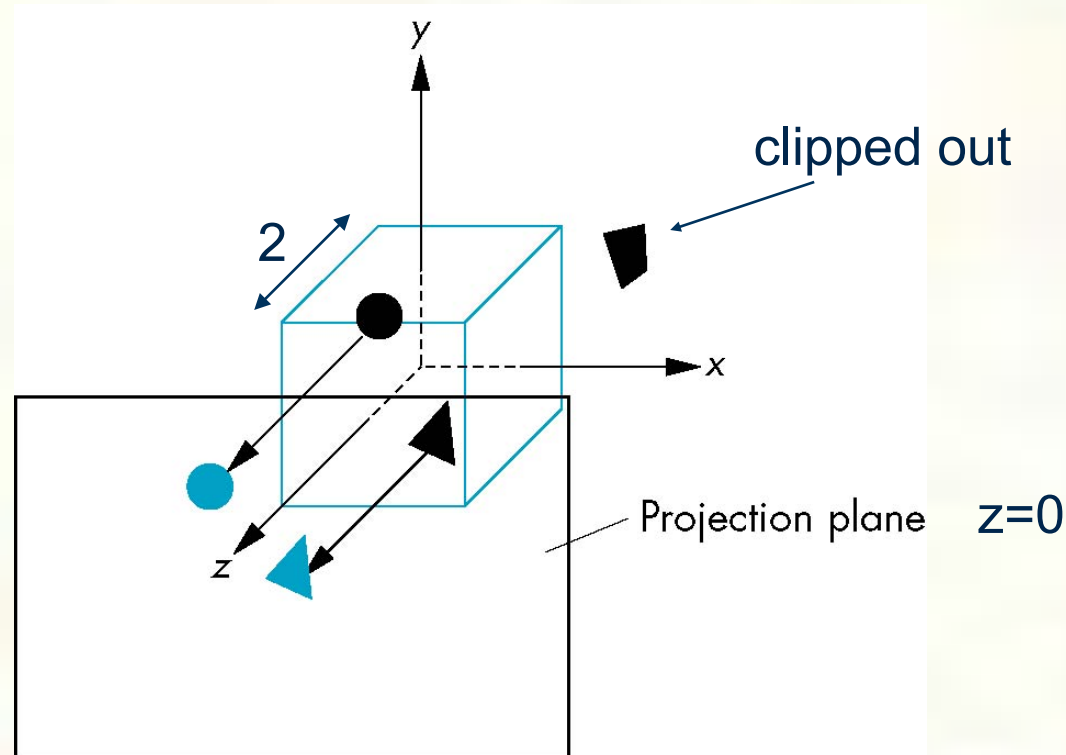
$$z_p = 0$$

- Most graphics systems use *view normalization*
 - All other views are converted to the default view by transformations that determine the projection matrix
 - Allows use of the same pipeline for all views



Default Projection

Default projection is orthographic

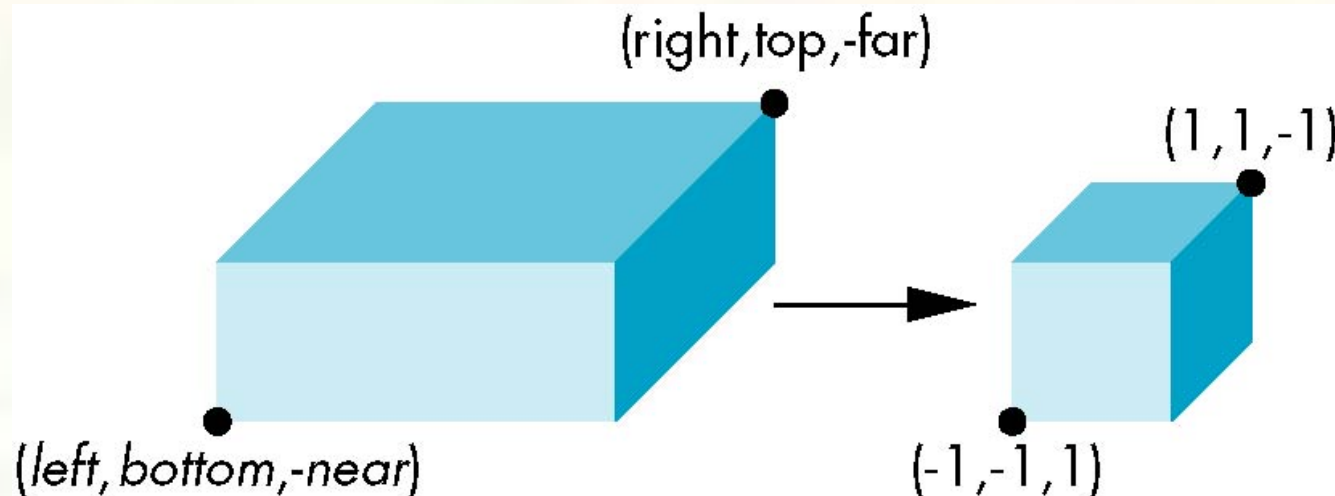




Orthogonal Normalization

`glOrtho (left, right, bottom, top, near, far)`

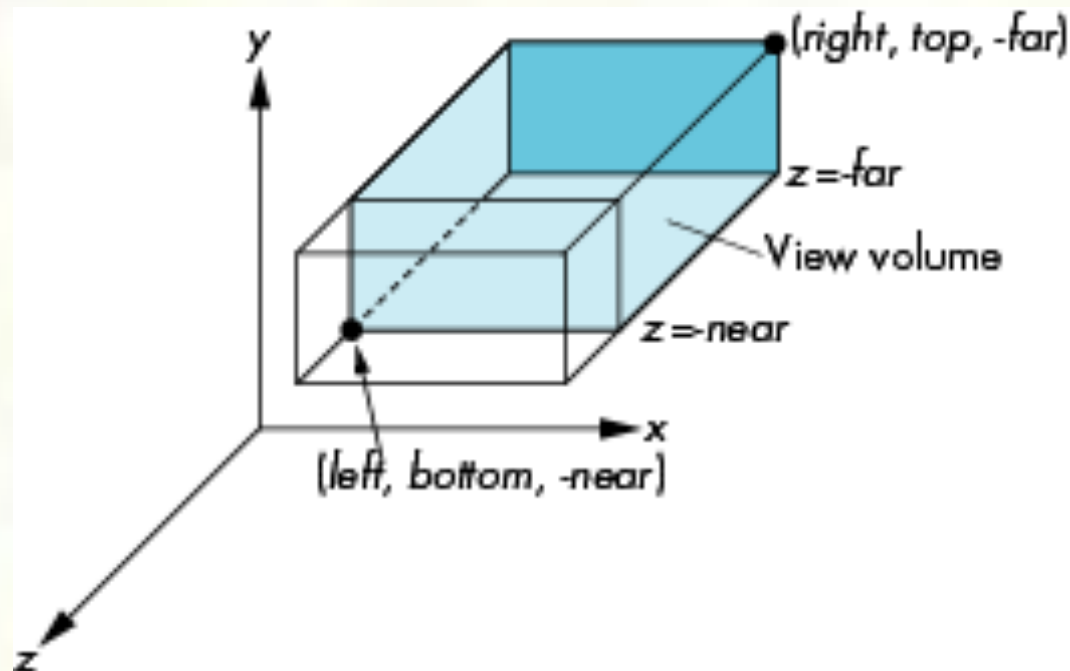
normalization \Rightarrow find transformation to convert specified clipping volume to default





