IC3 Software Model Checking on Control Flow Automata

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***Abstract*—In recent years, the inductive, incremental verification algorithm IC3 had a major impact on hardware model checking. Also with respect to software model checking, a number of adaptations of Boolean IC3 and combinations with CEGAR and ART-based techniques have been developed. However, most of them exploit the peculiarities of software programs, such as the explicit representation of control flow, only to a limited extent. In this paper, we propose a technique that supports this explicit representation in the form of control flow automata, and integrates it with symbolic reasoning about the data state space of the program. It thus provides a true lifting of IC3 from hardware to software model checking. By evaluating the approach on a number of case studies using a prototypical implementation, we demonstrate that our method shows promising results.**

I. Introduction

IC3 [1] is an incremental algorithm that has originally been designed for verifying invariant properties of finite transition systems. It constructs an over-approximation of the reachable state space by generating Boolean clauses that are inductive relative to stepwise reachability information. During this construction, candidate counterexamples are being disproved using Boolean SAT-solving techniques. This approach has turned out to be highly effective; in fact, it is considered to be one of (if not *the*) most important contribution of bit-level formal verification of hardware systems for the last decade.

There have been several attempts to lift Boolean IC3 to the domain of software model checking. As this setting usually induces infinite-state systems, more advanced symbolic reasoning techniques are required. The most prominent one is Satisfiability Modulo Theories (SMT). Here, sets of states are symbolically specified by first-order formulas over constraints from the respective theory, and SMT-solving techniques are employed to rule out spurious counterexamples.

One of the first integrations of SMT into IC3 has been presented in [2]. In addition to the generalisation of SAT to SMT solving, it exploits the partitioning of the program's state space as induced by its control flow graph. This is achieved by unwinding the latter into an Abstract Reachability Tree (ART) in which each node is associated with a control location and a formula, resulting in an "explicit-symbolic" approach named Tree-IC3. Candidate counterexamples are handled by computing under-approximations of pre-images.

The advantages in comparison to Boolean IC3 are twofold. First, Tree-IC3 eliminates the possible redundancy of subformulas partitioning of the control state space, the solver is exposed to simpler and smaller formulas.

On the downside, the key idea underlying IC3, relative inductiveness, cannot be directly applied in this setting due to the partitioned representation that leads to a path-wise unwinding of the transition system. The follow-up publication [3] therefore reverts to a monolithic transition relation, replacing the pre-image computation by (implicit) predicate abstraction. The latter is a standard abstraction technique [4] that partitions the state space according to the equivalence relation induced by a set of predicates. Its implicit variant [5] allows to express abstract transitions without explicitly computing the abstract system. In the IC3 setting, this avoids theory-specific generalisation techniques.

In [6], Horn clauses are employed to represent recursive predicate transformers. Proof obligations are generalised using a specialised interpolation procedure for linear arithmetic. However, the latter again does not exploit relative induction.

In summary, existing work exploits the peculiarities of software programs only to a limited extent to support IC3-style verification. In this paper, we develop an approach that combines the advantage of explicitly handling the control flow of a program, employing a corresponding automata model, with relative inductive reasoning over a symbolic representation of its data space. It thus provides a true lifting of IC3 from hardware to software model checking. We demonstrate the applicability and efficiency of our method by evaluating it on a number of case studies using a prototypical implementation.

The remainder of this paper is organised as follows. We start by introducing some general concepts in Section II. Section III sets the stage for our contribution with a description of the original IC3 algorithm, which is then extended in Section IV by taking control flow automata into account. Results of the experimental evaluation are given in Section V. Section VI concludes the paper with a summary and a description of future work.

II. Preliminaries

A *control flow automaton* (CFA) $A = (L, G)$ consists of a finite set of locations $L = \{0, \ldots, n\}$, modeling the program counter of a corresponding sequential code, and edges in $G \subseteq L \times FO \times L$ labeled with quantifier-free first-order formulas over the set Var of program variables and their next-state primed forms, Var' [7]. Priming of a formula, φ', is the same as priming every variable in φ. Such a formula can either encode a variable assignment, containing primed and unprimed variables, or an assume statement in which case it will only contain unprimed variables. We assume that for any two locations there exists at most one edge between them.

