Safety First: A Two-Stage Algorithm for LTL Games

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Motivation

- Recently, significant algorithmic advances in the game-theoretic approach to synthesis of reactive systems has renewed interest. Piterman 06, Piterman et al 06, Kupferman et al 06, Chatterjee et al 07, Bloem et al 07 are a few examples.
- Despite challenges in scalability, there is increasing hope that synthesis algorithms may be applied to the design and diagnosis of intricate, safety critical protocols.
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Outline

LTL Synthesis - Pnueli and Rosner (POPL'89)

- Automatically build design from specification
- Input:
  - Set of LTL formulae, e.g. $\mathsf{G}(req \rightarrow \mathsf{F}\, ack), \mathsf{G}(\neg req \rightarrow \mathsf{X}(\neg ack))$
  - Partition of the atomic propositions (input/output signals)
  - Environment controls inputs and system controls outputs
- The set of LTL formulae are converted to a non-terminating game with system as protagonist and environment as antagonist.
- Output: Automatically created functionally correct finite-state machine from the winning strategy of the system.
  - If such strategy doesn't exist then the specification is unrealizable.

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- The system's intended behavior is described by combination of LTL formulae or as ω- regular automata.
- In a naive approach, all formulae and automata are reduced to one deterministic automaton, whose transition structure provides the game graph.
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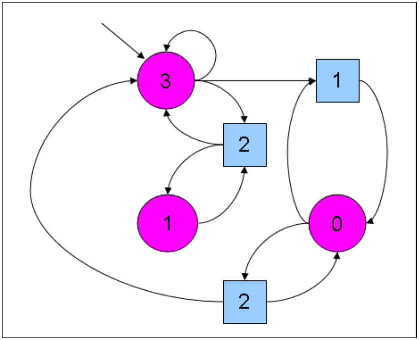
$player0 \rightarrow \Box$
wins if largest integer occuring infinitely often is even

$player1 \rightarrow \bigcirc$
wins if largest integer occuring infinitely often is odd



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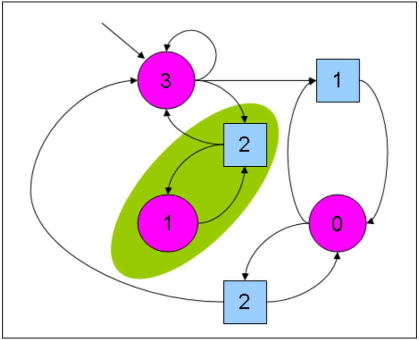

Game Graphs

- A game graph $G = ((S, E), S_0, S_1)$ is a directed graph (S, E) with a finite state space S, a set of edges E and a partition (S_0, S_1) of the state space belonging to player 0 and 1 respectively. We assume that every state has an outgoing edge.
- The game is started by placing a token in one of the S_{init} and then this token is moved along the edges, when the token is in a state $s \in S_1$, player 1 selects one of its outgoing edges and vice-versa. The result is an infinite path in the game graph termed as a play.
- A strategy for a player is a recipe that specifies how to extend finite path. Formally strategy for player i is a function $\sigma : S^*.S_i \rightarrow S$.

Parity Game

- For a game graph $G = (Q, E)$ and a parity function $\pi : Q \to [k]$, a parity acceptance condition requires that the maximal $\pi(s)$ occuring infinitely often is odd (even) for *player*$1(0)$.
- A generalized parity game for a game graph $G = (Q, E)$ and a set of parity functions $\{\pi_i | \pi_i : Q \to [k_i]\}$ is played between the conjunctive and disjunctive player. The conjunctive player wins if it has a strategy to win all the parity acceptance conditions while the disjunctive player wins if it has a strategy for some parity acceptance condition.

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Two Game Theoretic Approaches

- The standard approach which is the focus of this talk, requires the determinization of word automata.

$$LTL \rightarrow NBW \rightarrow DRW$$

- The Safraless Approach avoids determinization by working with Tree Automata.

$$LTL \rightarrow NGBW \rightarrow \overset{realizability}{UGCW} \rightarrow \overset{lang-empt}{UGCT} \rightarrow \overset{optimistic-reduction}{NBT}$$

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Specification of a simple 2-Client Arbiter

- Initially there are no acknowledgments.

$$\neg ack_0 \wedge \neg ack_1$$

- The acknowledgmnets are mutually exclusive.

$$\mathsf{G}(\neg ack_0 \vee \neg ack_1)$$

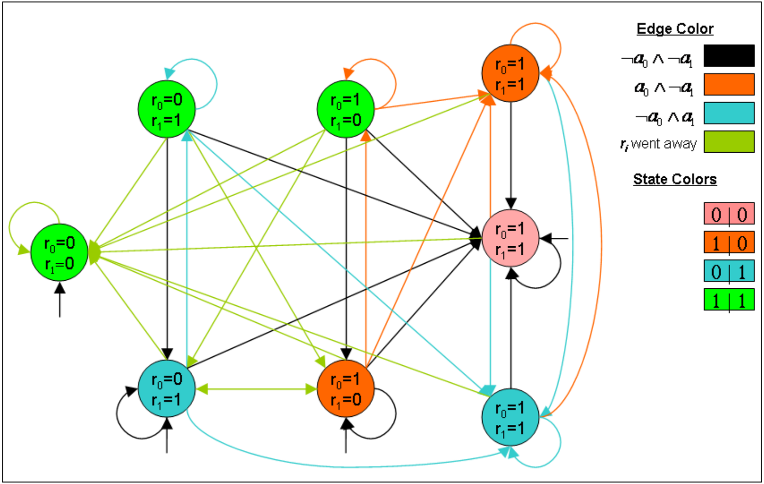
- There are no spurious acknowledgmnets.

$$\forall i\,.\,\mathsf{G}(\neg req_i \rightarrow \mathsf{X}(\neg ack_i))$$

- Every request will eventually be acknowledged

$$\forall i\,.\,\mathsf{G}(req_i \rightarrow \mathsf{F}\,ack_i)$$

Example: Game Graph and Synthesized Strategy



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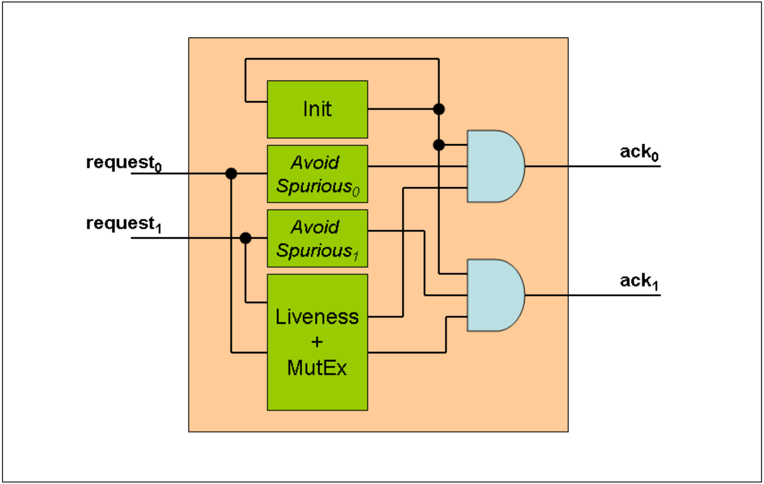
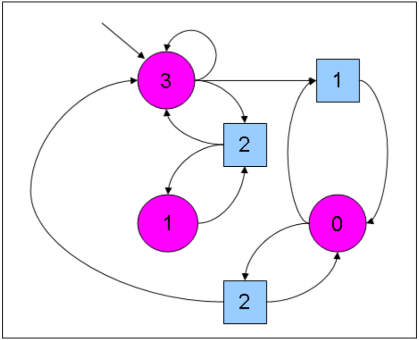