OpenGL Orthogonal Viewing

`glOrtho(left, right, bottom, top, near, far)`



`near` and `far` measured from camera



Homogeneous Representation

default orthographic projection

$$\begin{aligned}x_p &= x \\y_p &= y \\z_p &= 0 \\w_p &= 1\end{aligned}\quad \mathbf{p}_p = \mathbf{M}\mathbf{p}$$
$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In practice, we can let $\mathbf{M} = \mathbf{I}$ and set the z term to zero later



Orthographic Eye to NDC

- Two steps

- Move center to origin

$$T(-(\text{left}+\text{right})/2, -(\text{bottom}+\text{top})/2, -(\text{near}+\text{far})/2))$$

- Scale to have sides of length 2

$$S(2/(\text{left}-\text{right}), 2/(\text{top}-\text{bottom}), 2/(\text{near}-\text{far}))$$

$$\mathbf{P} = \mathbf{ST} = \begin{bmatrix} \frac{2}{\text{right} - \text{left}} & 0 & 0 & -\frac{\text{right} + \text{left}}{\text{right} - \text{left}} \\ 0 & \frac{2}{\text{top} - \text{bottom}} & 0 & -\frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} \\ 0 & 0 & \frac{2}{\text{near} - \text{far}} & -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



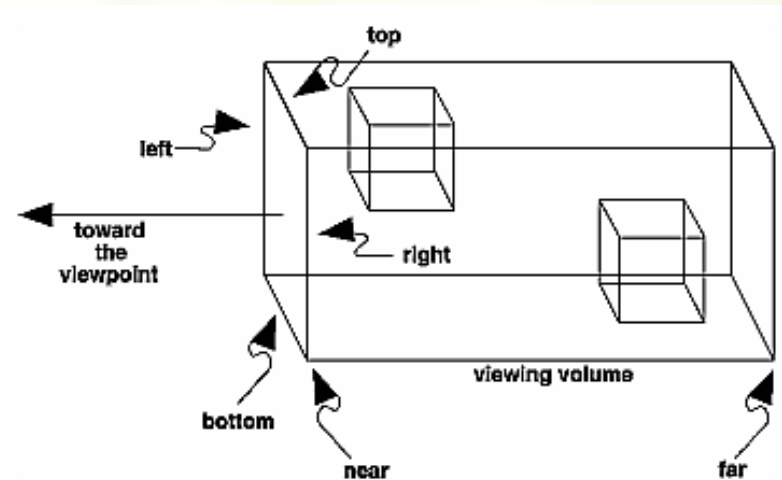
Orthographic Transform

- Prototype

- `glOrtho`(GLdouble left, GLdouble right, GLdouble bottom, GLdouble top, GLdouble near, GLdouble far)

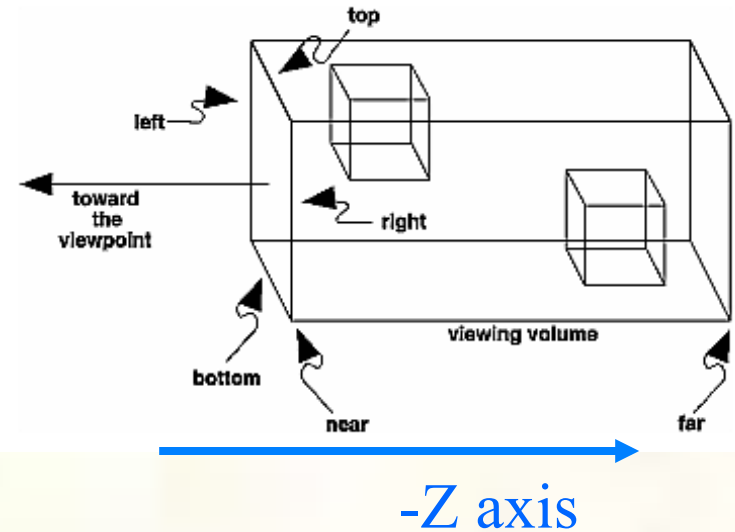
- Post-concatenates an orthographic matrix

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





glOrtho Example



■ Consider

■ `glLoadIdentity();`

`glOrtho(-20, 30, 10, 60, 15, -25)`

■ left=-20, right=30, bottom=10, top=50, near=15, far=-25

■ Matrix

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{25} & 0 & 0 & -\frac{1}{5} \\ 0 & \frac{1}{20} & 0 & -\frac{3}{2} \\ 0 & 0 & \frac{1}{20} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

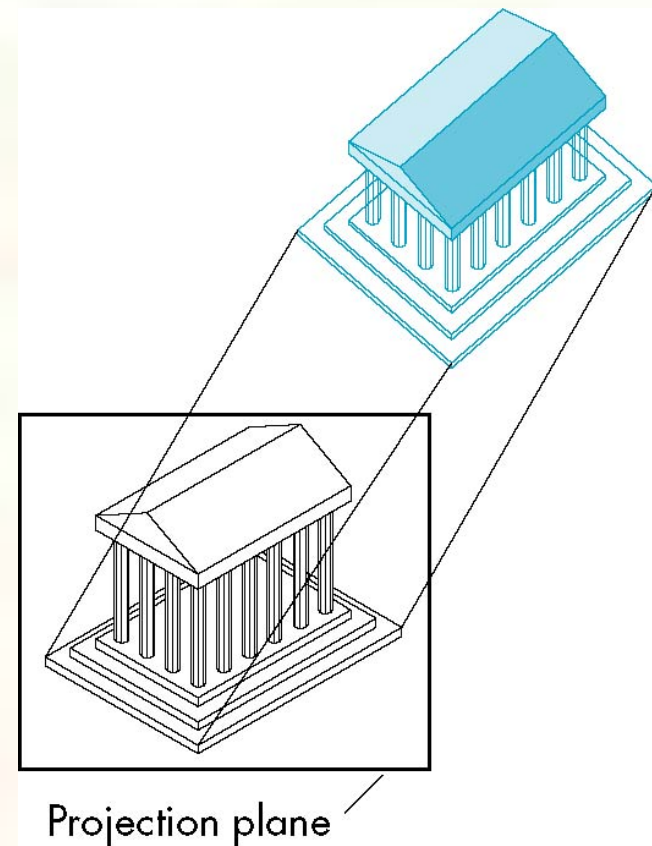
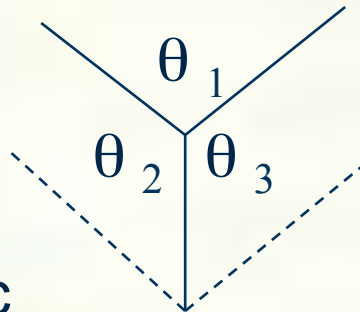