Given a subset of variables $X \subseteq \mathit{Var}$, a *cube* over X is defined as a conjunction of *literals*, each literal being a variable or its negation in the propositional case and a theory atom or its negation in the quantifier-free first-order case. The negation of a cube, i.e. a disjunction of literals, is called a *clause*.

A *program* $\mathcal{P} = (A, l_0, l_E)$ consists of a CFA A representing the control flow, as well as an initial location $l_0 \in L$ and an error location $l_E \in L$. This representation allows to encode arbitrary programs and assertions for safety verification. For every assertion that has to be verified at location l, split l and introduce an edge with the negated assertion to l_E and an edge with the positive assertion between the split nodes. This way, checking the violation of assertions becomes checking reachability of l_E in A.

Given two locations $l_1, l_2 \in L$, we define the *transition formula*

$$T_{l_1 \to l_2} = \begin{cases} (pc = l_1) \wedge t \wedge (pc' = l_2) & \text{, if } (l_1, t, l_2) \in G \\ \mathit{false} & \text{, otherwise.} \end{cases} \tag{1}$$

This yields the global transition formula as

$$T = \bigvee_{(l_1,t,l_2) \in G} T_{l_1 \to l_2}. \tag{2}$$

Definition 1 (Relative inductiveness [1]): Given a transition formula T, a formula φ is inductive relative to another formula ψ if

$$\psi \wedge \varphi \wedge T \Rightarrow \varphi' \tag{3}$$

is valid.

Starting from Def. 1 we can refine relative inductiveness to consider only a single transition rather than the whole transition relation.

Definition 2 (Edge-relative inductiveness): Given a CFA A and locations $l_1, l_2 \in L$, a formula φ is edge-relative inductive to another formula ψ if

$$\psi \wedge \varphi \wedge T_{l_1 \to l_2} \Rightarrow \varphi' \tag{4}$$

is valid.

Note that edge-relative inductiveness does also hold if $(l_1, t, l_2) \notin G$ for every t. In this case, $T_{l_1 \to l_2} = \mathit{false}$, which makes (4) hold trivially, i.e. if we are in a state satisfying φ and we cannot leave it via the considered edge, we remain in a state satisfying φ.

To handle the possibly infinite state space over Var, we use a symbolic representation through quantifier-free first-order formulas.

Definition 3 (Data region): A data region is represented by a quantifier-free FO formula s over Var and consists of all variable assignments σ satisfying s, i.e., $\{\sigma \mid \sigma \models s\}$.

Based on Def. 3 we can augment a data region with a control flow location $l \in L$. This information, sometimes referred to as atomic region [8] is called *region* in the following.

Definition 4 (Region): We define a region $r = (l, s)$ as a pair consisting of location $l \in L$ and data region s. Given such a region $r = (l, s)$, the corresponding formulas are defined as $\{\phi \mid \phi \equiv (pc = l \wedge s)\}$. Analogously the corresponding formulas for a negated region $\neg r$ are defined as $\{\phi \mid \phi \equiv \neg(pc = l \wedge s)\}$.

Given two regions r_1, r_2 and representatives of their corresponding propositional formulas φ_1, φ_2, then r_1 is inductive relative to r_2 iff φ_1 is inductive relative to φ_2. The analogue can be defined for the special case of edge-relative inductiveness.

Using their corresponding formula, we can use relative and edge-relative inductiveness for regions very similar to [1].

Even though it works for two non-negated regions as well, the IC3 algorithm only makes use of the case where we check whether a negated region $\neg r_2$ is inductive edge-relative to a non-negated region r_1. Therefore we will only consider this case in the following and inspect it in detail. We found two different cases whose premise can be statically determined and that simplify the SMT queries that we have to use.