Example: Game play & Strategy Computation for Player 1

◯ [Player 1] wins if the maximal $\pi(s)$ occuring infinitely often is odd.



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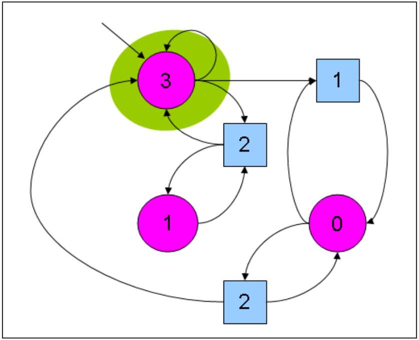

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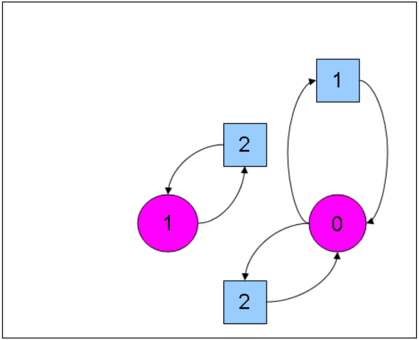

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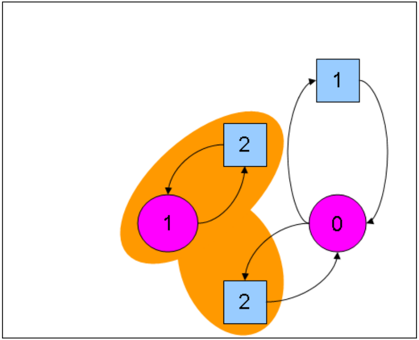

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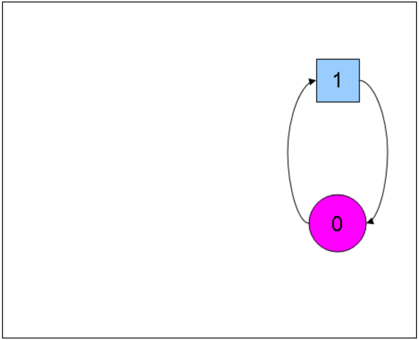

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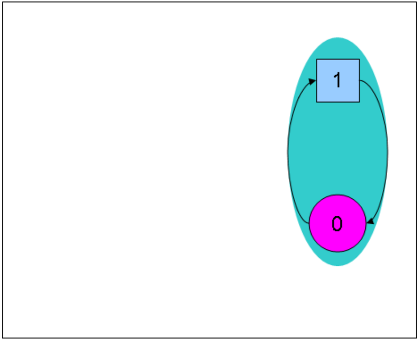

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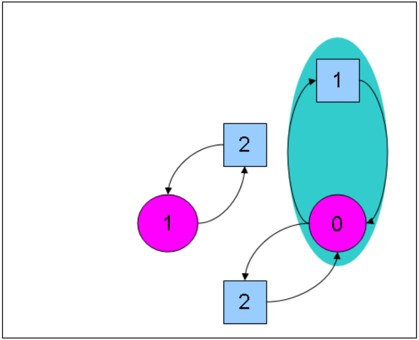

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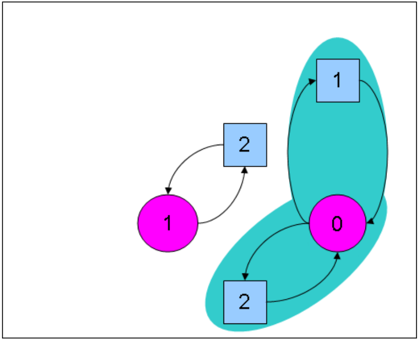

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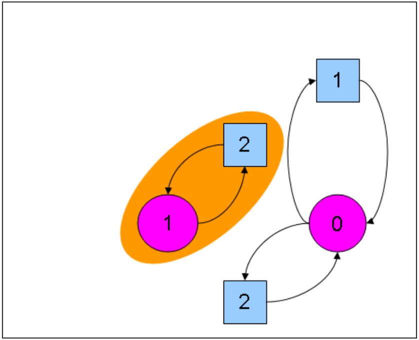

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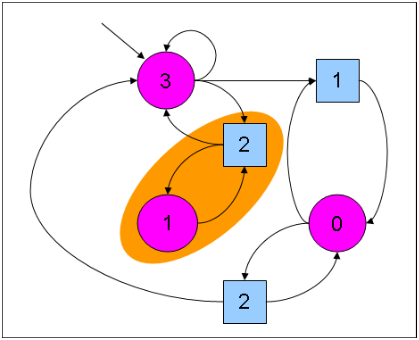

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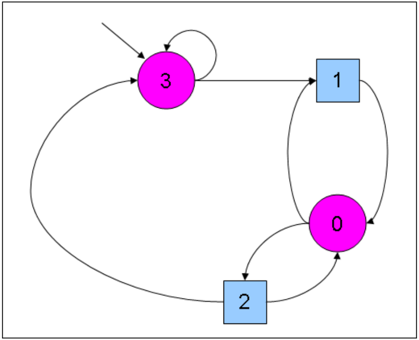

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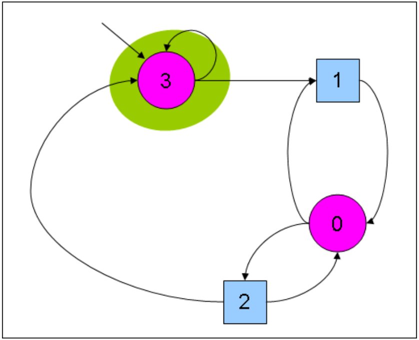