Axonometric Projections

Allow projection plane to move relative to object

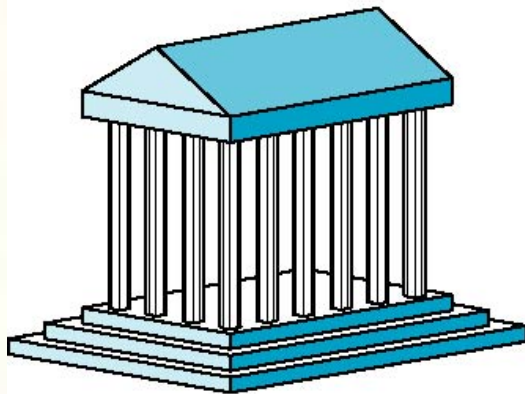
classify by how many angles of a corner of a projected cube are the same

none: trimetric
two: dimetric
three: isometric

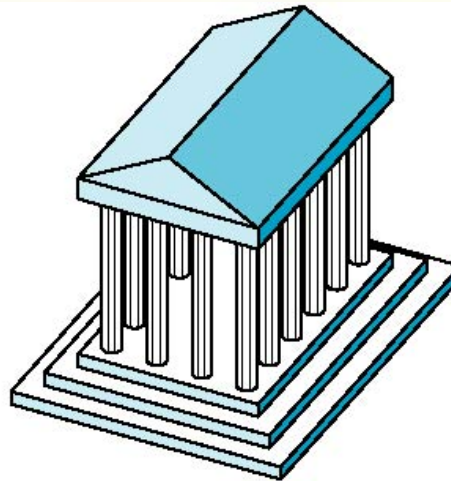




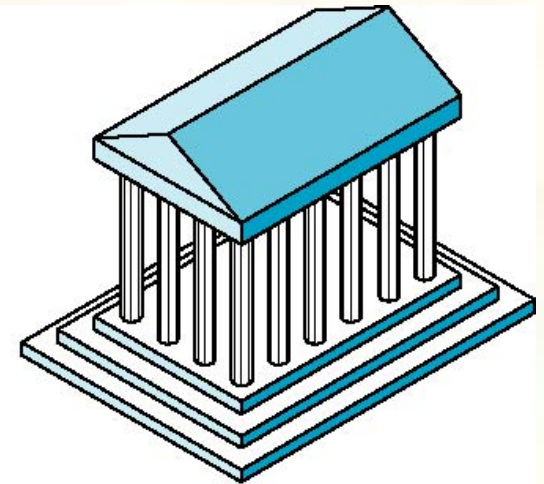
Types of Axonometric Projections



Dimetric



Trimetric



Isometric



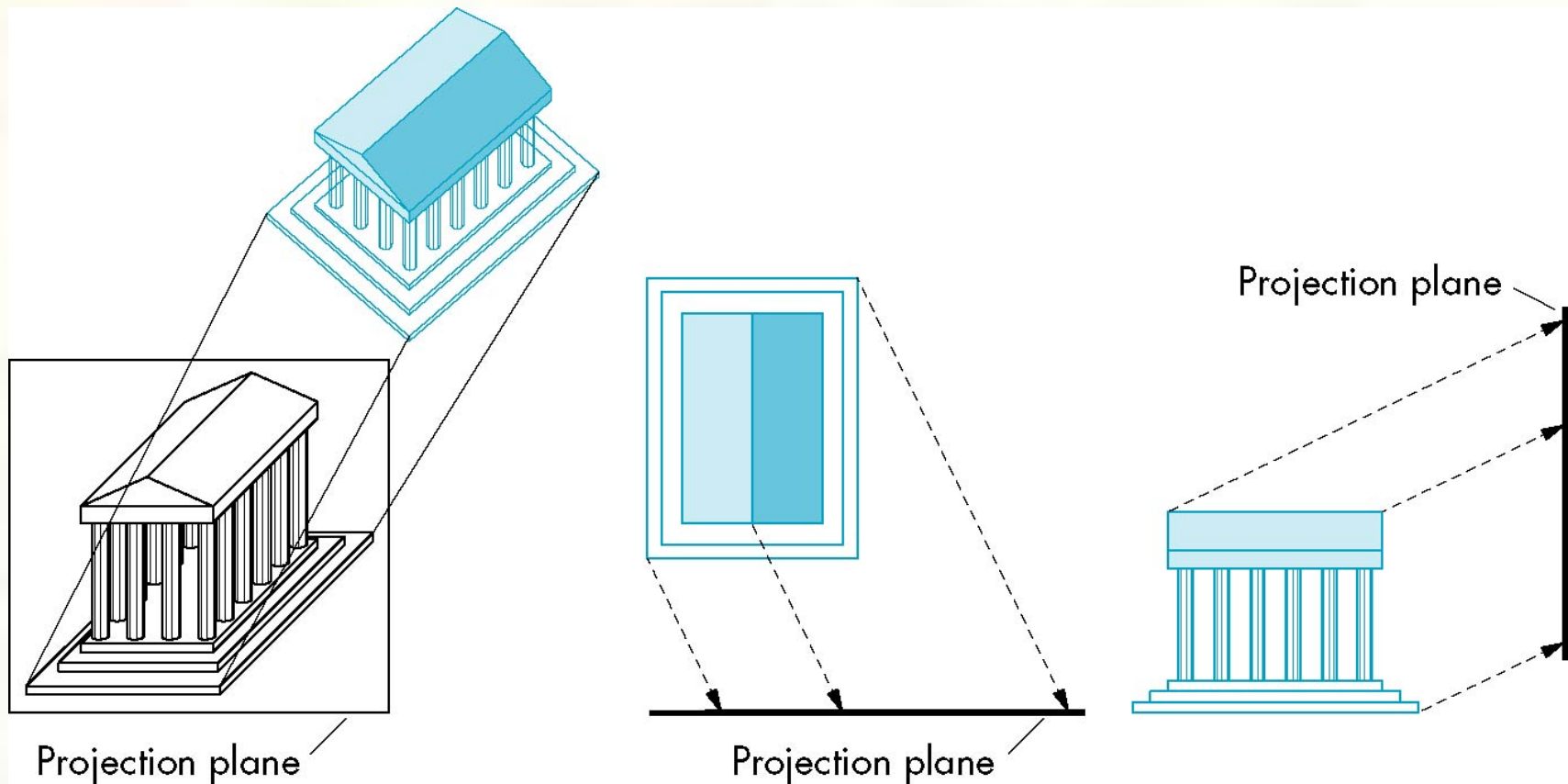
Advantages and Disadvantages

- Lines are scaled (*foreshortened*) but can find scaling factors
- Lines preserved but angles are not
 - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- Can see three principal faces of a box-like object
- Some optical illusions possible
 - Parallel lines appear to diverge
- Does not look real because far objects are scaled the same as near objects
- Used in CAD applications



