**CS354** **First Examination** **Spring 1997**

Each problem is worth 20 points.

1. You are working in a **left-handed** coordinate system. Give a set of three $4 \times 4$ homogeneous transformation matrices which will perform counterclockwise rotations of angle $\theta$ about each of the three principal axes as seen from an observer in the positive coordinate space looking at the origin in each case.

2. In general, given a transformation $T$ represented by matrix $M$, we must invert $M$ to find the matrix representation of the inverse of $T$. Suppose, however, that $M$ is represented as the compostion of transformations $M = M_1 \circ M_2 \circ \cdots \circ M_n$, where each $M_i$ is a simple translation, rotation about the origin, or scale about the origin. Give an expression for $M^{-1}$ that can be computed without explicitly inverting $M$.

3. You are trying to fit the square peg into the round hole in the sphere shown below. Ignoring the fact that if these objects were solid the peg may not fit into the hole, you want to make P1 coincide with P3 and the line segment P1P2 coincide with segment P3P4. Give a sequence of basic right-handed transformations expressed in terms of matrices with formulas in terms of the coordinates of P1, P2, P3 and P4 or actual numbers as appropriate for all entries in the matrices which will accomplish the goal.

4. You are modeling the Jack-o-lantern drawn below hierarchically, using the object definitions given below in terms of their master coordinate systems.

   (a) Draw a directed acyclic graph (DAG) structure for a model which allows both eyes to be rotated **independently** as well as allowing separate transformations of the nose and mouth, all with respect to the face coordinate system. Give the specific parameters of the transformation matrix in all instance transformations in your graph. Only the circle and triangle objects can be leaves of the DAG and each named object above can appear in only a single node of the DAG.

   (b) Draw a DAG which allows the eyes to be modified independently so that they can appear to be different from each other, e.g. one eye can wink while the other eye stays open. This will require changes to the internal structure of the object model for the winking eye. You should define a new object for the winking eye with new instances of circle, and provide the exact instance transformation matrices as in (a). The same constraints on leaves and DAG structure apply as in (a).

5. A perspective transformation can be performed on a point $P = [x, y, z, 1]$ in three space by multiplying $P$ by the matrix

$$\begin{vmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & d \\ 0 & 0 & e & f \end{vmatrix}$$

and then dividing all coordinates by the fourth coordinate. For OpenGL, we map $x$ and $y$ into the range $[-1, 1]$ and $z$ into the range $[0, 1]$. However, other systems such as the original version of GL map all three $x, y, z$ into the range $[-1, 1]$. Find the values of $a, b, c, d, e$, and $f$ that map all the coordinates into $[-1, 1]$. You may assume you are beginning with a frustum *centered on the positive $z$ axis in left-handed coordinates* with the near plane at $z = n$, the far plane at $z = f$, and the boundaries of the window in the near plane at $\pm s_u$ in the $x$ direction and $\pm s_v$ in the $y$ direction.