5. Shading

Reading

Required:
- Watt, sections 6.2-6.3

Optional:
- Watt, chapter 7.
**Introduction**

Affine transformations help us to place objects into a scene.

Before creating images of these objects, we’ll look at models for how light interacts with their surfaces.

Such a model is called a *shading model*.

Other names:
- Lighting model
- Light reflection model
- Local illumination model
- Reflectance model
- BRDF

**An abundance of photons**

Properly determining the right color is *really hard*.

Look around the room. Each light source has different characteristics. Trillions of photons are pouring out every second.

These photons can:
- interact with the atmosphere, or with things in the atmosphere
- strike a surface and
  - be absorbed
  - be reflected (scattered)
  - cause fluorescence or phosphorescence.
- interact in a wavelength-dependent manner
- generally bounce around and around
Break problem into two parts

Part 1:
What happens when photons interact with a particular point on a surface?

“Local illumination model”

Part 2:
How do photons bounce between surfaces? And, what is the final result of all of this bouncing?

“Global illumination model”

Today we’re going to focus on Part 1.

Strategy for today

We’re going to build up to an approximation of reality called the Phong illumination model.

It has the following characteristics:

- *not* physically based
- gives a first-order *approximation* to physical light reflection
- very fast
- widely used

We will assume *local illumination*, i.e., light goes: light source -> surface -> viewer.

No interreflections, no shadows.
Setup...

Given:
- a point \( P \) on a surface visible through pixel \( p \)
- The normal \( N \) at \( P \)
- The lighting direction, \( L \), and intensity, \( I_l \), at \( P \)
- The viewing direction, \( V \), at \( P \)
- The shading coefficients (material properties) at \( P \)

Compute the color, \( I \), of pixel \( p \).

Assume that the direction vectors are normalized:
\[
\|N\| = \|L\| = \|V\| = 1
\]

Iteration zero

The simplest thing you can do is...

Assign each polygon a single color:
\[
I = k_e
\]

where
- \( I \) is the resulting intensity
- \( k_e \) is the emissivity or intrinsic shade associated with the object

This has some special-purpose uses, but not really good for drawing a scene.

[Note: \( k_e \) is omitted in Watt.]
Iteration one

Let’s make the color at least dependent on the overall quantity of light available in the scene:

\[ I = k_a + k_a I_a \]

- \( k_a \) is the **ambient reflection coefficient.**
  - really the reflectance of ambient light
  - “ambient” light is assumed to be equal in all directions
- \( I_a \) is the **ambient intensity.**

Physically, what is “ambient” light?

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Wavelength dependence

Really, \( k_a \), \( k_a \), and \( I_a \) are functions over all wavelengths \( \lambda \).

Ideally, we would do the calculation on these functions. For the ambient shading equation, we would start with:

\[ I(\lambda) = k_a(\lambda)I_a(\lambda) \]

then we would find good RGB values to represent the spectrum \( I(\lambda) \).

Traditionally, though, \( k_a \) and \( I_a \) are represented as RGB triples, and the computation is performed on each color channel separately:

\[ I_R = k_{a,R} I_{a,R} \]
\[ I_G = k_{a,G} I_{a,G} \]
\[ I_B = k_{a,B} I_{a,B} \]
Diffuse reflectors

Diffuse reflection occurs from dull, matte surfaces, like latex paint, or chalk.

These **diffuse** or **Lambertian** reflectors reradiate light equally in all directions.

Picture a rough surface with lots of tiny **microfacets**.

Diffuse reflectors

...or picture a surface with little pigment particles embedded beneath the surface (neglect reflection at the surface for the moment):

The microfacets and pigments distribute light rays in all directions.

Embedded pigments are responsible for the coloration of diffusely reflected light in plastics and paints.

Note: the figures above are intuitive, but not strictly (physically) correct.
Diffuse reflectors, cont.

The reflected intensity from a diffuse surface does not depend on the direction of the viewer. The incoming light, though, does depend on the direction of the light source:

\[ I = k_d I_s + k_d I_i \cos(\theta) + (x) \]

where:
- \( k_d \) is the diffuse reflection coefficient
- \( I_s \) is the intensity of the light source
- \( N \) is the normal to the surface (unit vector)
- \( L \) is the direction to the light source (unit vector)
- \((x)\) means max \( \{0, x\} \)

[Note: Watt uses \( I_i \) instead of \( I_s \).]