Oblique Projection

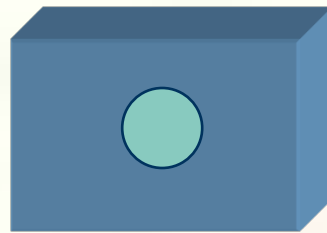
Arbitrary relationship between projectors and projection plane





Advantages and Disadvantages

- Can pick the angles to emphasize a particular face
 - Architecture: plan oblique, elevation oblique
- Angles in faces parallel to projection plane are preserved while we can still see “around” side

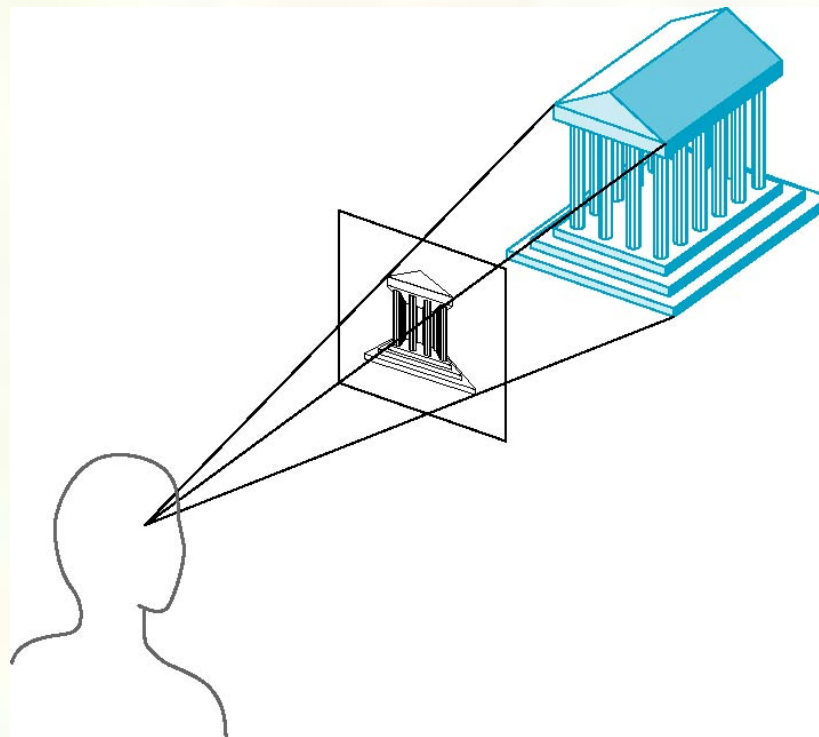


- In physical world, cannot create with simple camera; possible with bellows camera or special lens (architectural)



Perspective Projection

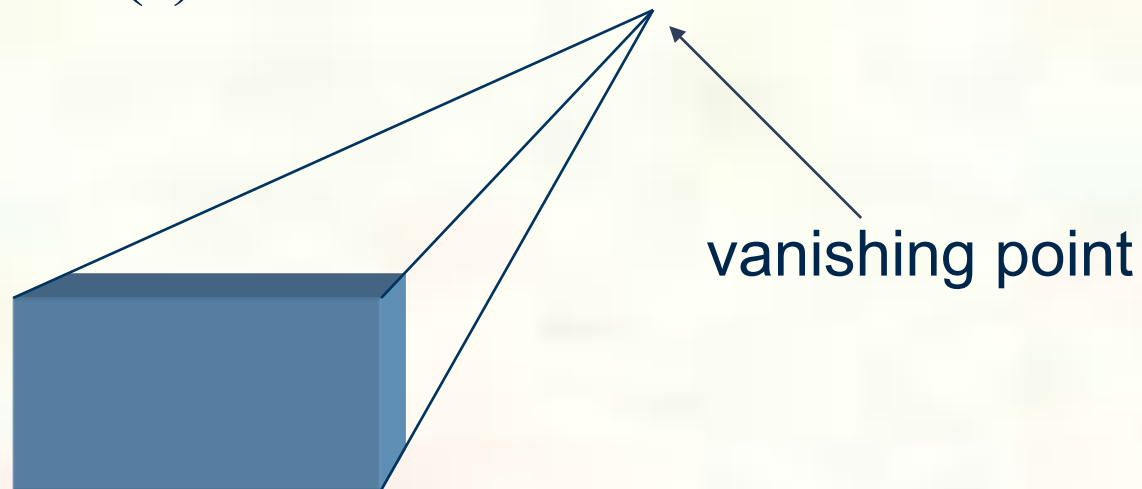
Projectors converge at center of projection





Vanishing Points

- Parallel lines (not parallel to the projection plan) on the object converge at a single point in the projection (the *vanishing point*)
- Drawing simple perspectives by hand uses these vanishing point(s)





Three-Point Perspective

- No principal face parallel to projection plane
- Three vanishing points for cube





Two-Point Perspective

- On principal direction parallel to projection plane
- Two vanishing points for cube





