

# Shading





# Introduction

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- Affine transformations help us to place objects into a scene.
- Before creating images of these objects, we'll look at models for how light interacts with their surfaces.
- Such a model is called a **shading model**.
- Other names:
  - Lighting model
  - Light reflection model
  - Local illumination model
  - Reflectance model
  - BRDF



# An abundance of photons

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- Properly determining the right color is *really hard*.
- Look around the room. Each light source has different characteristics. Trillions of photons are pouring out every second.
- These photons can:
  - interact with the atmosphere, or with things in the atmosphere
  - strike a surface and
    - be absorbed
    - be reflected (scattered)
    - cause fluorescence or phosphorescence.
  - interact in a wavelength-dependent manner
  - generally bounce around and around



# Break problem into two parts

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- Part 1:  
What happens when photons interact with a particular point on a surface?
- “Local illumination model”
- Part 2:  
How do photons bounce between surfaces?  
And, what is the final result of all of this bouncing?
- “Global illumination model”
  
- Today we’re going to focus on Part 1.



# Strategy for today

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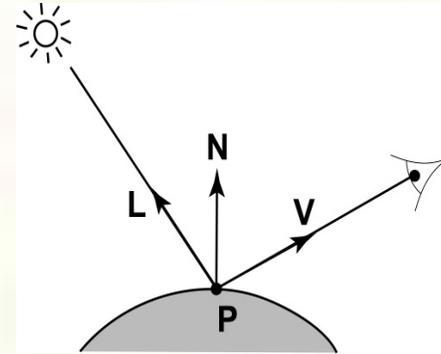
- We're going to build up to an *approximation* of reality called the **Phong illumination model**.
- It has the following characteristics:
  - *not* physically based
  - gives a first-order *approximation* to physical light reflection
  - very fast
  - widely used
- We will assume **local illumination**, i.e., light goes: light source -> surface -> viewer.
- No interreflections, no shadows.



# Setup...

## ■ Given:

- a point  $\mathbf{P}$  on a surface visible through pixel  $p$
- The normal  $\mathbf{N}$  at  $\mathbf{P}$
- The lighting direction,  $\mathbf{L}$ , and intensity,  $I_\ell$ , at  $\mathbf{P}$
- The viewing direction,  $\mathbf{V}$ , at  $\mathbf{P}$
- The shading coefficients (material properties) at  $\mathbf{P}$



## ■ Compute the color, $I$ , of pixel $p$ .

## ■ Assume that the direction vectors are normalized:

$$\|\mathbf{N}\| = \|\mathbf{L}\| = \|\mathbf{V}\| = 1$$



# Iteration zero

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- The simplest thing you can do is...
- Assign each polygon a single color:  $I = k_e$   
where
  - $I$  is the resulting intensity
  - $k_e$  is the **emissivity** or intrinsic shade associated with the object
- This has some special-purpose uses, but not really good for drawing a scene.
- [Note:  $k_e$  is omitted in Watt.]



# Iteration one

- Let's make the color at least dependent on the overall quantity of light available in the scene:

$$I = k_e + k_a I_a$$

- $k_a$  is the **ambient reflection coefficient**.
  - really the reflectance of ambient light
  - “ambient” light is assumed to be equal in all directions
- $I_a$  is the **ambient intensity**.
- Physically, what is “ambient” light?



# Wavelength dependence

- Really,  $k_e$ ,  $k_a$ , and  $I_a$  are functions over all wavelengths  $\lambda$ .
- Ideally, we would do the calculation on these functions.  
We would start with:

$$I(\lambda) = k_e(\lambda) + k_a(\lambda)I_a(\lambda)$$

- then we would find good RGB values to represent the spectrum  $I(\lambda)$ .
- Traditionally, though,  $k_e$ ,  $k_a$  and  $I_a$  are represented as RGB triples, and the computation is performed on each color channel separately:  $I_R = k_{e,R} + k_{a,R}I_{a,R}$