Iteration two

The incoming energy is proportional to ______, giving the diffuse reflection equations:

\[ I = k_o + k_d I_s + k_d I_i \cos(\theta) + (x) \]

\[ = k_o + k_d I_s + k_d I_i (\hat{L} \cdot \hat{N}) \]

where:
- \( k_o \) is the diffuse reflection coefficient
- \( I_i \) is the intensity of the light source
- \( N \) is the normal to the surface (unit vector)
- \( L \) is the direction to the light source (unit vector)
- \((x)\) means max \( \{0, x\} \)
Specular reflection

Specular reflection accounts for the highlight that you see on some objects. It is particularly important for smooth, shiny surfaces, such as:

- metal
- polished stone
- plastics
- apples
- skin

Properties:

- Specular reflection depends on the viewing direction $V$.
- For non-metals, the color is determined solely by the color of the light.
- For metals, the color may be altered (e.g., brass)

Specular reflection “derivation”

For a perfect mirror reflector, light is reflected about $N$, so

$$I = \begin{cases} I_v & \text{if } V = R \\ 0 & \text{otherwise} \end{cases}$$

For a near-perfect reflector, you might expect the highlight to fall off quickly with increasing angle $\phi$.

Also known as:

- “rough specular” reflection
- “directional diffuse” reflection
- “glossy” reflection
Derivation, cont.

One way to get this effect is to take \((R \cdot V)^{ns}\), raised to a power \(ns\).

As \(ns\) gets larger,

- the dropoff becomes {more, less} gradual
- gives a {larger, smaller} highlight
- simulates a {more, less} mirror-like surface

Iteration three

The next update to the Phong shading model is then:

\[
l = (k_e + k_s I_s) + k_d I_s (N \cdot L) + k_l I_s (V \cdot R)_s
\]

where:

- \(k_s\) is the specular reflection coefficient
- \(ns\) is the specular exponent or shininess
- \(R\) is the reflection of the light about the normal (unit vector)
- \(V\) is viewing direction (unit vector)

[Note: Watt uses \(n\) instead of \(ns\).]
What is incoming light intensity?

So far we’ve just been considering what happens at the surface itself.

How does incoming light intensity change as light moves further away?

Intensity drop-off with distance

OpenGL supports different kinds of lights: point, directional, and spot.

For point light sources, the laws of physics state that the intensity of a point light source must drop off inversely with the square of the distance.

We can incorporate this effect by multiplying I, by $\frac{1}{d^2}$.

Sometimes, this distance-squared dropoff is considered too “harsh.” A common alternative is:

$$f_{\text{atten}}(d) = \frac{1}{a + bd + cd^2}$$

with user-supplied constants for $a$, $b$, and $c$.

[Note: not discussed in Watt.]
**Iteration four**

Since light is additive, we can handle multiple lights by taking the sum over every light.

Our equation is now:

\[
I = k_a + k_d \sum_l \left( \frac{\mathbf{n} \cdot (\mathbf{L} + \mathbf{R})}{d} \right)_l \left[ k_d (\mathbf{N} \cdot \mathbf{L})_l + k_s (\mathbf{N} \cdot \mathbf{R})_l \right]^{n_s}
\]

This is the Phong illumination model.

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**Choosing the parameters**

Experiment with different parameter settings. To get you started, here are a few suggestions:

- Try \( n_s \) in the range \([0, 100]\)
- Try \( k_a + k_d + k_s < 1 \)
- Use a small \( k_a \) (~0.1)

<table>
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<th>( n_s )</th>
<th>( k_d )</th>
<th>( k_s )</th>
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<tr>
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<td>large</td>
<td>Small, color of metal</td>
<td>Large, color of metal</td>
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<td>medium</td>
<td>Medium, color of plastic</td>
<td>Medium, white</td>
</tr>
<tr>
<td>Planet</td>
<td>0</td>
<td>varying</td>
<td>0</td>
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</tbody>
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The Phong illumination model is really a function that maps light from incoming (light) directions to outgoing (viewing) directions:

\[ f_r(\omega_{\text{in}}, \omega_{\text{out}}) \]

This function is called the **Bi-directional Reflectance Distribution Function (BRDF)**.

Here’s a plot with \( \omega_{\text{in}} \) held constant:

Physically valid BRDF’s obey Helmholtz reciprocity:

\[ f_r(\omega_{\text{in}}, \omega_{\text{out}}) = f_r(\omega_{\text{out}}, \omega_{\text{in}}) \]

and should conserve energy (no light amplification).

How do we express Phong model using explicit BRDF?

\[ l = k_a + k_d \sum f_{\text{atten}}(d_j) I_j \left[ k_o (N \cdot L)_j + k_d (V \cdot R)_j \right] \]
More sophisticated BRDF’s

Cook and Torrance, 1982

Summary

Local vs. Global Illumination Models

Local Illumination Models:

- Phong – Physically inspired, but not truly physically correct.

- Arbitrary BRDFs

In applying the Phong model, we assumed unshadowed “point” light sources.
Next time: Ray tracing

Topics:

How do we model the transport of light within the scene?

How do we determine which surfaces are visible from the eye, or shadowed from a light?

Read:

• Watt, sections 1.3-1.4, 12.1-12.5.1.


Optional:

• A. Glassner. An Introduction to Ray Tracing. Academic Press, 1989. [In the graphics research lab, ACES 2.102]