One-Point Perspective

- One principal face parallel to projection plane
- One vanishing point for cube



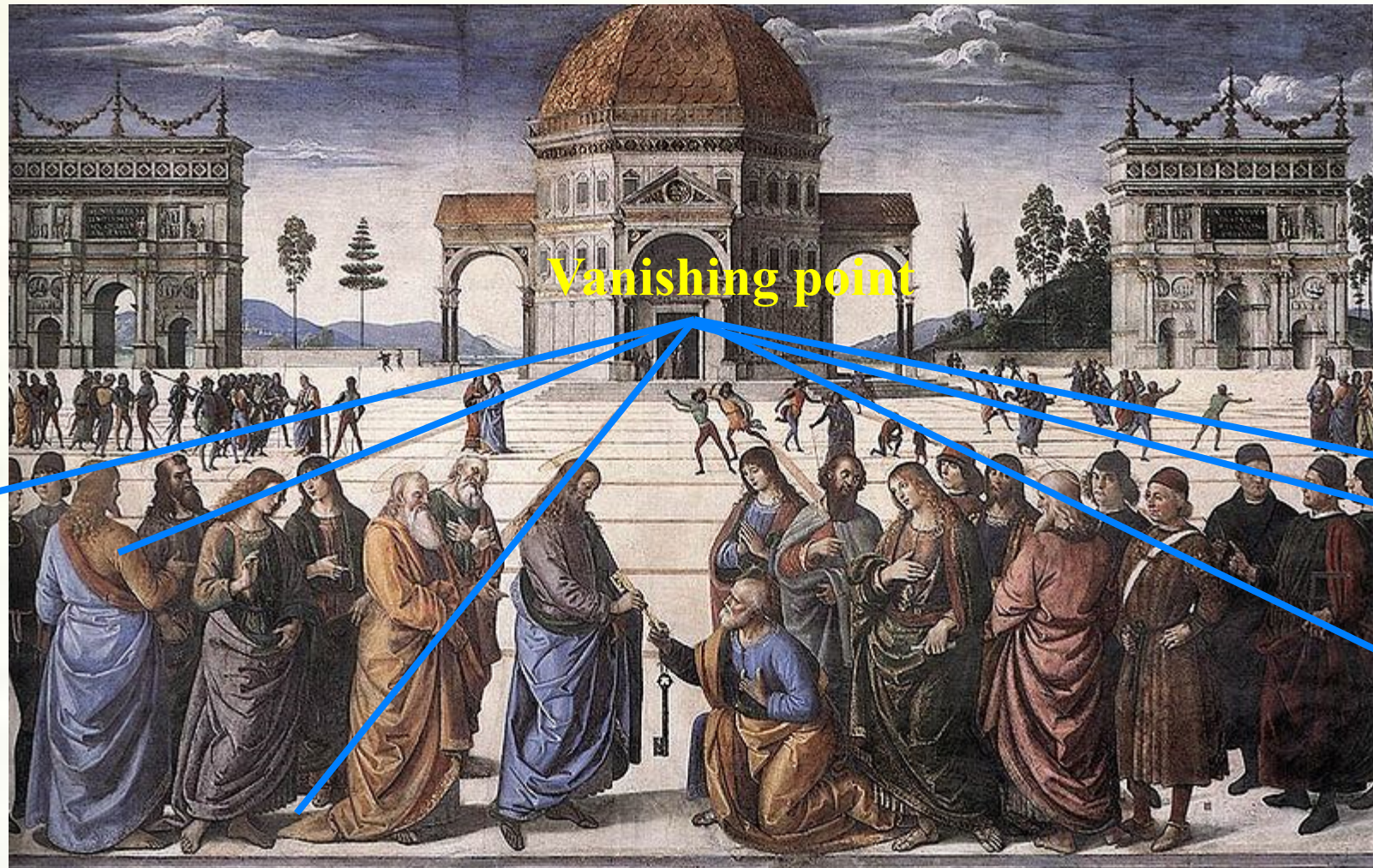


Perspective in Art History



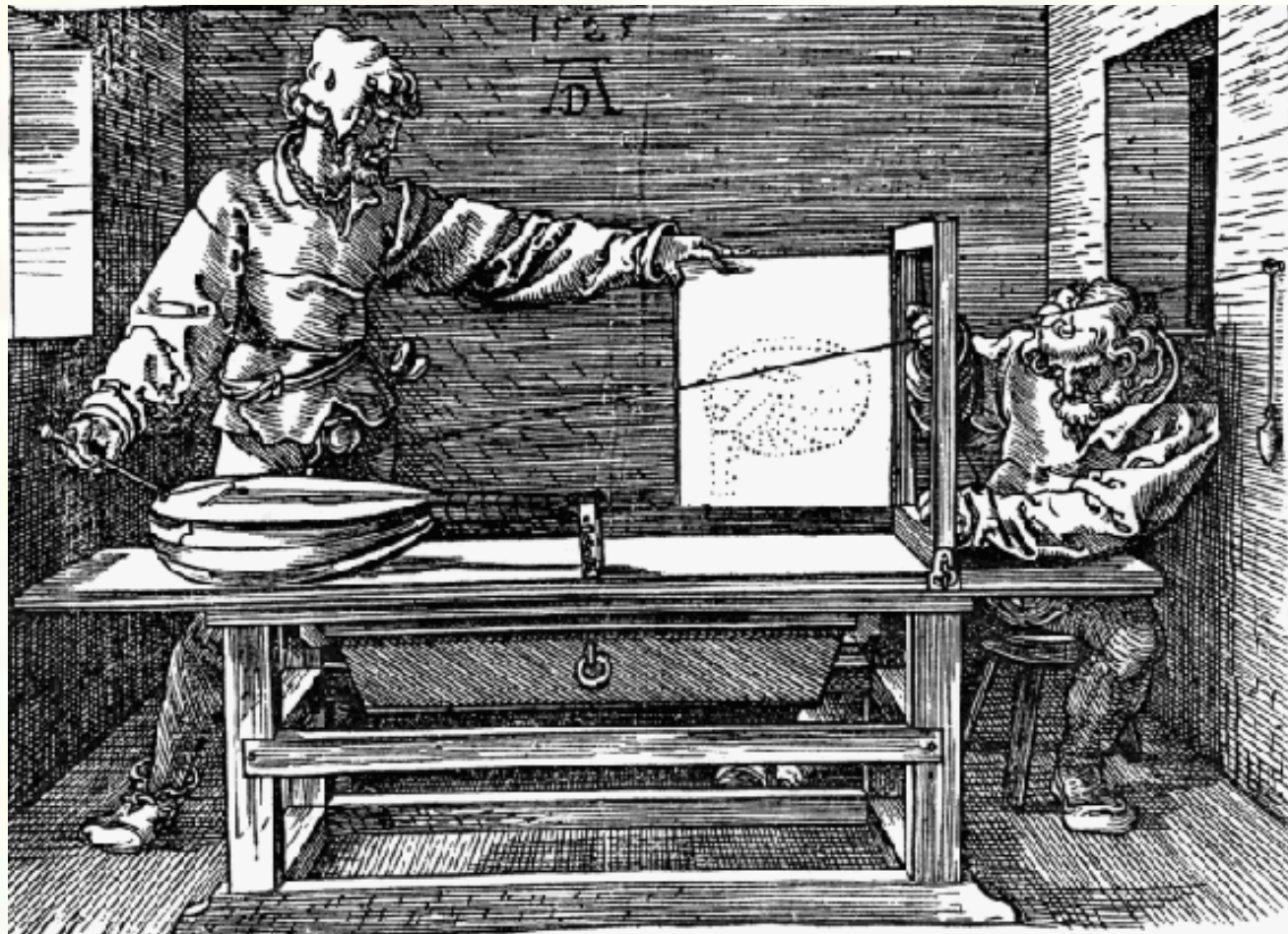


Perspective in Art History





Humanist Analysis of Perspective



[Albrecht Dürer, 1471]



Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (*diminution*)
 - Looks realistic
- Equal distances along a line are not projected into equal distances (*nonuniform foreshortening*)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand than parallel projections (but not more difficult by computer)

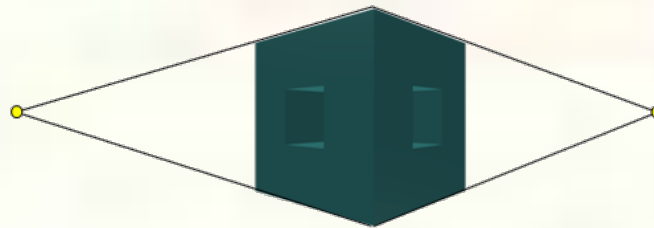


1-, 2-, and 3-point Perspective

- A 4x4 matrix can represent 1, 2, or 3 vanishing points
 - As well as zero for orthographic views