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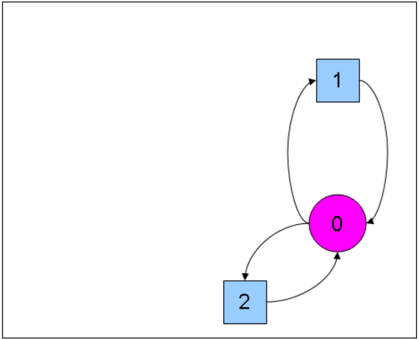

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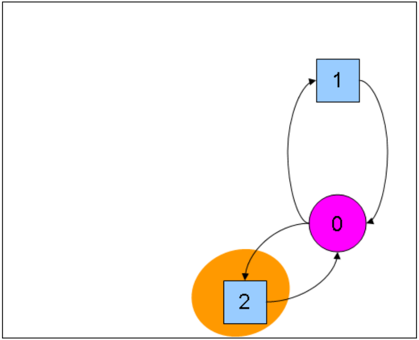

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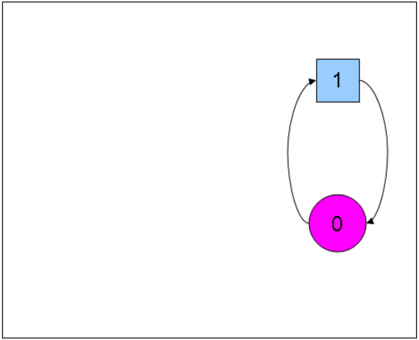

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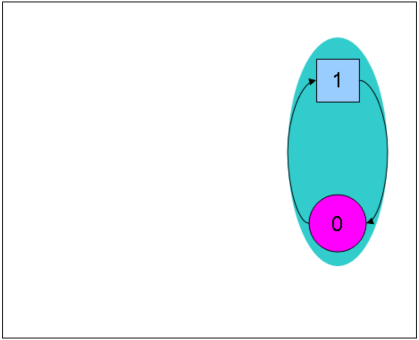

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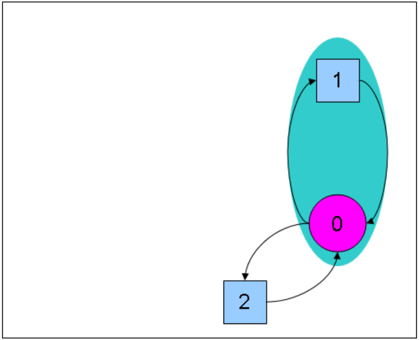

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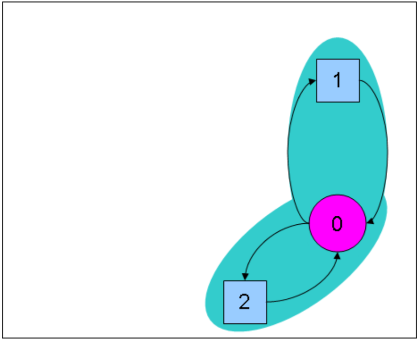

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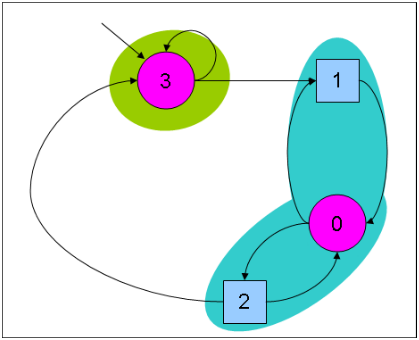

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◯ [Conjunctive Player] wins if it has a strategy to win all the parity functions

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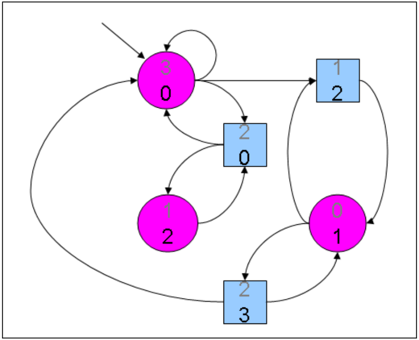

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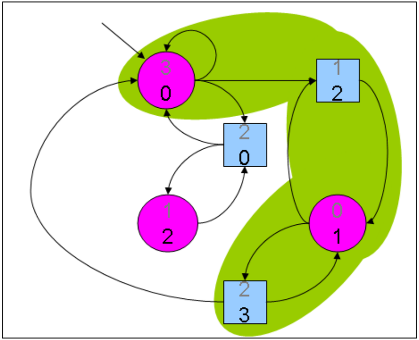

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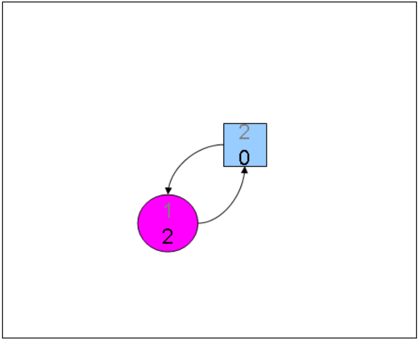

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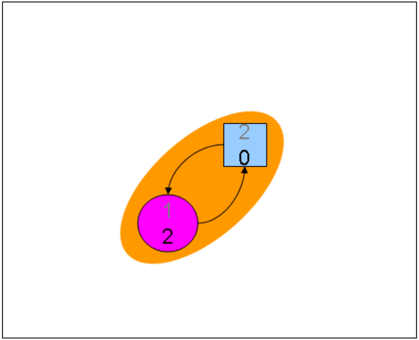

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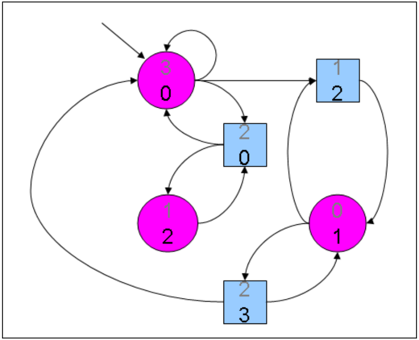

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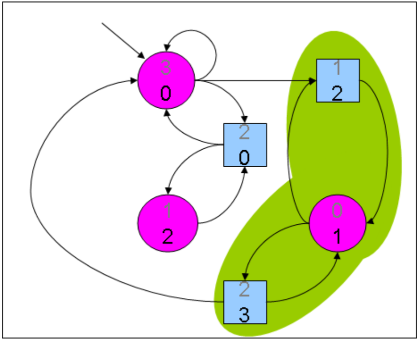

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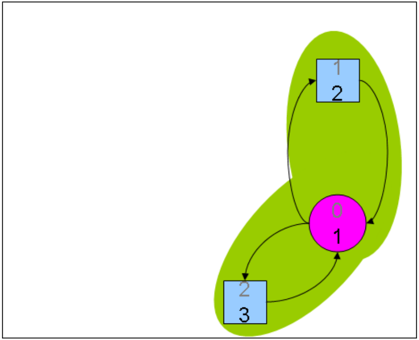