Lemma 1 (Relative inductive regions): Assuming two regions $r_1 = (l_1, s_1)$, $\neg r_2 = \neg(l_2, s_2)$, we can reduce edge-relative inductiveness of $\neg r_2$ to r_1 to

$$s_1 \wedge T_{l_1 \to l_2} \Rightarrow \neg s_2' \qquad \text{, if } l_2 \neq l_1 \tag{5}$$

$$s_1 \wedge \neg s_2 \wedge T_{l_1 \to l_2} \Rightarrow \neg s_2' \qquad \text{, if } l_2 = l_1 \tag{6}$$

Proof 1: Given two regions $r_1 = (l_1, s_1)$ and $r_2 = (l_2, s_2)$ with corresponding formulas φ_1 and φ_2, we have:

$$\varphi_1 \equiv (pc = l_1 \wedge s_1) \qquad \neg\varphi_2 \equiv \neg(pc = l_2 \wedge s_2)$$

Def. 2 yields:

$$\begin{aligned} & (pc = l_1 \wedge s_1) \wedge \neg(pc = l_2 \wedge s_2) \wedge T_{l_1 \to l_2} \\ & \Rightarrow \neg(pc' = l_2 \wedge s_2') \\ \equiv\; & (pc = l_1 \wedge s_1) \wedge (pc \neq l_2 \vee \neg s_2) \wedge T_{l_1 \to l_2} \\ & \Rightarrow (pc' \neq l_2 \vee \neg s_2') \end{aligned}$$

If $l_1 \neq l_2$, this is equisatisfiable to

$$\begin{aligned} & (\mathit{true} \wedge s_1) \wedge (\mathit{true} \vee \neg s_2) \wedge T_{l_1 \to l_2} \Rightarrow (\mathit{false} \vee \neg s_2') \\ \equiv\; & s_1 \wedge T_{l_1 \to l_2} \Rightarrow \neg s_2' \end{aligned}$$

Otherwise, we obtain

$$\begin{aligned} & (\mathit{true} \wedge s_1) \wedge (\mathit{false} \vee \neg s_2) \wedge T_{l_1 \to l_2} \Rightarrow (\mathit{false} \vee \neg s_2') \\ \equiv\; & s_1 \wedge \neg s_2 \wedge T_{l_1 \to l_2} \Rightarrow \neg s_2' \end{aligned}$$

□

In Proof 1, we can use the presented equisatisfiable transformations because the transition from l_1 and l_2 implicitly contains the atoms $(pc = l_1)$ and $(pc' = l_2)$. Therefore in the first case, where $l_1 \neq l_2$, the atoms $(pc = l_1)$ and $(pc' \neq l_2)$ can be rewritten to *true*, while the atom $(pc' \neq l_2)$ can be rewritten to *false*. The analogous holds for the case where $l_1 = l_2$.

Given a program $\mathcal{P}$, we can define a finite *path* π of length n as a sequence $l_0, l_1, \ldots, l_n$, s.t. for every $0 \leq i < n$ there exists an edge $(l_i, t_i, l_{i+1}) \in G$ in $\mathcal{P}$. A path π is called *feasible* iff for every l_j in π we can construct a region (l_j, s_j) that is non-empty, i.e. $s_j \not\equiv false$, s.t. $s_i \wedge T_{l_i \to l_{i+1}} \Rightarrow s_{i+1}$ for $0 \leq i < n$.

III. Original IC3 Algorithm

Let $S = (X, I, T)$ be a transition system over a set X of Boolean variables, and $I(X)$ and $T(X, X')$ two propositional formulas respectively describing the initial condition and the transition relation over variables in X and next-state primed successors X'. Given a propositional property $P(X)$, we want to verify that every state in S that is reachable from a state in I satisfies P. Sometimes also an inverted formulation is used like in [9] where $\neg P$ states are *bad states* and we want to show that no bad state is reachable from the initial states. The main idea of the IC3 algorithm [1] and the earlier *finite state inductive strengthening* (FSIS [10]) is that if P is *inductive*, i.e. $I \Rightarrow P$ and $P \wedge T \Rightarrow P'$, then P is also an invariant on S. However, even if P is an invariant on S it may not be inductive. Therefore the goal of IC3 and FSIS is to produce a so called *inductive strengthening* F of property P, s.t. $F \wedge P$ is inductive. This means that we can restrict the set of states in P to a smaller set of states in the intersection of F and P, which still contains all states reachable from I, but excludes unreachable states that will lead to a violation of induction. While FSIS tries to come up with such a strengthening in one step, IC3 proceeds in a more relaxed approach and constructs F *incrementally*.