1-point perspective



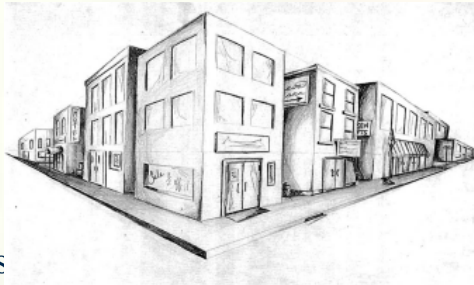
2-point perspective



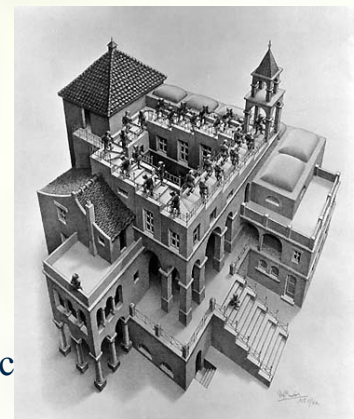
3-point perspective



of Texas



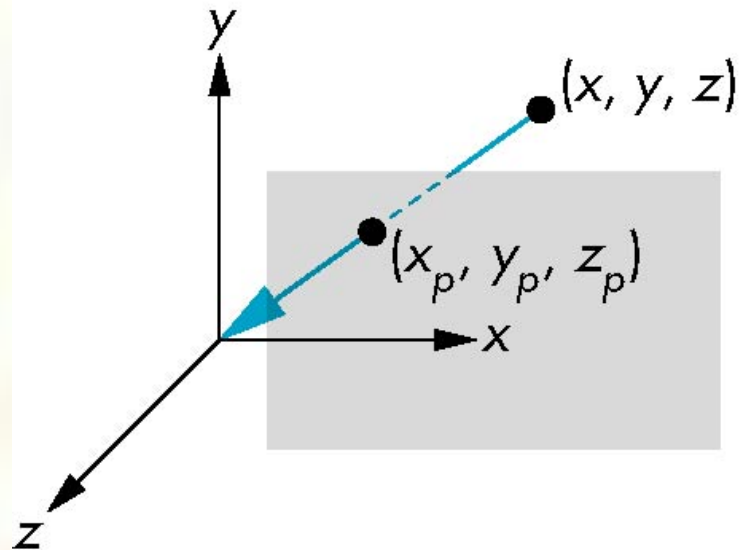
r Graphic





Simple Perspective

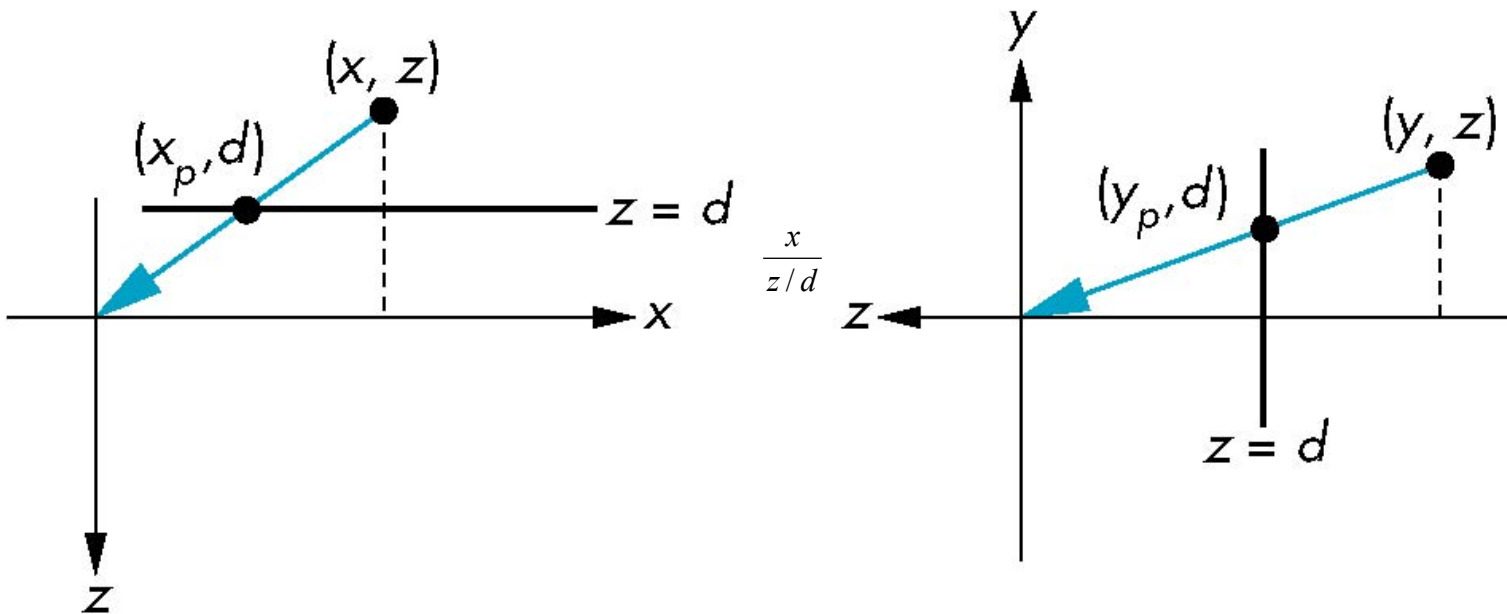
- Center of projection at the origin
- Projection plane $z = d, d < 0$