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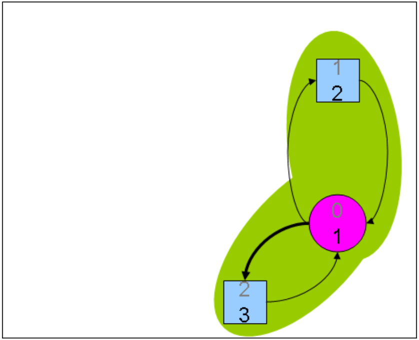

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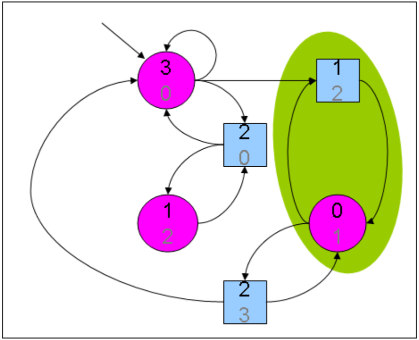

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- Generalized parity game is an NP-Complete problem and the current algorithm (Chatterjee et. al 07) is computationally very expensive.
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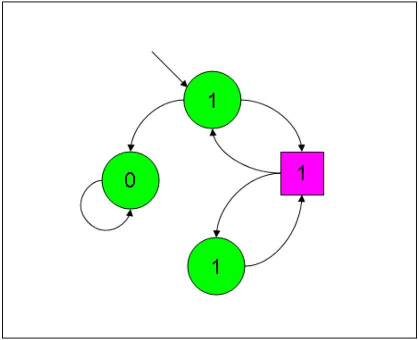
Safety Properties

A safety condition for a game graph $G = (Q, E)$ is a function $\pi : Q \rightarrow \{0, 1\}$ such that there is no transition $(u, v) \in E$ such that $\pi(u) = 0$ and $\pi(v) = 1$.



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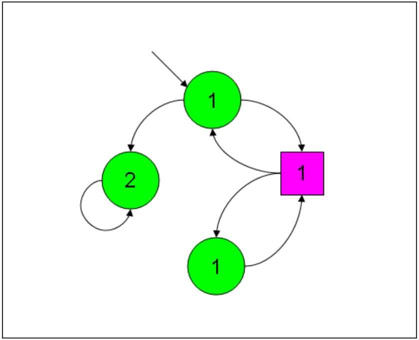
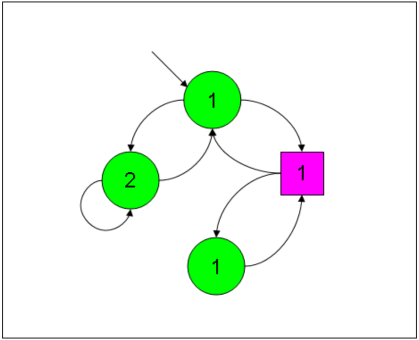


Persistence Properties

A persistence condition for a game graph $G = (Q, E)$ is a function $\pi : Q \to \{1, 2\}$.



The Claim

- What is so unique about persistence properties?
- The winning states for persistence properties can be categorized into persistent and transient states.
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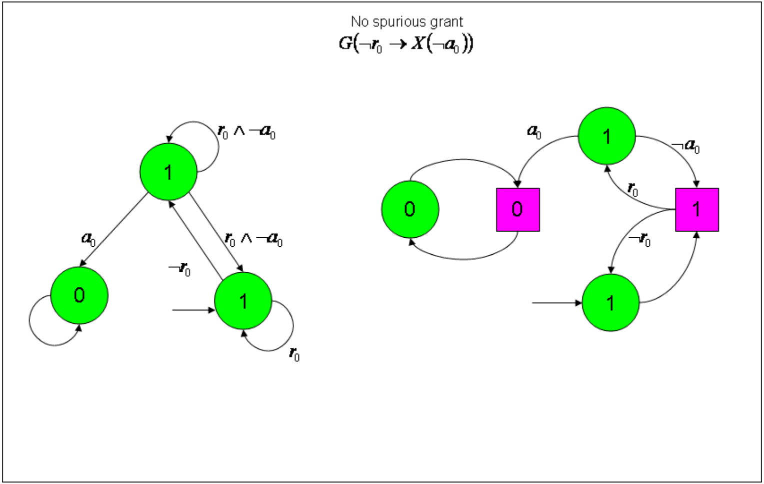
Input/Output based game → State based game



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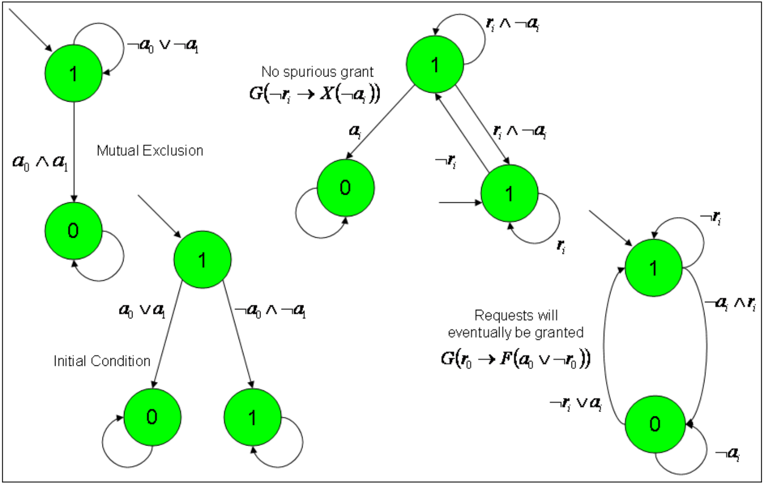

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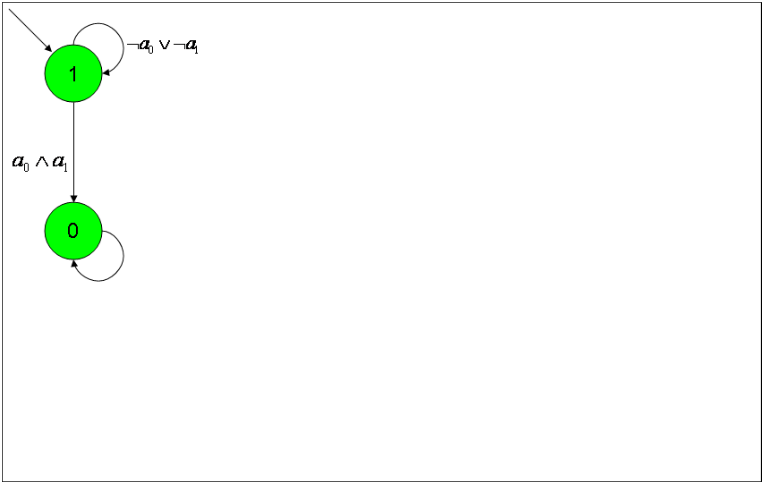

