Subdivision surfaces
Recommended:

Building complex models

We can extend the idea of subdivision from curves to surfaces...
Subdivision surfaces

- Chaikin’s use of subdivision for curves inspired similar techniques for subdivision surfaces.
- Iteratively refine a control polyhedron (or control mesh) to produce the limit surface using splitting and averaging steps.

\[ \sigma = \lim_{j \to \infty} M^j \]
There are a variety of ways to subdivide a polygon mesh.

A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:
Loop averaging step

Once again we can use masks for the averaging step:

\[
Q \leftarrow \frac{\alpha(n)Q + Q_1 + \cdots + Q_n}{\alpha(n) + n}
\]

where

\[
\alpha(n) = \frac{n(1 - \beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi/n))^2}{32}
\]

These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as $G^1$ continuity for surfaces.
Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.

How do we compute the normal?

\[
\mathbf{Q}^\infty = \frac{\varepsilon(n)\mathbf{Q} + \mathbf{Q}_1 + \cdots + \mathbf{Q}_n}{\varepsilon(n) + n}
\]

\[
\mathbf{T}_1^\infty = \tau_1(n)\mathbf{Q}_1 + \tau_2(n)\mathbf{Q}_2 + \cdots + \tau_n(n)\mathbf{Q}_n
\]

\[
\mathbf{T}_2^\infty = \tau_n(n)\mathbf{Q}_1 + \tau_1(n)\mathbf{Q}_2 + \cdots + \tau_{n-1}(n)\mathbf{Q}_n
\]

where \(\varepsilon(n) = \frac{3n}{\beta(n)}\) and \(\tau_i(n) = \cos(2\pi i / n)\)
Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

■ Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
■ Compute two tangent vectors using the tangent masks.
■ Compute the normal from the tangent vectors.
■ Push the resulting points to the limit positions. Use the evaluation mask.
■ Render!
Adding creases without trim curves

- In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.
- For subdivision surfaces, we can just modify the subdivision mask:

This gives rise to $G^0$ continuous surfaces (i.e., having positional but not tangent plane continuity)
Creases without trim curves, cont.

Here’s an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):
Face schemes

- 4:1 subdivision of triangles is sometimes called a **face scheme** for subdivision, as each face begets more faces.

- An alternative face scheme starts with arbitrary polygon meshes and inserts vertices along edges and at face centroids:

  ![Original](image1.png) ![After splitting](image2.png)

- **Catmull-Clark subdivision:**

  ![Example](image3.png)

- **Note:** after the first subdivision, all polygons are quadrilaterals in this scheme.
Subdivision can be equivalent to tensor-product patches!

For a regular quadrilateral mesh, Catmull-Clark subdivision produces the same surface as tensor-product cubic B-splines!

But – it handles irregular meshes as well.

There are similar correspondences between other subdivision schemes and other tensor-product patch schemes.

These correspondences can be proven (but we won’t do it…)}
Vertex schemes

- In a **vertex scheme**, each vertex begets more vertices. In particular, a vertex surrounded by \( n \) faces is split into \( n \) sub-vertices, one for each face:

- **Doo-Sabin subdivision:**

- The number edges (faces) incident to a vertex is called its **valence**. Edges with only once incident face are on the **boundary**. After splitting in this subdivision scheme, all non-boundary vertices are of valence 4.
Interpolating subdivision surfaces

- Interpolating schemes are defined by
  - splitting
  - averaging only new vertices
- The following averaging mask is used in butterfly subdivision:

```
  0
  0   0
0 -t   0   0
  0   0   0
  0   t   0
  0   0   0
  0   0   0
  0
```

- Setting $t=0$ gives the original polyhedron, and increasing small values of $t$ makes the surface smoother, until $t=1/8$ when the surface is provably $G^1$.

There are several variants of Butterfly subdivision.
Next class: Projections & Z-Buffers

Topics:
- How do projections from 3D world to 2D image plane work?
- How does the Z-buffer visibility algorithm (used in today’s graphics hardware) work?

Read:
• Watt, Section 5.2.2 – 5.2.4, 6.3, 6.6 (esp. intro and subsections 1, 4, and 8–10)

Optional:
• Foley, et al, Chapter 5.6 and Chapter 6