Anti-aliased and accelerated ray tracing
Reading

- **Required:**
  - Watt, sections 12.5.3 – 12.5.4, 14.7

- **Further reading:**
Aliasing in rendering

One of the most common rendering artifacts is the “jaggies”. Consider rendering a white polygon against a black background:

We would instead like to get a smoother transition:
Anti-aliasing

**Q:** How do we avoid aliasing artifacts?

1. Sampling:
2. Pre-filtering:
3. Combination:

**Example - polygon:**

![Example Diagram]
Polygon anti-aliasing

Without antialiasing

With antialiasing

Magnification
Antialiasing in a ray tracer

- We would like to compute the average intensity in the neighborhood of each pixel.

- When casting one ray per pixel, we are likely to have aliasing artifacts.
- To improve matters, we can cast more than one ray per pixel and average the result.
- A.k.a., **super-sampling and averaging down**.
Speeding it up

- Vanilla ray tracing is really slow!
- Consider: \( m \times m \) pixels, \( k \times k \) supersampling, and \( n \) primitives, average ray path length of \( d \), with 2 rays cast recursively per intersection.
- Complexity =
- For \( m=1,000,000, k = 5, n = 100,000, d=8 \)…very expensive!!
- In practice, some acceleration technique is almost always used.
- We’ve already looked at reducing \( d \) with adaptive ray termination.
- Now we look at reducing the effect of the \( k \) and \( n \) terms.
Antialiasing by adaptive sampling

- Casting many rays per pixel can be unnecessarily costly.
- For example, if there are no rapid changes in intensity at the pixel, maybe only a few samples are needed.
- Solution: **adaptive sampling**.

**Q:** When do we decide to cast more rays in a particular area?
Let’s say you were intersecting a ray with a polyhedron:

- **Straightforward method**
  - intersect the ray with each triangle
  - return the intersection with the smallest $t$-value.

- **Q:** How might you speed this up?
Ray Tracing Acceleration Techniques

**Approaches**

- **Faster Intersection**
  - Uniform grids
  - Spatial hierarchies
  - k-d, oct-tree, bsp
  - Hierarchical grids
  - Hierarchical bounding volumes (HBV)
  - Tighter bounds
  - Faster intersector

- **Fewer Rays**
  - Early ray termination
  - Adaptive sampling

- **Generalized Rays**
  - Beam tracing
  - Cone tracing
  - Pencil tracing
Another approach is **uniform spatial subdivision**.

**Idea:**
- Partition space into cells (voxels)
- Associate each primitive with the cells it overlaps
- Trace ray through voxel array *using fast incremental arithmetic* to step from cell to cell
Uniform Grids

- Preprocess scene
- Find bounding box
Uniform Grids

- Preprocess scene
  - Find bounding box
  - Determine resolution
    \[ n_v = n_x n_y n_z \propto n_o \]
    \[ \max(n_x, n_y, n_z) = d \sqrt[3]{n_o} \]
Uniform Grids

- Preprocess scene
  - Find bounding box
  - Determine resolution
  - Place object in cell, if object overlaps cell

\[
\max(n_x, n_y, n_z) = d \frac{3}{2} n_o
\]
Uniform Grids

- Preprocess scene
  - Find bounding box
  - Determine resolution
  - Place object in cell, if object overlaps cell
  - Check that object intersects cell

\[
\max(n_x, n_y, n_z) = d^{\frac{3}{n_o}}
\]
Uniform Grids

- Preprocess scene
- Traverse grid
  - 3D line – 3D-DDA
  - 6-connected line
Caveat: Overlap

- **Optimize for objects that overlap multiple cells**

- Traverse until $t_{\text{min}}(\text{cell}) > t_{\text{max}}(\text{ray})$
- **Problem:** Redundant intersection tests:
- **Solution:** Mailboxes
  - Assign each ray an increasing number
  - Primitive intersection cache (mailbox)
    - Store last ray number tested in mailbox
    - Only intersect if ray number is greater
Non-uniform spatial subdivision

- Still another approach is **non-uniform spatial subdivision**.

- Other variants include k-d trees and BSP trees.

- Various combinations of these ray intersections techniques are also possible. See Glassner and pointers at bottom of project web page for more.
Non-uniform spatial subdivision

- Best approach - k-d trees or perhaps BSP trees
  - More adaptive to actual scene structure
  - BSP vs. k-d tradeoff between speed from simplicity and better adaptability
Spatial Hierarchies

Letters correspond to planes (A)

Point Location by recursive search
Spatial Hierarchies

Letters correspond to planes (A, B)

Point Location by recursive search
Spatial Hierarchies

Letters correspond to planes (A, B, C, D)

Point Location by recursive search
Variations

kd-tree  oct-tree  bsp-tree
Ray Traversal Algorithms

- Recursive inorder traversal
- [Kaplan, Arvo, Jansen]

\[
\begin{align*}
\text{Intersect}(L, t_{\text{min}}, t_{\text{max}}) \quad & \quad \text{Intersect}(L, t_{\text{min}}, t^*) \quad \text{Intersect}(R, t_{\text{min}}, t_{\text{max}}) \\
\text{Intersect}(R, t^*, t_{\text{max}}) & \quad \text{Intersect}(R, t^*, t_{\text{min}})
\end{align*}
\]
Build Hierarchy Top-Down

Choose splitting plane
• Midpoint
• Median cut
• Surface area heuristic
Surface Area and Rays

- Number of rays in a given direction that hit an object is proportional to its projected area.

- The total number of rays hitting an object is given by Crofton’s Theorem:
  - For a convex body:
    \[ \overline{A} = \frac{S}{4} \]

- For example: sphere
  \[ S = 4\pi r^2 \quad \overline{A} = A = \pi r^2 \]
Surface Area and Rays

- The probability of a ray hitting a convex shape that is completely inside a convex cell equals

\[
\Pr[r \cap S_o \mid r \cap S_c ] = \frac{S_o}{S_c}
\]
Surface Area Heuristic

\[ C = t_t + p_a N_a t_i + p_b N_b t_i \]

- Intersection time: \( t_i \)
- Traversal time: \( t_t \)

\[ t_i = 80t_t \]
Surface Area Heuristic

\[ p_a = \frac{S_a}{S} \quad \quad p_b = \frac{S_b}{S} \]
Hierarchical bounding volumes

- We can generalize the idea of bounding volume acceleration with **hierarchical bounding volumes**.

- Key: build balanced trees with **tight bounding volumes**.

  Many different kinds of bounding volumes.  
  Note that bounding volumes can overlap.