This incremental construction is based on a sequence of *frames* $F_0, \ldots, F_k$ for which

$$\begin{aligned} I &\Rightarrow F_0 & & \\ F_i &\Rightarrow F_{i+1} & &, \text{for } 0 \leq i < k \\ F_i &\Rightarrow P & &, \text{for } 0 \leq i \leq k \\ F_i \wedge T &\Rightarrow F'_{i+1} & &, \text{for } 0 \leq i < k \end{aligned}$$

has to hold in order to produce an inductive invariant. The algorithm starts with two initial checks for 0- and 1-step reachable states in $\neg P$ and afterwards initializes the first frame F_0 to I. The rest of the algorithm can be divided into an inner and an outer loop, sometimes also referred to as *blocking* and *propagation* phases, respectively.

The outer loop iterates over the maximal frame index k, looking for states in F_k that can reach $\neg P$, so called *counterexamples to induction* (CTI). If such a CTI exists, it is analyzed in the inner loop, the blocking phase. If no such CTI exists, IC3 tries to propagate clauses learned in frame F_i forward to F_{i+1}. In the end it checks for termination, which is given if $F_i = F_{i+1}$ for some $0 \leq i < k$.

The objective of the blocking phase is to decide whether a CTI is reachable from I or not. For this purpose, it maintains a set of pairs of frame indices and states, called *proof obligations*. From this set it picks the pair (i, s) with the smallest frame index i. If there is more than one pair with this frame index, the choice between those is arbitrary. For the chosen state, IC3 checks whether $\neg s$ is relative inductive to F_{i-1}, using (3). If it is relative inductive, we can block s in frames F_j for $0 \leq j \leq i+1$. But rather than just adding $\neg s$, IC3 first tries to obtain a clause that is a subset of $\neg s$ and therefore excludes more states. This clause, called a *generalization* of $\neg s$, is then added to the frames and afterwards the pair (i, s) in the set of proof obligations is replaced by $(i+1, s)$. If s is not relative inductive to F_{i-1} this means that there exists an F_{i-1} predecessor p that can reach s. IC3 therefore adds $(i-1, p)$ to the set of proof obligations. The blocking phase terminates if either there exists an s in the set of proof obligations that is relative inductive to an initial state at index 0, in which case there exists a counterexample path, or for every proof obligation the frame index $i > k$, i.e. there exists a $j \geq 0$, s.t. every predecessor of the original CTI is inductive relative to F_j.

IV. IC3 on Control Flow Automata

In this section we will present our IC3 algorithm for control flow automata as annotated pseudocode, give a short explanation and a proof of partial correctness, followed by an example showing the benefits of our method.

The most straight-forward way to lift IC3 to software model checking is to encode the control flow in an additional pc variable representing the program location, as presented in [2]. However, this approach introduces some tedious handling of the implicit pc variable and is not very competitive. One reason is that the control flow of the input program already gives a very clear structure to the system, which is completely disregarded when encoded inside a global transition formula. Thus our approach tries to exploit as much of the structure given in the input program as possible.

In [1] Bradley draws an analogy between the way IC3 proves properties on a transition system and how a human analyzes a system - by producing a set of lemmas s.t. each holds relative to a previous one and that all together imply the property. It is that stepwise approach that makes IC3 so competitive and which motivated us to apply our version of IC3 directly to a control flow automaton as an explicit representation of a program's possible execution steps.

With respect to the definition of programs we follow the notion of [9] and reason about error states, rather than property states.

The explicit representation of edges leads to a situation where we can reduce the possible transitions for a region $r = (l, s)$ to those that are available from l in the program $\mathcal{P}$, which allows us to formulate significantly smaller solver queries. Explicit initial and error states enable us to statically check 0-step and (potential) 1-step reachability, as well as to avoid initial and error conditions, which in turn reduces the size of the solver queries even further.