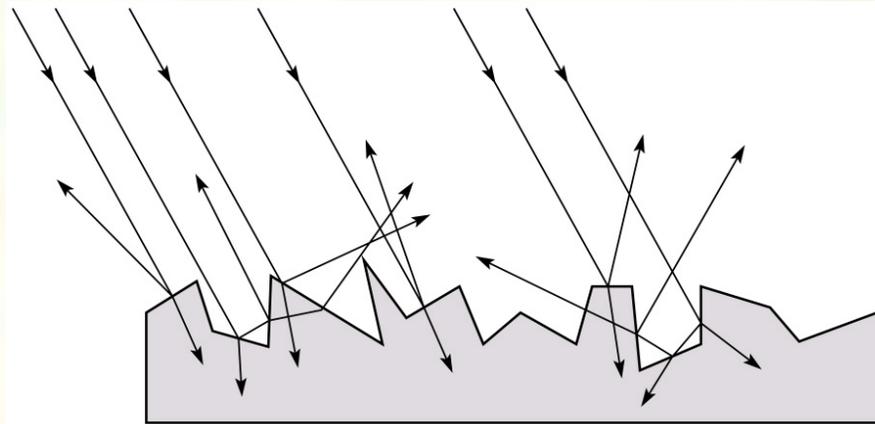
$$I_G = k_{e,G} + k_{a,G}I_{a,G}$$

$$I_B = k_{e,B} + k_{a,B}I_{a,B}$$



# Diffuse reflectors

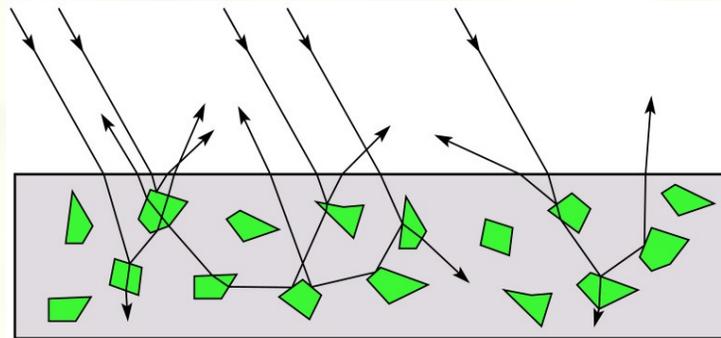
- Diffuse reflection occurs from dull, matte surfaces, like latex paint, or chalk.
- These **diffuse** or **Lambertian** reflectors reradiate light equally in all directions.
- Picture a rough surface with lots of tiny **microfacets**.





# Diffuse reflectors

- ...or picture a surface with little pigment particles embedded beneath the surface (neglect reflection at the surface for the moment):

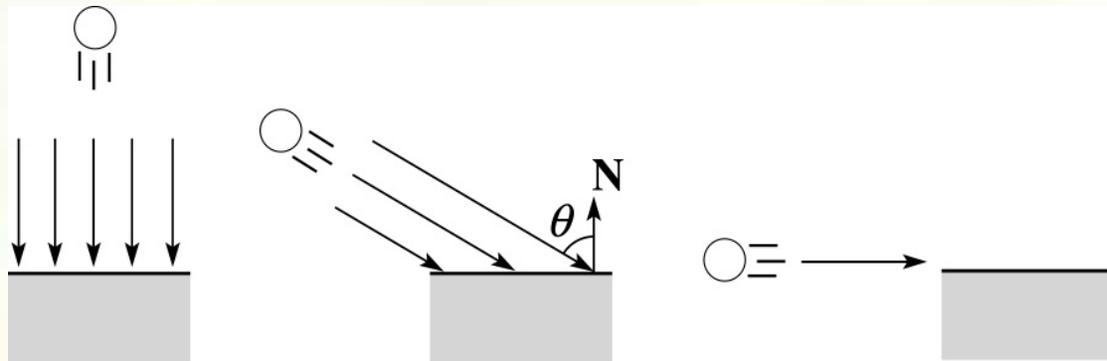


- The microfacets and pigments distribute light rays in all directions.
- Embedded pigments are responsible for the coloration of diffusely reflected light in plastics and paints.
- Note: the figures above are intuitive, but not strictly (physically) correct.