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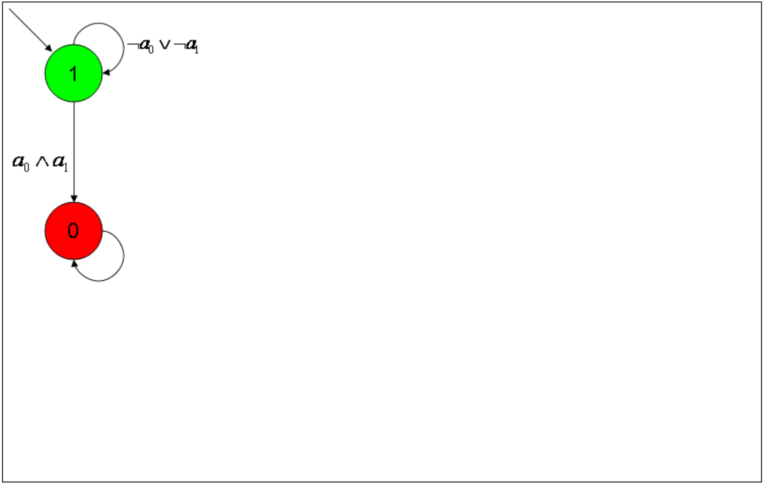

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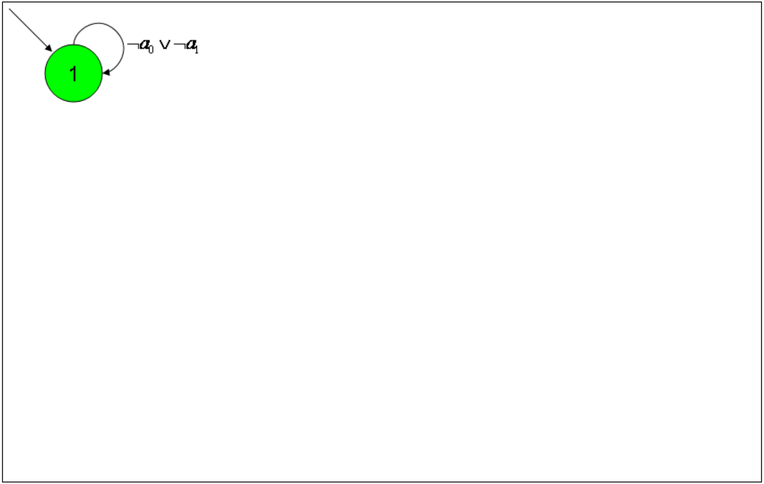

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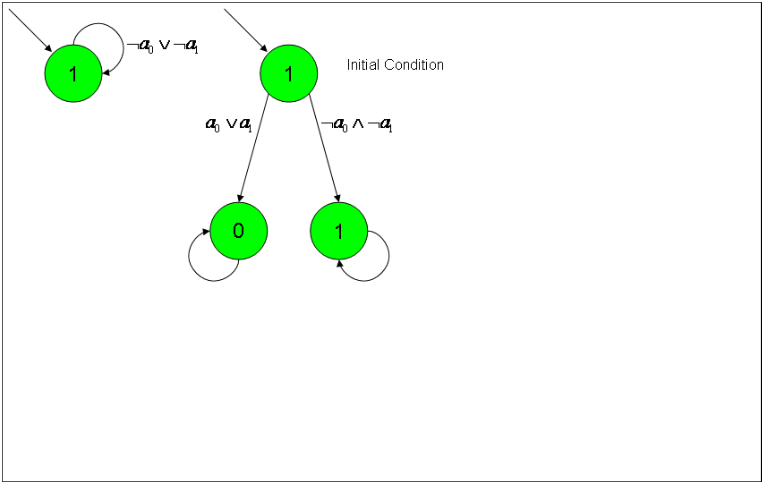

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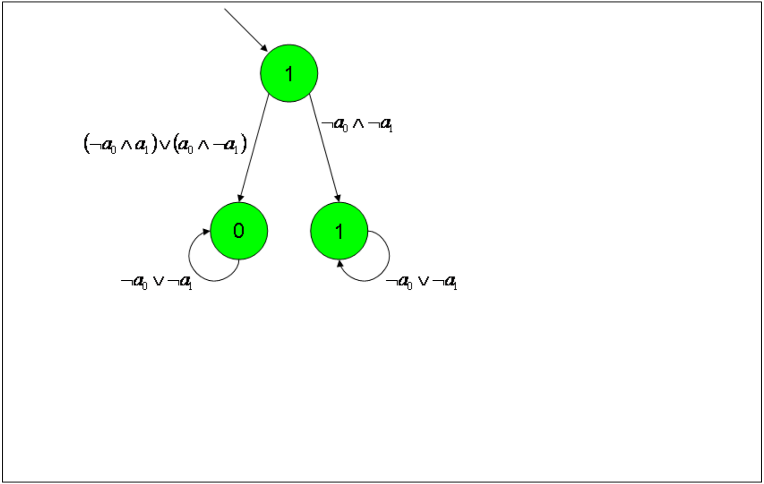

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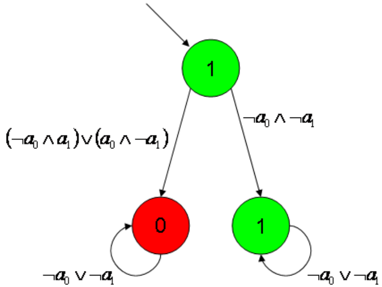

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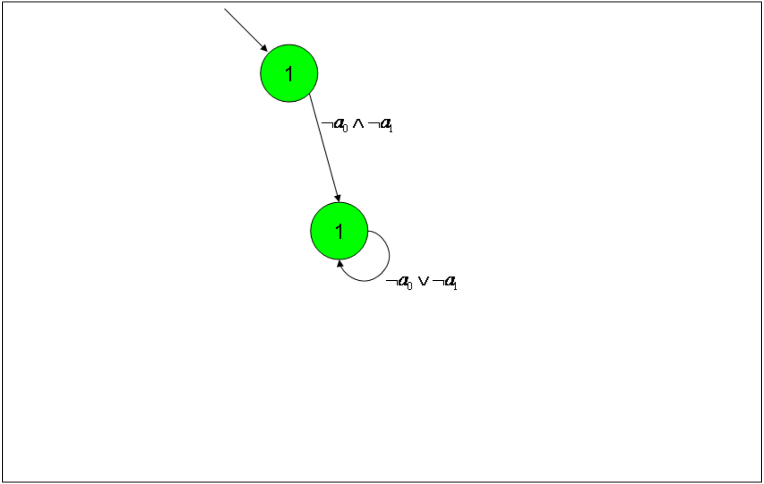

