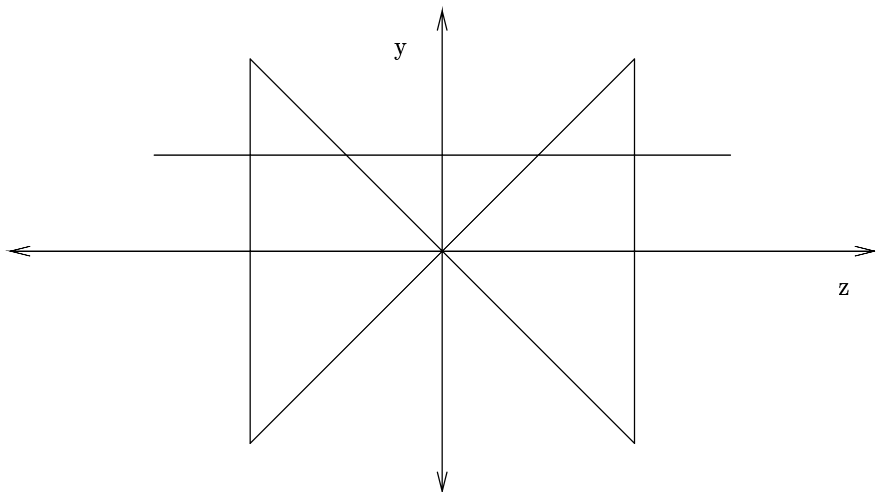
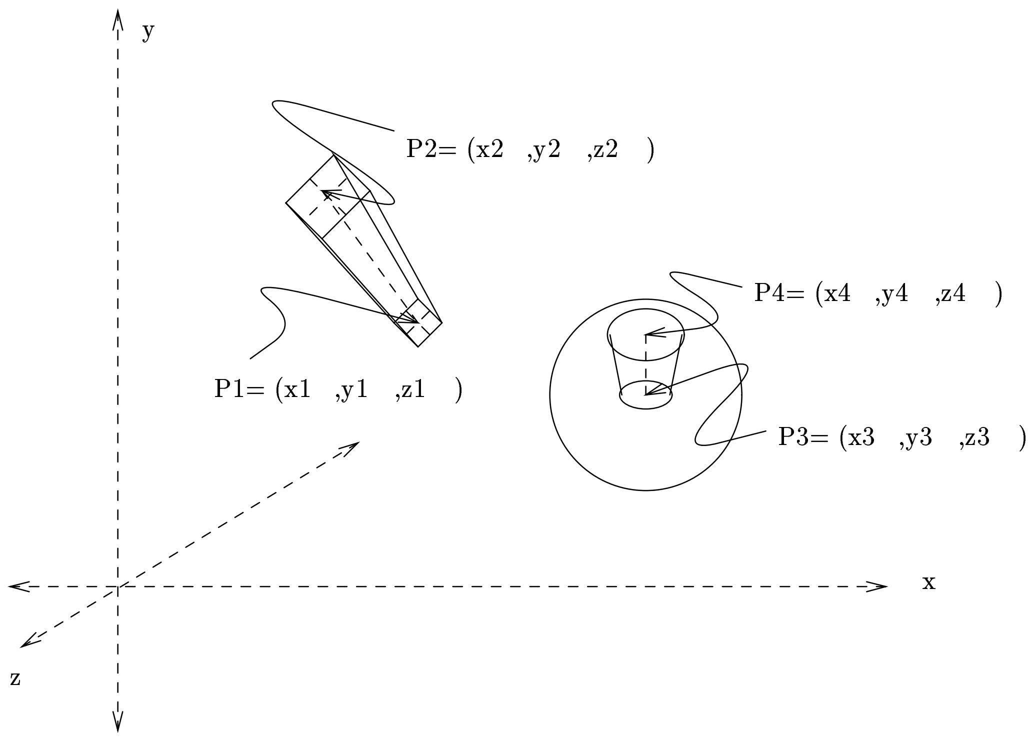
CS354 **Final Examination** **Spring 1995**

Each problem is worth 20 points.

1. You are working in a **left-handed** coordinate system. Give a set of three 4×4 homogeneous transformation matrices which will perform counterclockwise rotations of angle θ about each of the three principal axes as seen from an observer in the positive coordinate space looking at the origin in each case.