In analogy to bit-level IC3, we construct frame sequences $F_0, \ldots, F_k$, but instead of using global frames, we use *location-local* frames $F_{(i,l)}$ in every $l \in G$. We interpret those $F_{(i,l)}$ as the set of, possibly overapproximating, data regions reachable in at most i steps at location l.

Algorithm 1 Outer loop

```
Ensure: ret. value iff $l_E$ is reachable
  function BOOL PROVE
    if $l_0 = l_E$ or $((l_0, t, l_E) \in G$ and $sat(t))$ then
      return false
    initialize frames
    for $k = 1$ to $\ldots$ do
      if not STRENGTHEN($k$) then
        return false
      propagate
      if termination then
        return true
```

Alg. 1 works more or less like the original function *prove* in [1]. In the initial checks, we can reformulate the 0-step reachability query $I \wedge \neg P$ to the simple check whether $l_0 = l_E$. Also for 1-step counterexamples, originally $I \wedge T \wedge \neg P'$, we can still check the necessary syntactic reachability condition statically. If there exists an edge $e = (l_0, t, l_E)$, then we have to use the solver to check satisfiability of t. After those initial checks are completed we initialize frames F_0 and F_1. Exploiting the fact that only l_0 is initial, we can set $F_{(0,l_0)}$ to *true* and $F_{(0,l)}$ to *false* for every $l \neq l_0$. After the initial phase, the algorithm starts the main loop with frame limit k and tries to strengthen the new frame set. If the blocking phase succeeds and finds a strengthening for k, the propagation phase starts and tries to push learned data regions forward. The inner loop ends with checking termination. Here we have to modify the original termination condition $F_i = F_{i+1}$, for some i, to $F_{(i,l)} = F_{(i+1,l)}$ for some i and every $l \in L \setminus \{l_E\}$.

Algorithm 2 Strengthening

```
Require: (a) $k \geq 1$
Require: (b) $\forall i \geq 0, l \in L, F_{(i,l)} \Rightarrow F_{(i+1,l)}$
Require: (c) $\forall 0 \leq i < k, l, l' \in L$, s.t. $(l, t, l') \in G$, $F_{(i,l)} \wedge T_{l \to l'} \Rightarrow F'_{(i+1,l')}$
Ensure: $\forall i \geq 0, l \in L, F_{(i,l)} \Rightarrow F_{(i+1,l)}$
Ensure: if ret. value then $\forall 0 \leq i < k, l, l' \in L$, s.t.$(l, t, l') \in G$, $F_{(i,l)} \wedge T_{l \to l'} \Rightarrow F'_{(i+1,l')}$
Ensure: if $\neg$ret. value, there exists a counterexample path
  function BOOL STRENGTHEN($k$: int)
    while $\exists l$, s.t. $sat(F_{(k,l)} \wedge T_{l \to l_E})$ do
      @assert (b),(c)
      s := predecessor data region
      if not BACKWARDBLOCK($k, l, s$) then
        return false
      @assert $s \not\models F_{(k,l)}$
    return true
```

The function STRENGTHEN in Alg. 2 also works similarly to the original IC3. The main difference here is in the condition of the while loop which checks whether there exists $l \in L$ s.t. $e = (l, t, l_E) \in G$ and t is satisfiable under $F_{(k,l)}$. If this is the case, there exists a CTI. Note that in this paper we do not tackle the whole topic of generalization and restrict ourselves to computing weakest preconditions (WP) of the error state w.r.t. to t. While this might not offer the best performance possible, it is a safe approximation, as the WP is the smallest overapproximation of CTI states. The so extracted predecessor s is then analyzed in the function BACKWARDBLOCK, shown in Alg. 3.

