Each problem is worth 25 points.

1. You are going to implement a graphics system with the triangular clipping window given below instead of the usual rectangular one. You may assume this window is always located at the position shown.

\[
\begin{array}{c}
\text{(0,0)} \\
\text{(-1,0)} \quad \text{(-1,0)} \\
\text{(0,1)} \\
\text{(1,0)} \\
\end{array}
\]

(a) Describe an “outcode” scheme similar to that used in the Cohen-Sutherland clipping algorithm for testing visibility of points and lines with respect to this window. Draw the regions and the codes associated with each region on the figure, and give criteria for assigning values to each bit of the code.

(b) Give a trivial acceptance test for a point. Give a trivial acceptance test for a line segment. Give trivial rejection tests for both points and line segments.

(c) Show how to compute the intersection of a line segment with the upper right boundary of the triangular window.
2. You are devising a system for producing three-dimensional perspective images, and are now working on figuring out how to provide a *perspective transformation* which preserves all three dimensions of information while producing the proper distortions of objects to make them appear in perspective. You have decided to do this work in the coordinate system shown below.

(a) Write a pair of algebraic equations giving the \( x \) and \( y \) transformations for a perspective projection in this coordinate system.

(b) Now suppose you want to normalize this coordinate system to provide a *clipping coordinate system* with the eye at \( z = -1 \) and with the yon plane at \( z = 0 \) and with the planes bounding the viewing volume having slopes of \( \pm 1 \). Give a sequence of \( 4 \times 4 \) matrices which will perform this normalization. Assume the yon plane in eye coordinates (not shown) is at a distance \( F \) from the eye position.

(c) Give a set of comparison operations which would perform point clipping against this viewing volume in clipping coordinates.

(d) Your perspective transformation will be performed in clipping coordinates, and will use homogeneous techniques to actually perform the perspective divisions. Give a \( 4 \times 4 \) homogeneous transformation matrix which will perform a perspective transformation in clipping coordinates giving resulting \( x \) and \( y \) values in a range of \( \pm 1 \) and \( z \) values in a range of \(-1\) to \(0\).
3. Parametric equations for two three-dimensional line segments are given below.

\[
\begin{align*}
[x \ y \ z] &= [5 \ 10 \ 7] + t_1[3 \ 8 \ 2] \\
[x \ y \ z] &= [3 \ 9 \ 1] + t_2[4 \ 2 \ 7]
\end{align*}
\]

(a) What is the intersection of the orthographic projections of the lines defined by these line segments onto the \(x - y\) plane?

(b) What is the intersection of the orthographic projections of the lines defined by these line segments onto the \(y - z\) plane?

(c) Does the intersection in (a) fall within the projections of the line segments?

(d) Does the intersection in (b) fall within the projections of the line segments?

(e) Do the lines intersect? If so, what is their intersection? If not, why not?
4. You are trying to fit the square peg into the round hole in the sphere as shown. Ignoring the fact that if these objects were solid the peg may not fit into the hole, you want to make P1 coincide with P3 and the line segment P1P2 coincide with segment P3P4. You will do this by applying an affine transformation to the peg only, the sphere will not move. Give a sequence of basic right-handed transformations expressed in terms of matrices with formulas in terms of the coordinates of P1, P2, P3 and P4 or actual numbers as appropriate for all entries in the matrices which will accomplish the goal.