# Diffuse reflectors, cont.

- The reflected intensity from a diffuse surface does not depend on the direction of the viewer. The incoming light, though, does depend on the direction of the light source:





# Iteration two

- The incoming energy is proportional to  $\cos(\theta)$ , giving the diffuse reflection equations:

$$\begin{aligned} I &= k_e + k_a I_a + k_d I_\ell \cos(\theta)_+ \\ &= k_e + k_a I_a + k_d I_\ell (\mathbf{N} \cdot \mathbf{L})_+ \end{aligned}$$

where:

- $k_d$  is the **diffuse reflection coefficient**
- $I_\ell$  is the intensity of the light source
- $\mathbf{N}$  is the normal to the surface (unit vector)
- $\mathbf{L}$  is the direction to the light source (unit vector)
- $(x)_+$  means  $\max\{0, x\}$

[Note: Watt uses  $I_i$  instead of  $I_\ell$ .]



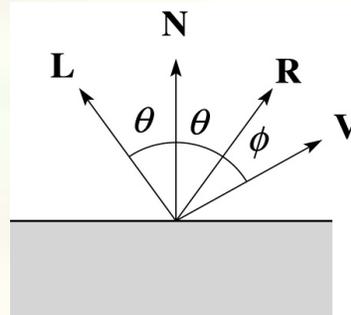
# Specular reflection

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- **Specular reflection** accounts for the highlight that you see on some objects.
- It is particularly important for *smooth, shiny* surfaces, such as:
  - metal
  - polished stone
  - plastics
  - apples
  - skin
- **Properties:**
  - Specular reflection depends on the viewing direction  $V$ .
  - For non-metals, the color is determined solely by the color of the light.
  - For metals, the color may be altered (e.g., brass)



# Specular reflection “derivation”



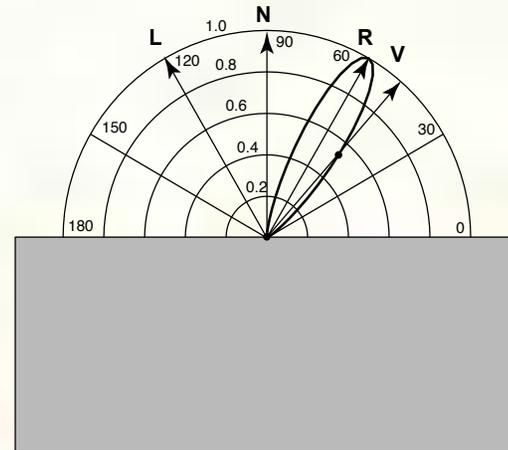
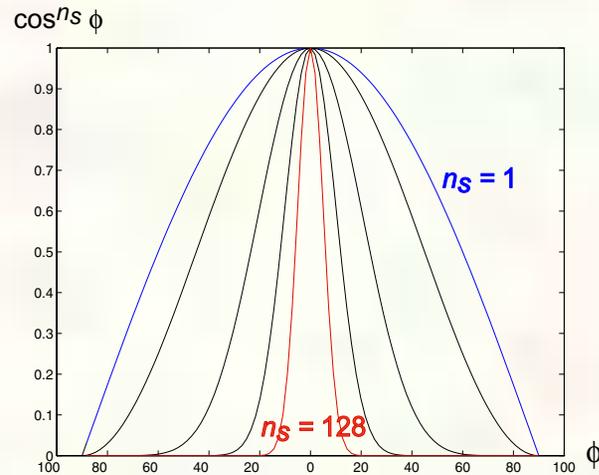
- For a perfect mirror reflector, light is reflected about  $\mathbf{N}$ , so

$$I = \begin{cases} I_\ell & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

- For a near-perfect reflector, you might expect the highlight to fall off quickly with increasing angle  $\phi$ .
- Also known as:
  - “rough specular” reflection
  - “directional diffuse” reflection
  - “glossy” reflection