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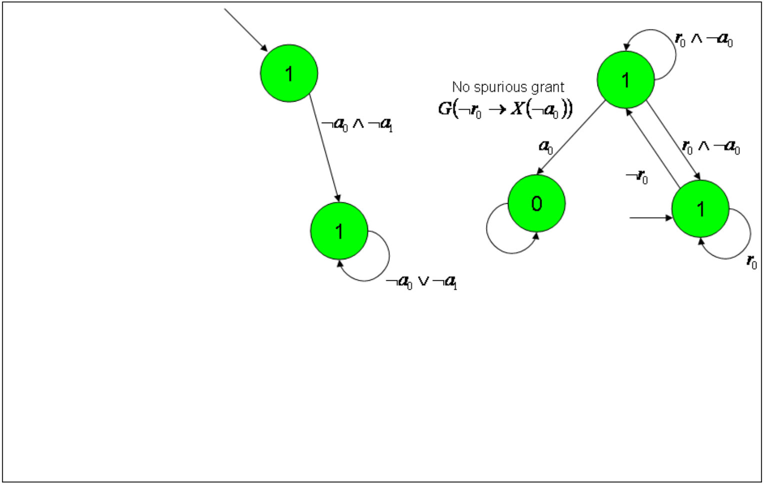

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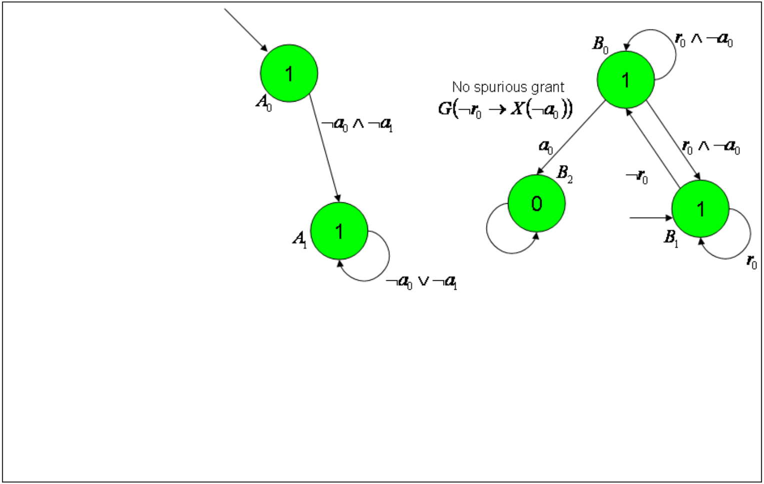

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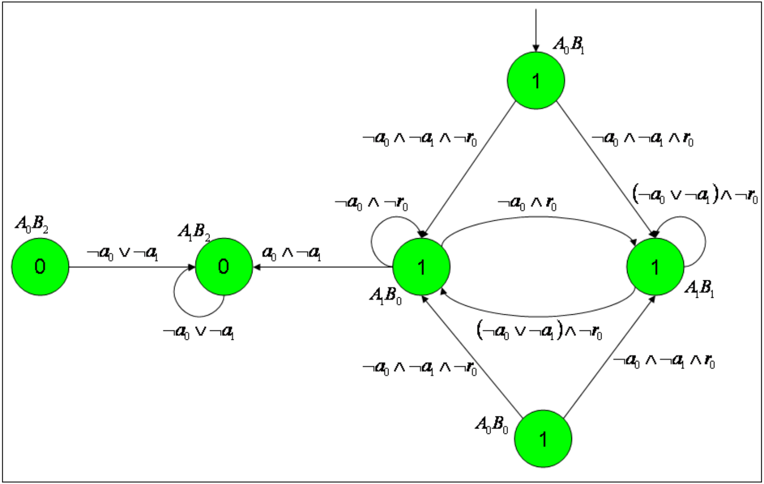

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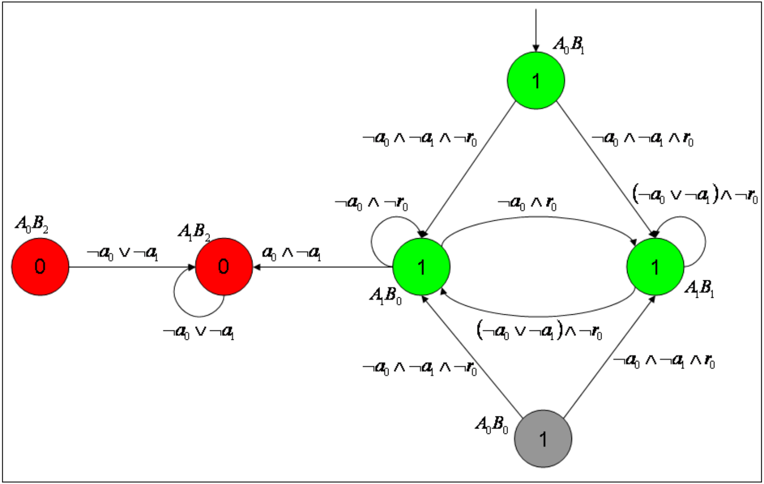

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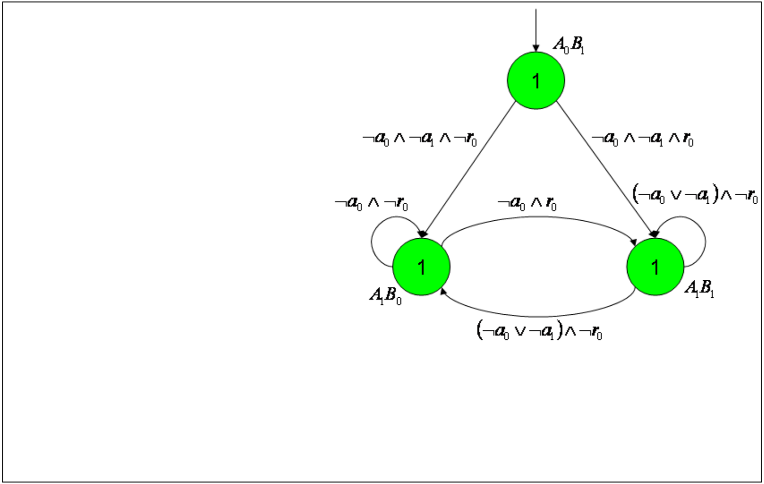

