Hierarchical Modeling
Reading

- Angel, sections 9.1 - 9.6
  [reader pp. 169-185]

- OpenGL Programming Guide, chapter 3
  Focus especially on section titled “Modelling Transformations”.
Hierarchical Modeling

Consider a moving automobile, with 4 wheels attached to the chassis, and lug nuts attached to each wheel:
Symbols and instances

- Most graphics APIs support a few geometric primitives:
  - spheres
  - cubes
  - triangles

- These symbols are instanced using an instance transformation.
Use a series of transformations

- Ultimately, a particular geometric instance is transformed by one combined transformation matrix:

- But it’s convenient to build this single matrix from a series of simpler transformations:

- We have to be careful about how we think about composing these transformations.

  (Mathematical reason: Transformation matrices don’t commute under matrix multiplication)
Two ways to compose xforms

- **Method #1:**
  Express every transformation with respect to global coordinate system:

- **Method #2:**
  Express every transformation with respect to a “parent” coordinate system created by earlier transformations:

The goal of this second approach is to build a series of transforms. Once they exist, we can think of points as being “processed” by these xforms as in Method #1.
#1: Xform for global coordinates

FinalPosition = M_1 * M_2 * ... * M_n * InitialPosition

Note: Positions are column vectors:

\[
\begin{bmatrix}
  x \\
  y \\
  z \\
  1
\end{bmatrix}
\]
#2: Xform for coordinate system

FinalPosition = \( M_1 \times M_2 \times \ldots \times M_n \times \text{InitialPosition} \)
Xform direction for coord. sys

FinalPosition = \( M_1 \times M_2 \times \ldots \times M_n \times \) InitialPosition

Translate/Rotate:
FROM previous coord sys
TO new one
with transformation expressed in
the ‘previous’ coordinate system.
Connecting primitives
3D Example: A robot arm

Consider this robot arm with 3 degrees of freedom:
- Base rotates about its vertical axis by $\theta$
- Upper arm rotates in its $xy$-plane by $\phi$
- Lower arm rotates in its $xy$-plane by $\psi$

**Q:** What matrix do we use to transform the base?
**Q:** What matrix for the upper arm?
**Q:** What matrix for the lower arm?
Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```c
Matrix M_model;
main()
{
    . . .
    robot_arm();
    . . .
}
robot_arm()
{
    M_model = R_y(theta);
    base();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);
    upper_arm();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi) *T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

Do the matrix computations seem wasteful?
Instead of recalculating the global matrix each time, we can just update it \textit{in place} by concatenating matrices on the right:

```c
Matrix M_model;
main()
{
  . . .
  M_model = Identity();
  robot_arm();
  . . .
}
robot_arm()
{
  M_model *= R_y(theta);
  base();
  M_model *= T(0,h1,0)*R_z(phi);
  upper_arm();
  M_model *= T(0,h2,0)*R_z(psi);
  lower_arm();
}
```
OpenGL maintains a global state matrix called the **model-view matrix**, which is updated by concatenating matrices on the **right**.

```c
main()
{
  ...
  glMatrixMode( GL_MODELVIEW );
  glLoadIdentity();
  robot_arm();
  ...
}
robot_arm()
{
  glRotatef( theta, 0.0, 1.0, 0.0 );
  base();
  glTranslatef( 0.0, h1, 0.0 );
  glRotatef( phi, 0.0, 0.0, 1.0 );
  lower_arm();
  glTranslatef( 0.0, h2, 0.0 );
  glRotatef( psi, 0.0, 0.0, 1.0 );
  upper_arm();
}
```
Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:

- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

How might we draw the tree for the robot arm?
A complex example: human figure

Q: What’s the most sensible way to traverse this tree?
Human figure implementation, OpenGL

```c
figure()
{
    torso();
    glPushMatrix();
    glTranslate( ... );
    glRotate( ... );
    head();
    glPopMatrix();
    glPushMatrix();
    glPushMatrix();
    glTranslate( ... );
    glRotate( ... );
    left_upper_arm();
    glPushMatrix();
    glTranslate( ... );
    glRotate( ... );
    left_lower_arm();
    glMatrix();
    glPopMatrix();
    glPopMatrix();
    ...
}
```
Animation

- The above examples are called **articulated models**:
  - rigid parts
  - connected by joints
- They can be animated by specifying the joint angles (or other display parameters) as functions of time.
Key-frame animation

- The most common method for character animation in production is **key-frame animation**.
  - Each joint specified at various **key frames** (not necessarily the same as other joints)
  - System does interpolation or **in-betweening**

- Doing this well requires:
  - A way of smoothly interpolating key frames: **splines**
  - A good interactive system
  - A lot of skill on the part of the animator
Scene graphs

- The idea of hierarchical modeling can be extended to an entire scene, encompassing:
  - many different objects
  - lights
  - camera position

- This is called a **scene tree** or **scene graph**.