Perspective Equations

Consider top and side views



$$x_p = \frac{x}{z/d}$$

$$y_p = \frac{y}{z/d}$$

$$z_p = d$$



Homogeneous Form

consider $\mathbf{q} = \mathbf{M}\mathbf{p}$ where

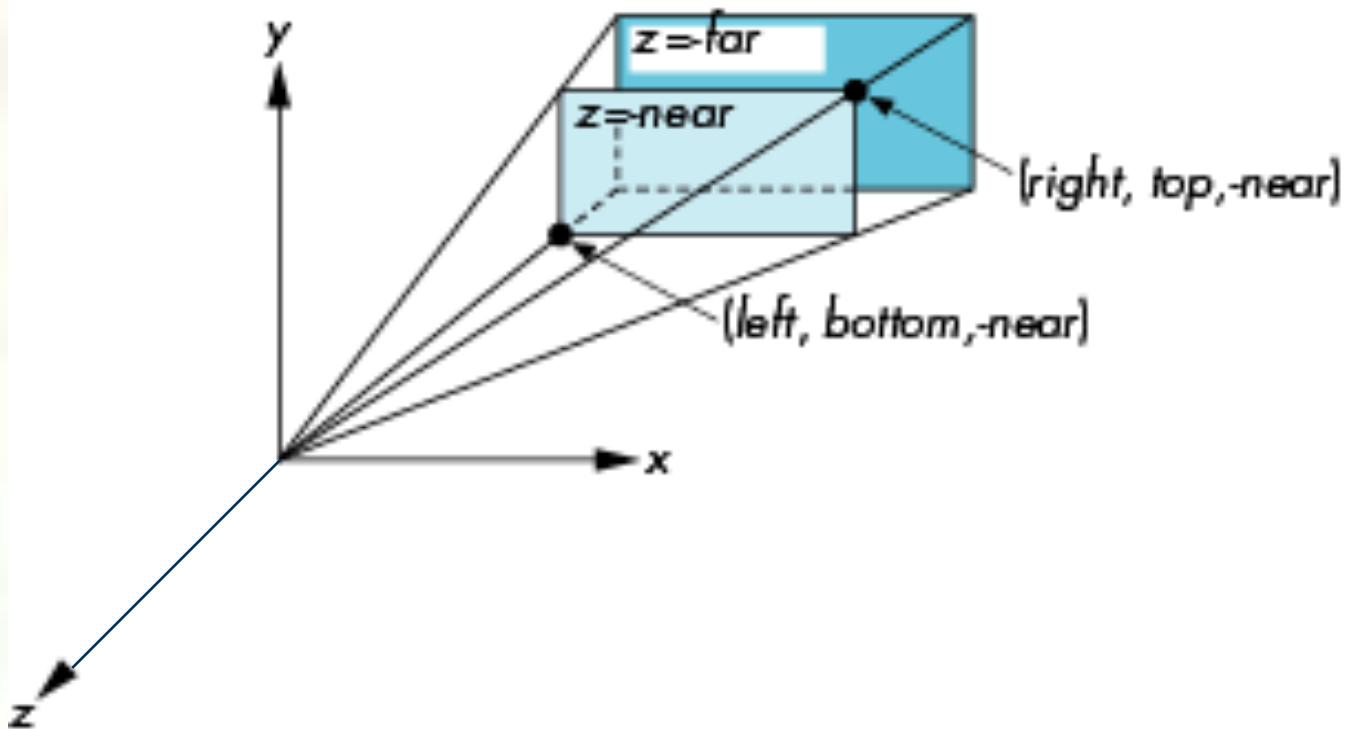
$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$



OpenGL Perspective

```
glFrustum(left, right, bottom, top, near, far)
```

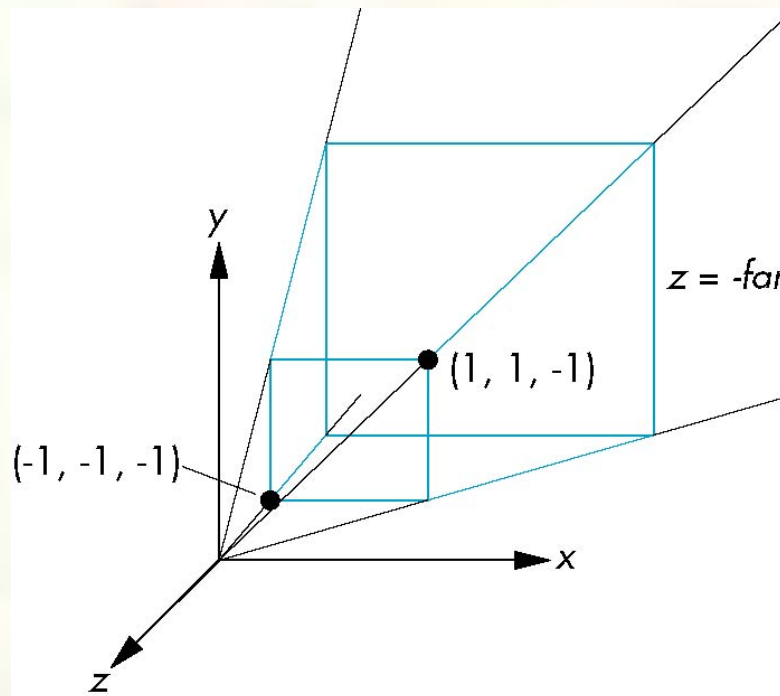




Simple Perspective

Consider a simple perspective with the COP at the origin, the near clipping plane at $z = -1$, and a 90 degree field of view determined by the planes