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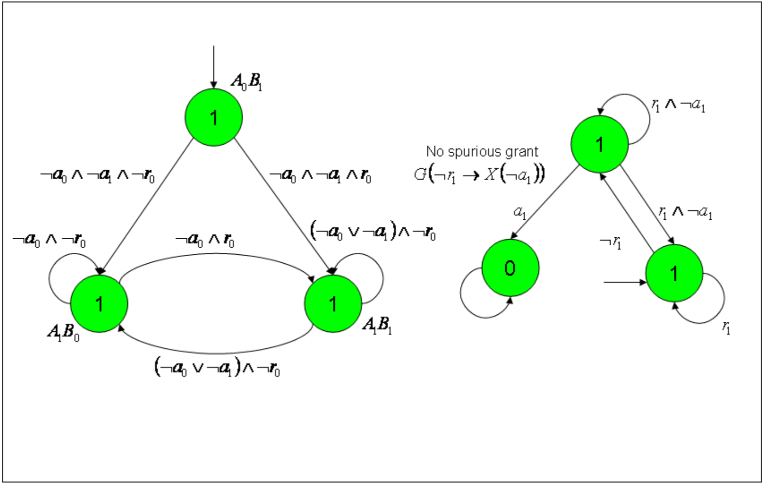

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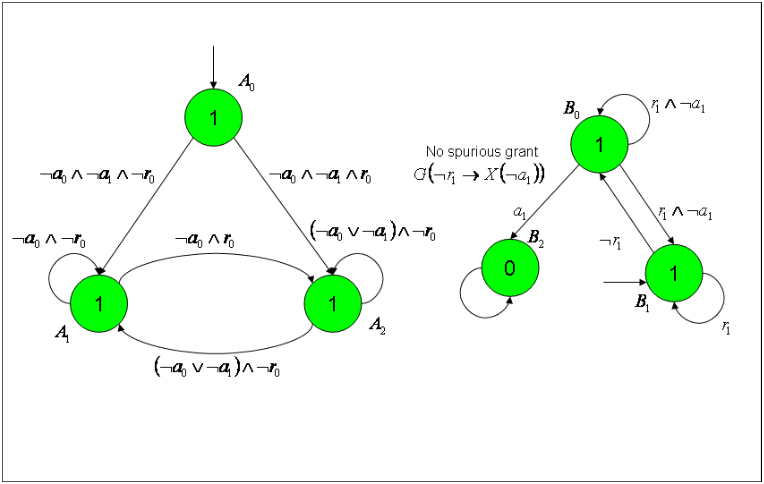

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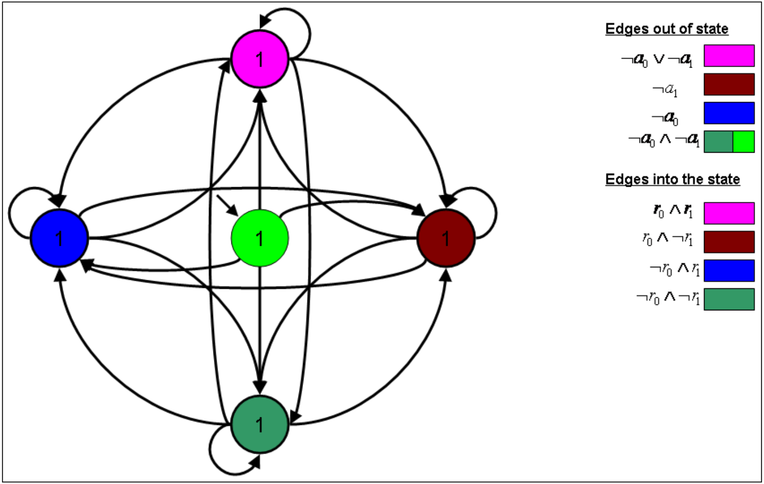

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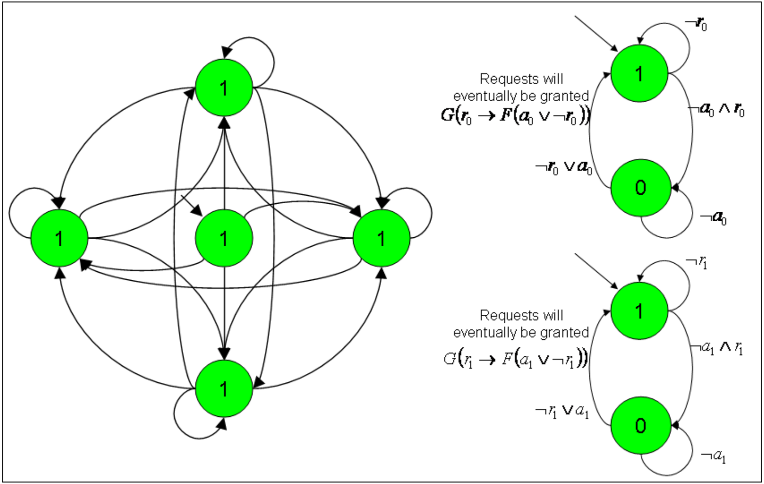

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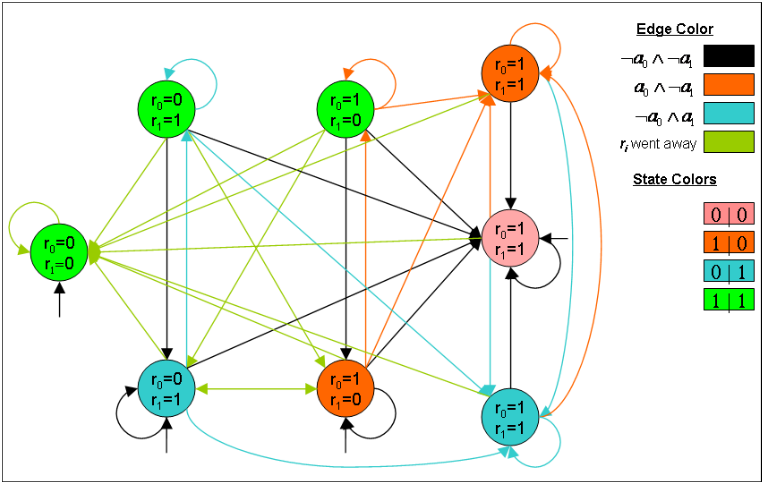

How significant is the improvement?

- The complexity of "classical" algorithm of (Chatterjee et al 07) is given by

$$O(m \cdot n^{2d}) \cdot \binom{d}{d_1, d_2, \ldots, d_k},$$

$$d_i = \lceil k_i/2 \rceil$$

  If π_k is a safety condition, solving the game in two stages leads to a better bound for the second stage, $O(m \cdot n^{2d-2}) \cdot \binom{d-1}{d_1, \ldots, d_{k-1}}$, while the first stage runs in $O(m \cdot n^2)$.