Algorithm 3 Inner loop

```
Require: (b),(c)
Require: $sat(F_{(\hat{i},\hat{l}')} \wedge \hat{s} \wedge T_{\hat{l}' \to l_E})$
Ensure: if ret. value, then $\neg\hat{s}$ is inductive relative to $F_{(\hat{i}-1,l)}$, $\forall l$, s.t. $(l, t, \hat{l}') \in G$
Ensure: if ret. value, then (b),(c)
Ensure: if $\neg$ret. value, there exists a feasible path $l_0 \rightsquigarrow \hat{l}'$
  function BOOL BACKWARDBLOCK($\hat{i}$: int, $\hat{l}'$: location, $\hat{s}$: data region)
    $Q.add(\hat{i}, \hat{l}', \hat{s})$
    while $|Q| > 0$ do
      @assert $\forall (i, l', s) \in Q. 0 \leq i \leq k$
      @assert $\forall (i, l', s) \in Q. \exists$ path $(l', s) \rightsquigarrow (l_E, true)$
      $(i, l', s) = Q.pop$
      if $i = 0$ then
        return false
      else
        @assert $(l', \neg s)$ is inductive relative to $F_{(j,l)}$,
          $\forall 0 \leq j < i$, $l \in L \backslash \{l_E\}$
        for each $l$, s.t. $(l, t, l') \in G$ do
          if $l = l'$ and $sat(F_{(i-1,l)} \wedge \neg s \wedge T_{l \to l'} \wedge s')$ then
            generate predecessor $c$ of $s$
            @assert $\forall (i, l', s) \in Q, c \neq s$
            add $(i-1, l, c)$ and $(i, l', s)$ to $Q$
          else if $l \neq l'$ and $sat(F_{(i-1,l)} \wedge T_{l \to l'} \wedge s')$ then
            generate predecessor $c$ of $s$
            @assert $\forall (i, l', s) \in Q, c \neq s$
            add $(i-1, l, c)$ and $(i, l', s)$ to $Q$
          else
            block $s$ in frames $F_{(j,l')}$ for $0 \leq j \leq i$
      return $true$
```

While Alg. 1 and 2 are based on [1], we decided to present the inner loop similarly to the representation in [9] as we found it easier to comprehend. The function BACKWARDBLOCK gets as parameter a frame index $\hat{i}$, a location $\hat{l}' \in L$ and a data region $\hat{s}$. Following [9] we add this initial proof obligation to a priority queue Q, s.t. the obligation with the lowest i will get popped first. While the queue is non-empty, we start the inner loop by picking the obligation with the smallest i. If $i =$ 0, we can immediately stop with a counterexample because l' has to be initial. If it were not initial, the previous proof obligation at level 1 would have included the frame $F_{(0,l)}$, which is *false* for every non-initial location l. If $i \neq 0$ we have to check whether the region $\neg(l', s)$ is inductive edge-relative to $F_{(i-1,l)}$ of any predecessor l by solving the query (5) or (6), depending on whether $l = l'$ or not. If $\neg(l', s)$ is inductive edge-relative to $F_{(i-1,l)}$ for every predecessor l, we can block s in all $F_{(j,l')}, 0 \leq j \leq i$. If, on the other hand, $\neg(l', s)$ is not inductive edge-relative to $F_{(i-1,l)}$ for some l, then there must exist a predecessor that can reach (l', s). We therefore take the WP c of s w.r.t. the transition formula and add the new proof obligation $(i-1, l, c)$ to the obligation queue. We also add the old obligation (i, l', s) to the queue for future re-inspection. The inner loop terminates with *true* in case that Q is empty or with *false* in case there exists an obligation at frame 0.

Note that Alg. 3 slightly differs from the idea of blocking a region r iff r is edge-relative inductive to all incoming edges. However, the presented algorithm behaves correctly: Due to the ordering of the obligation queue, the algorithm proceeds in a kind of depth-first search manner. Even if r has been blocked at level i via one edge, another obligation r', that is a predecessor of r, at level $i-1$ will be chosen. Here we can distinguish two cases: Either r' is the last step on a counterexample path from l_0 to r at level i or it is not, in which case all regions explored will be blocked. In both cases r at level i will not be reconsidered before backtracking to an obligation at level $i+1$, in which case r at level i has been blocked.

In the following we will show that our algorithm is partially correct, i.e. it is correct given its termination. We construct our proof bottom-up by first proving that Alg. 3 is correct. As we use weakest preconditions, we can actually show full correctness, because termination is achieved for the following reason: Given an initial obligation (i, l, s) we have to construct at most n^i proof obligations, where n is the maximal in-degree of locations in L. This upper bound can be established because a WP of s covers all, possibly infinitely many, predecessor data regions that can reach s w.r.t. the transition. Therefore we cover all data regions in one step and only have to construct predecessor regions until we reach frame level 0, i.e., after doing this i times.