# Derivation, cont.



- One way to get this effect is to take  $(\mathbf{R} \cdot \mathbf{V})$ , raised to a power  $n_s$ .
- As  $n_s$  gets larger,
  - the dropoff becomes {more,less} gradual
  - gives a {larger,smaller} highlight
  - simulates a {more,less} mirror-like surface



# Iteration three

- The next update to the Phong shading model is then:

$$I = k_e + k_a I_a + k_d I_\ell (\mathbf{N} \cdot \mathbf{L})_+ + k_s I_\ell (\mathbf{R} \cdot \mathbf{V})_+^{n_s}$$

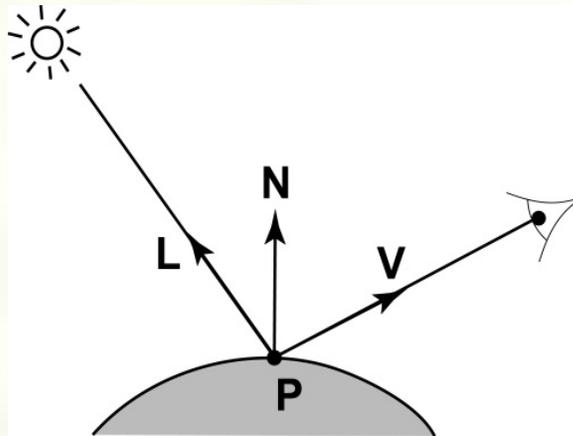
where:

- $k_s$  is the **specular reflection coefficient**
- $n_s$  is the **specular exponent** or **shininess**
- $\mathbf{R}$  is the reflection of the light about the normal (unit vector)
- $\mathbf{V}$  is viewing direction (unit vector)

[Note: Watt uses  $n$  instead of  $n_s$ .]



# What is incoming light intensity?



So far we've just been considering what happens at the surface itself.

How does incoming light intensity change as light moves further away?



# Intensity drop-off with distance

- OpenGL supports different kinds of lights: point, directional, and spot.
- For point light sources, the laws of physics state that the intensity of a point light source must drop off inversely with the square of the distance.
- We can incorporate this effect by multiplying  $I_\ell$  by  $1/d^2$ .
- Sometimes, this distance-squared dropoff is considered too “harsh.” A common alternative is:

$$f_{atten}(d) = \frac{1}{a + bd + cd^2}$$

with user-supplied constants for  $a$ ,  $b$ , and  $c$ .

[Note: not discussed in Watt.]



# Iteration four

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- Since light is additive, we can handle multiple lights by taking the sum over every light.
- Our equation is now:

$$I = k_e + k_a I_a + \sum_j f_{atten}(d_j) I_{\ell_j} \left[ k_d (\mathbf{N} \cdot \mathbf{L}_j)_+ + k_s (\mathbf{R}_j \cdot \mathbf{V})_+^{n_s} \right]$$

- This is the Phong illumination model.



# Choosing the parameters

- Experiment with different parameter settings. To get you started, here are a few suggestions:
  - Try  $n_s$  in the range [0,100]
  - Try  $k_a + k_d + k_s < 1$
  - Use a small  $k_a$  ( $\sim 0.1$ )

|         | $n_s$  | $k_d$                          | $k_s$                       |
|---------|--------|--------------------------------|-----------------------------|
| Metal   | large  | Small,<br>color of<br>metal    | Large,<br>color of<br>metal |
| Plastic | medium | Medium,<br>color of<br>plastic | Medium,<br>white            |
| Planet  | 0      | varying                        | 0                           |