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Outline

Methodology

- Identify the safety/persistent properties in the specification.
- Translate each property into a deterministic automaton.
- Compose the automaton with already existing game-graph and then playing the 2-player game on the relevant section of the graph.
- Determinize all the remaining non-safety/non-persistent properties and then compose with the game-graph and play the final generalized parity game on the relevant section of the graph.
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Algorithm

1. $\textsc{Safety-First}(G, SPECIFICATION)$
2. $(SAFETY, NON-PERSISTENT) \leftarrow SPECIFICATION$
3. **foreach** $\varphi \in SAFETY$
   3.1 $G = G \parallel automaton_{det}^{\varphi}$
   3.2 $(Q_{sys}, E_{sys}) \leftarrow \textsc{Chatterjee}(G, \varphi)$
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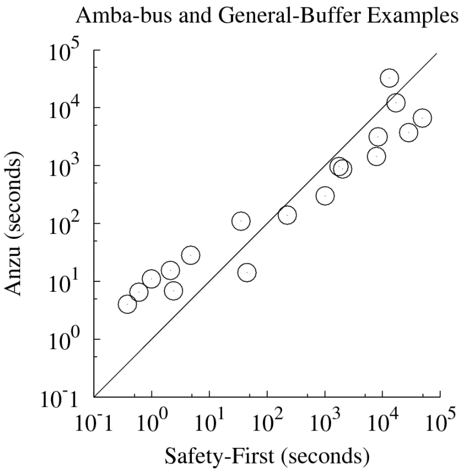
Amba-bus and General-Buffer Examples
Anzu (seconds)
Safety-First (seconds)
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
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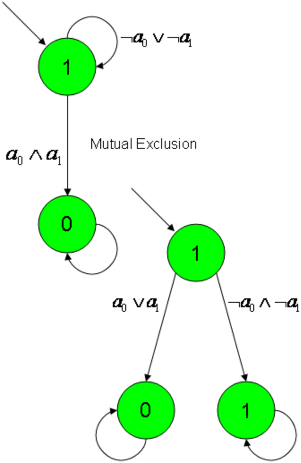

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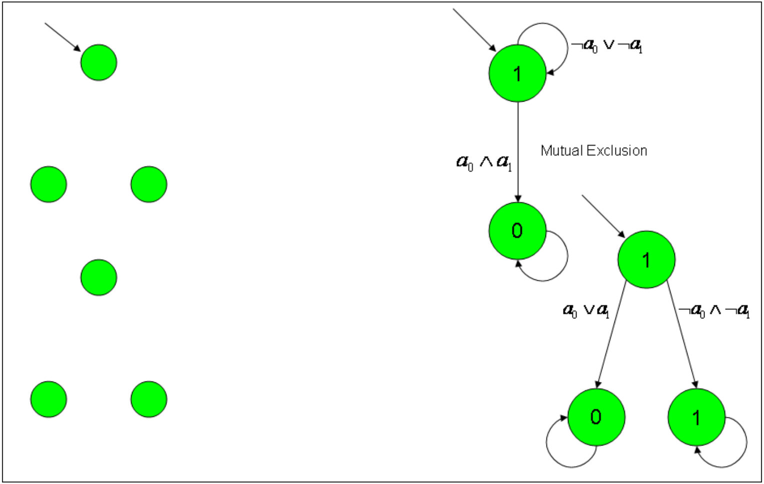

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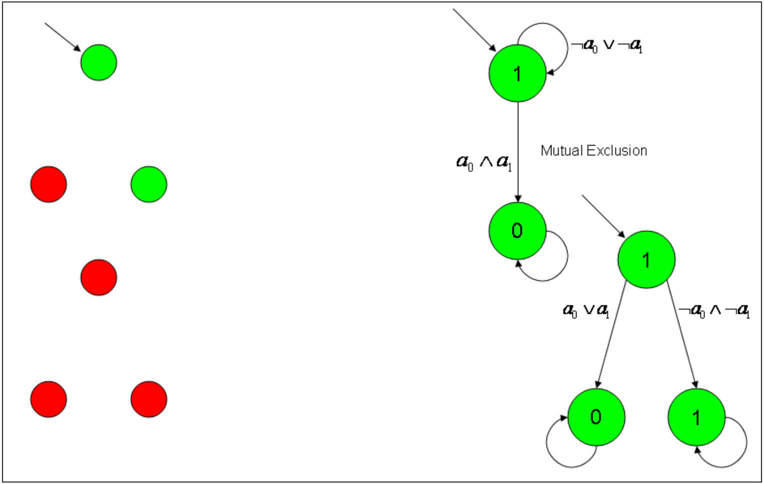

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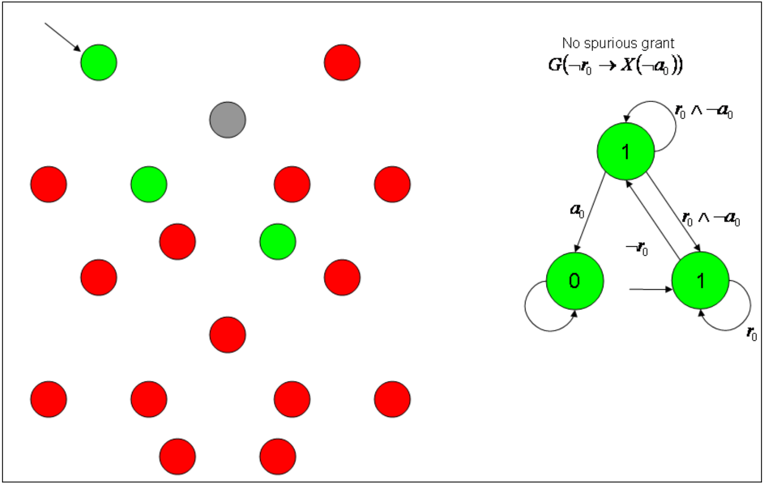

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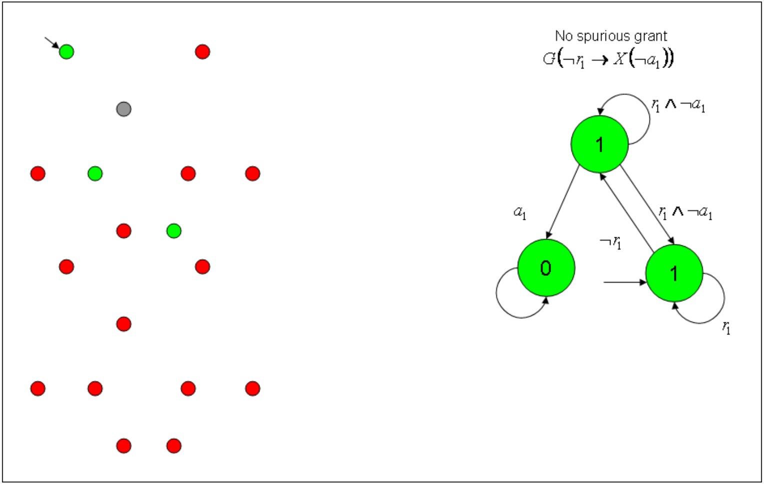

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