Lemma 2: Function BACKWARDBLOCK returns true iff $\neg\hat{s}$ is inductive relative to $F_{(\hat{i}-1,l)}$ for all locations l that are a predecessor of $\hat{l}'$.

Proof 2: Function BACKWARDBLOCK starts the while loop by examining the proof obligation in the queue that has the lowest frame index i. If the frame index is zero, then $\hat{l}'$ must be l_0, because $F_{(0,I)}$ is the only region with frame index 0 to which any other region is relatively inductive, by construction. This way, there must exist a feasible path from l_0 to $\hat{l}'$.

If there exists no feasible path from l_0 to $\hat{l}'$ given $F_0, ..., F_k$, then every path of length j ending in $\hat{l}'$ starts in a location l, s.t. the region $(l, \neg s)$ is inductive relative to $F_{(k-j-1,l^x)}$ for all l^x, s.t.$(l^x, t, l) \in G$. This means that every proof obligation added to Q is ultimately inductive relative to its predecessors and thus also $\neg\hat{s}$ is inductive relative to $F_{(\hat{i}-1,l)}$.

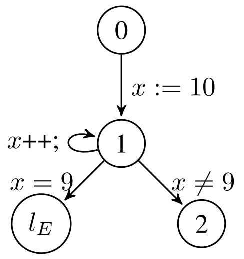


Fig. 1. Example for non-termination of Alg. 1

We continue by proving correctness of Alg. 2. Here we can guarantee termination for the same reason as for BACKWARDBLOCK. Because we only search for CTIs, we have at most n_E of them, where n_E is the number of predecessor locations of l_E in $\mathcal{P}$. As we compute exact pre-images by weakest preconditions, every data region has exactly one predecessor data region per edge.

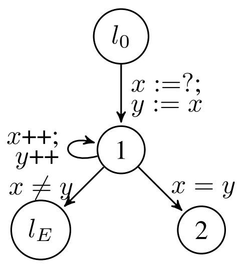
Lemma 3: Function STRENGTHEN terminates with result true iff there exists an inductive strengthening for the frames $F_{(k,l)}$ in all locations l in the CFA.

Proof 3: Assume a call of function STRENGTHEN returns false, then there must have been a call to BACKWARDBLOCK with some (k, l, s), such that BACKWARDBLOCK returned false. From Lemma 2 we know that in this case, there exists a feasible path of length k from l_0 to l that ends up in data region s. Because l is a predecessor of l_E and s is a precondition under $T_{l \to l_E}$, there exists a counterexample path of length $k+1$. Otherwise every call of BACKWARDBLOCK returned true, which means that every predecessor location (and data region) of l_E is unreachable in the current frame sequence. Thus every predecessor of l_E was excluded from their frames at level k which yields an inductive strengthening for F_k.

After proving correctness of Alg. 2 and 3, we have to drop termination for Alg. 1. The reason is that there might exist infinite ascending or descending chains that we cannot generalize. This is exemplified in the simple program in Fig. 1. While there exist an inductive strengthening, e.g., $(x \geq 10)$, for every maximal frame index k our algorithm will block a region $(1, 9-(k-1))$ of which there can be infinitely many for unbounded integers, such that there will never exist an i for which $F_{(i,1)} = F_{(i+1,1)}$. Note that in the following, as all statements over $F_{(i,l)}$ always concern all non-error locations, we will slightly abuse notation and use F_i in place of $F_{(i,l)}, \forall l \in L \backslash \{l_E\}$.

Lemma 4: In case function PROVE terminates, it returns *true* iff there exists an inductive strengthening F for P, s.t. $F \wedge P$ is inductive.