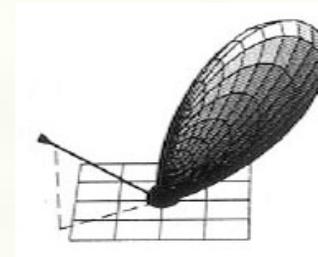


# BRDF

- The Phong illumination model is really a function that maps light from incoming (light) directions to outgoing (viewing) directions:

$$f_r(\omega_{in}, \omega_{out})$$

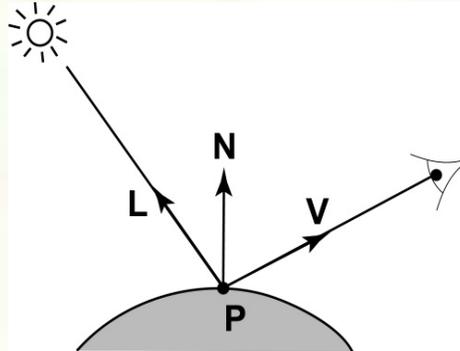
- This function is called the **Bi-directional Reflectance Distribution Function (BRDF)**.
- Here's a plot with  $\omega_{in}$  held constant:



- Physically valid BRDF's obey Helmholtz reciprocity:

$$f_r(\omega_{in}, \omega_{out}) = f_r(\omega_{out}, \omega_{in})$$

and should conserve energy (no light amplification).



$$f_r(\omega_{in}, \omega_{out}) = f_r(\mathbf{L}, \mathbf{V})$$

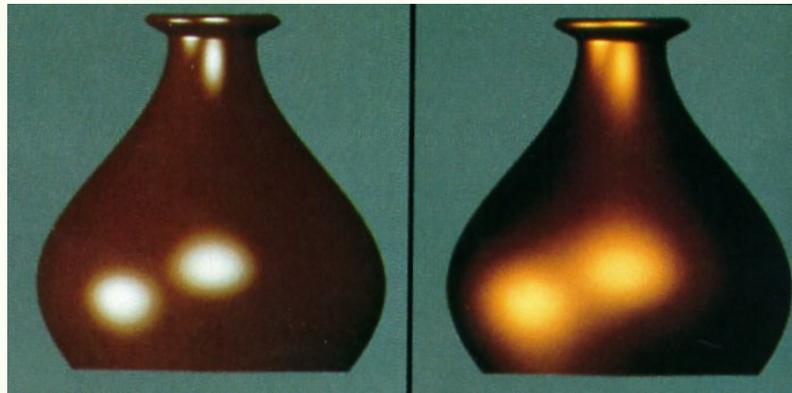
How do we express Phong model using explicit BRDF?

$$I = k_e + k_a I_a + \sum_j f_{atten}(d_j) I_{\ell_j} \left[ k_d (\mathbf{N} \cdot \mathbf{L}_j)_+ + k_s (\mathbf{R}_j \cdot \mathbf{V})_+^{n_s} \right]$$

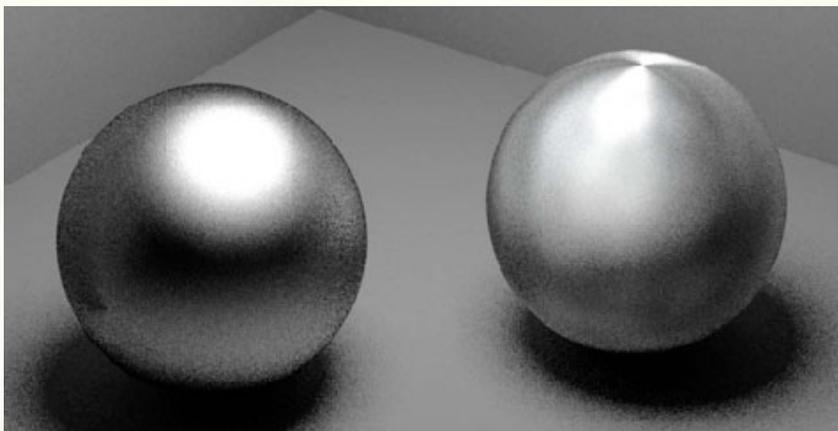


# More sophisticated BRDF's

Cook and Torrance, 1982



Westin, Arvo, Torrance 1992





# Summary

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- Local vs. Global Illumination Models
- Local Illumination Models:
  - Phong – Physically inspired, but not truly physically correct.
  - Arbitrary BRDFs
- In applying the Phong model, we assumed unshadowed “point” light sources.