2. Consider the simple model with only a single line segment and the viewing volume in *clipping coordinates* as shown in cross section from the point of view of an observer on the positive x axis. Suppose that you *do not clip* the line segment before you project it. You project each point on the line to the window using the perspective projection, and then display it in a viewport whose boundaries are $xmin = ymin = 0$ and $xmax = ymax = 0.5$ on a screen with coordinates ranging from (0,0) to (511,511). Sketch the viewport, labelling the coordinates of any visible endpoint of the line segment and any intersection of the line segment with the viewport boundaries.



3. You are trying to fit the square peg into the round hole in the sphere as shown. Ignoring the fact that if these objects were solid the peg may not fit into the hole, you want to make P1 coincide with P3 and the line segment P1P2 coincide with segment P3P4. Give a sequence of basic right-handed transformations expressed in terms of matrices with formulas in terms of the coordinates of P1, P2, P3 and P4 or actual numbers as appropriate for all entries in the matrices which will accomplish the goal.

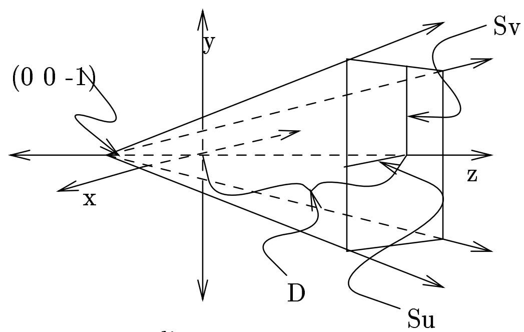
4. Consider the matrix and points defined in **eye coordinates** shown below. Assume that the corners of the window are at (1,1,1), (1,-1,1), (-1,1,1), and (-1,-1,1) and that there are no hither or yon planes.

Points	Transformation Matrix
$P_1 = (0.7, -0.3, 0.5)$	1 0 0 0
$P_2 = (-0.7, 0.8, 1.5)$	0 1 0 0
$P_3 = (-1.05, 1.2, 2.25)$	0 0 0 1
	0 0 1 0

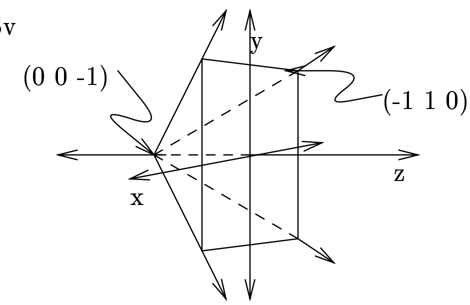
- (a) Give the values of the z coordinates of the points after they have been transformed by this matrix.

- (b) Now project the points from homogeneous space into three dimensions. Give the resulting three space coordinates for the points.

- (c) Assuming all points are opaque, tell which are visible and why or why not.



eye coordinates



clipping coordinates

5. You are devising a system for producing three-dimensional perspective images, and are now working on figuring out how to provide a *perspective transformation* which preserves all three dimensions of information while producing the proper distortions of objects to make them appear in perspective. You have decided to do this work in the convenient *eye coordinate system* given.

   - (a) Write a pair of algebraic equations giving the x and y transformations for a perspective projection in this coordinate system.

   - (b) Now suppose you want to normalize this coordinate system to provide a *clipping coordinate system* with the eye at $z = -1$ and with the yon plane at $z = 0$ and with the planes bounding the viewing volume having slopes of ± 1. Give a sequence of 4×4 matrices which will perform this normalization. Assume the yon plane in eye coordinates (not shown) is at a distance F from the origin

   - (c) Give a set of comparison operations which would perform point clipping against this viewing volume in clipping coordinates. Assume that there are hither and yon planes as well as the slanted planes shown, and that these are at distances H and F from the origin in *eye coordinates* respectively.

   - (d) Your perspective transformation will be performed in clipping coordinates, and will use homogeneous techniques to actually perform the perspective divisions. Give a 4×4 homogeneous transformation matrix which will perform a perspective transformation in clipping coordinates giving resulting x and y values in a range of ± 1 and z values in a range of -1 to 0.