$$x = \pm z, y = \pm z$$





Simple Eye to NDC

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

after perspective division, the point $(x, y, z, 1)$ goes to

$$x' = x/z$$

$$y' = y/z$$

$$z' = -(\alpha + \beta/z)$$

which projects orthogonally to the desired point regardless of α and β



Picking α and β

If we pick

$$\alpha = \frac{\text{near} + \text{far}}{\text{far} - \text{near}}$$

$$\beta = \frac{2\text{near} * \text{far}}{\text{near} - \text{far}}$$

the near plane is mapped to $z = -1$

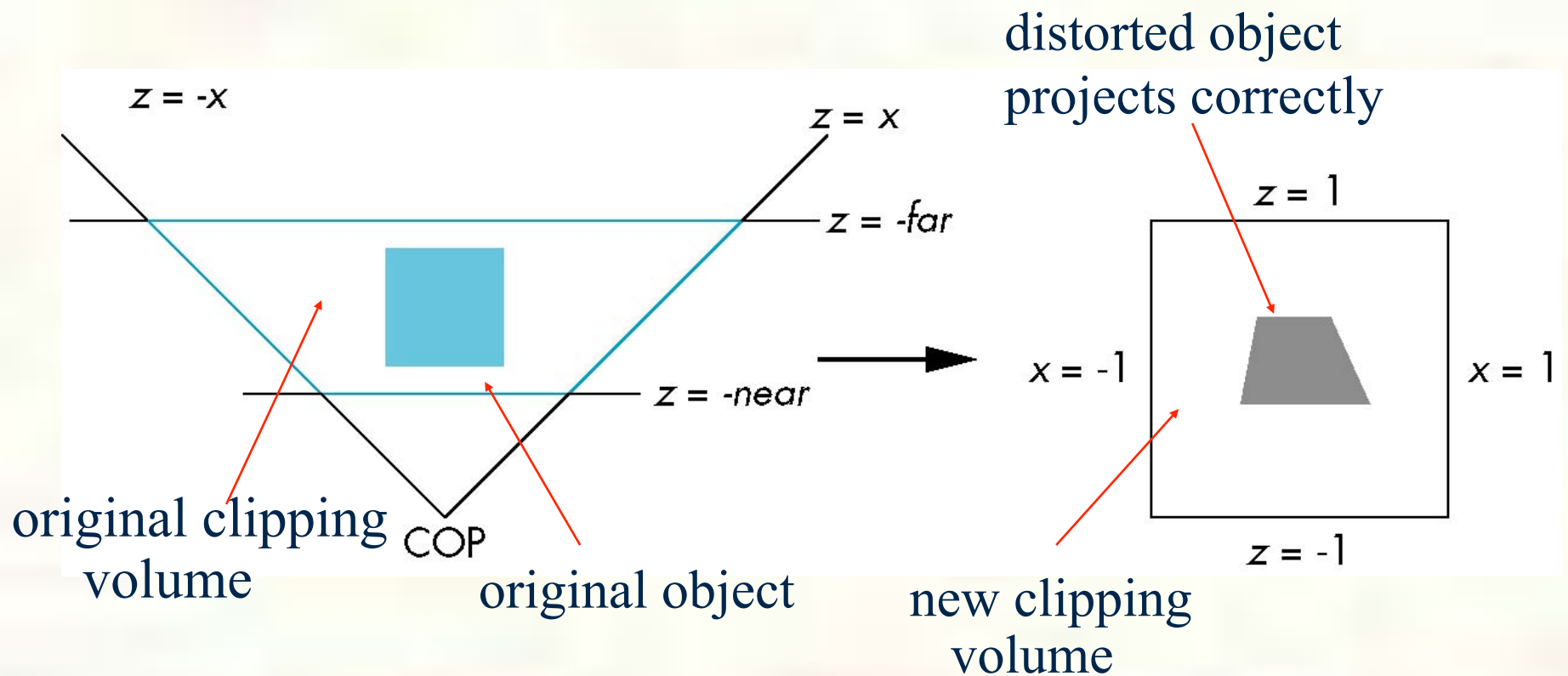
the far plane is mapped to $z = 1$

and the sides are mapped to $x = \pm 1, y = \pm 1$

If we start from the simple eye frustum, we end up with the NDC clipping cube



Normalization Transformation





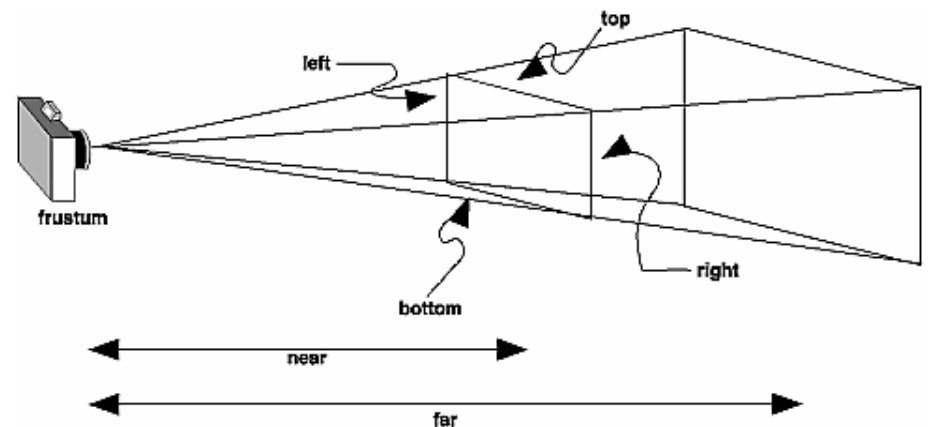
Frustum Transform

- Prototype

- `glFrustum(GLdouble left, GLdouble right, GLdouble bottom, GLdouble top, GLdouble near, GLdouble far)`

- Post-concatenates a frustum matrix

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$





glFrustum Matrix

■ Projection specification

■ `glLoadIdentity();`

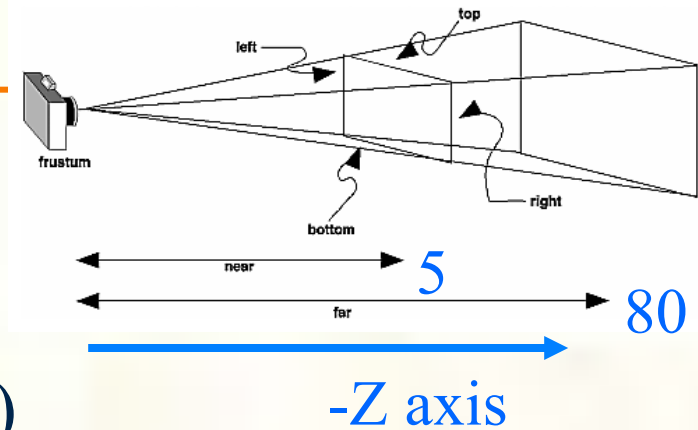
`glFrustum(-4, +4, -3, +3, 5, 80)`

■ left=-4, right=4, bottom=-3, top=3, near=5, far=80

■ Matrix

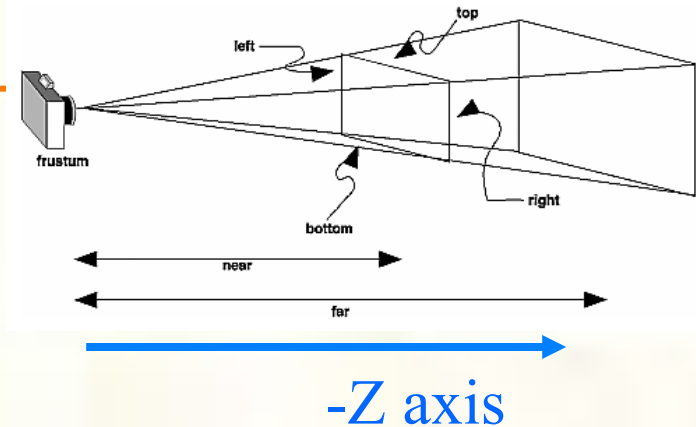
symmetric left/right & top/bottom so zero

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & 0 & 0 & 0 \\ 0 & \frac{5}{3} & 0 & 0 \\ 0 & 0 & -\frac{85}{75} & -\frac{800}{75} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$





glFrustum Example



■ Consider

■ `glLoadIdentity();`

`glFrustum(-30, 30, -20, 20, 1, 1000)`

■ left=-30, right=30, bottom=-20, top=20, near=1, far=1000

■ Matrix

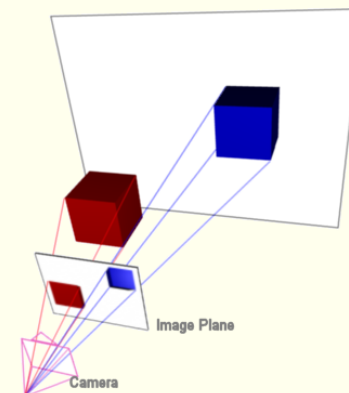
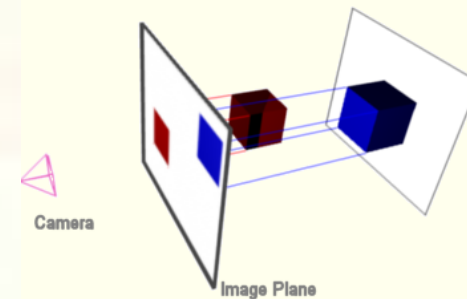
symmetric left/right & top/bottom so zero

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{30} & 0 & 0 & 0 \\ 0 & \frac{1}{20} & 0 & 0 \\ 0 & 0 & -\frac{1001}{999} & -\frac{2000}{999} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



glOrtho and glFrustum

- These OpenGL commands provide a parameterized transform mapping eye space into the “clip cube”
- Each command
 - `glOrtho` is orthographic
 - `glFrustum` is single-point perspective





Next Lecture

- *More viewing*
- *Transform from object to eye space*