Proof 4: Assume PROVE terminates with true, then every call of STRENGTHEN for every $j < k$ must have returned true and there must exist a frame with index $i < k$, s.t. $F_i = F_{i+1}$, i.e. the frame F_i is inductive, because $F_i \wedge T \Rightarrow F'_i$. Therefore there cannot exist a counterexample path of length k (more precise of length i) or less and there cannot exist one of length greater than k, because F_i is inductive.



l: \ i:	0	1	2
l_0	true	true	true
1	false	$x = y$	$x = y$

Fig. 2. Example program and resulting frames

Now assume that PROVE returns false: Then there must exist a k, s.t. for no $i < k$, F_i is inductive and STRENGTHEN for k returns false, i.e. there exists a path of length k from the initial to the error state.

Theorem 1: If the algorithm terminates, it returns true iff P is an invariant on S.

Proof 5: By Lem. 2-4 the theorem holds. □

Example

In the remainder of this section we show by example how our lifting of IC3 to control flow automata works and what its benefits are. We start with the program from Fig. 2 as input. Note that in our presentation we omit the computation steps for $l = 2$, as l_E is not reachable from that location. As shown in Alg. 1 we start by the two static checks for 0- and 1-step counterexamples. As they are both obviously not satisfied, we proceed to initializing $F_{(0,l_0)}$ to $true$ and $F_{(0,1)}$ to $false$. $F_{(1,l)}$ is set to $true$ for both locations $l \in \{l_0, 1\}$.

We now start our algorithm with $k = 1$ and try to construct a strengthening. There exists exactly one l s.t. $(l, t, l_E) \in G$, namely $l = 1$, which yields the initial proof obligation $(1, 1, x \neq y)$ for the priority queue Q in Alg. 3. As i is not 0, we start the blocking phase by searching for a predecessor of 1 and find location l_0, which means we have to apply query (5) and check $sat(F_{(0,l_0)} \wedge y' = x' \wedge x' \neq y')$, which is obviously not satisfiable. Next we check $l = 1$. As this is a self-loop we can use the stronger query (6) to check $sat(F_{(0,1)} \wedge x = y \wedge x' = x + 1 \wedge y' = y + 1 \wedge x' \neq y')$.

This query shows two improvements over existing techniques: First, we initialized $F_{(0,1)}$ to $false$ because it is not initial, which allows us to block the obligation one step earlier and also make the query non-satisfiable immediately. Second, even without any information learned in $F_{(0,1)}$ the query is not satisfiable due to the stronger edge-relative inductiveness of (6).

We continue the execution by blocking $x \neq y$ in $F_{(1,1)}$, i.e. add $\neg(x \neq y)$ to $F_{(1,1)}$. Afterwards there is no entry in Q and we leave BACKWARDBLOCK. As by the blocking there is no more CTI at index 1, we also leave STRENGTHEN and continue with major iteraton $k = 2$.

The function call STRENGTHEN(2) enters BACKWARDBLOCK with the initial obligation $(2, 1, x \neq y)$. For predecessor l_0, we check $sat(F_{(1,l_0)} \wedge y' = x' \wedge x' \neq y')$, which is again unsatisfiable, as well as $sat(F_{(1,1)} \wedge x = y \wedge x' = x + 1 \wedge y' = y + 1 \wedge x' \neq y')$ for location 1, which is not satisfiable either. We can therefore block $x \neq y$ in frame $F_{(2,1)}$, too.

Again we have an empty Q and no more CTI at level 2. We check for termination and find $F_{(1,l_0)} = F_{(2,l_0)}$ as well as $F_{(1,1)} = F_{(2,1)}$, which means that we found an inductive strengthening $F = (1, x = y)$ s.t. l_E is not reachable in the CFA.

V. Benchmarks

In this section we start with details of our implementation, followed by an evaluation and end with a discussion of the presented results.

Implementation

We implemented our IC3CFA algorithm on top of an existing proprietary model checking framework. A flow chart of the framework is shown in Fig. 3. The framework makes use of the LLVM project enable parsing a wide range of input languages and translate them into the LLVM intermediate representation (IR) [11]. To close the gap between compiler oriented semantics of C as assumed by LLVM and verification semantics that was encountered in the development of the UFO model checker we use the approach of initializing variables with a call to an external function as presented in [12]. We translate this IR into our own intermediate verification language (IVL) that is more suitable for verification, e.g. no SSA form, no three-adress code. On the IVL code we execute some optimization stages that are also widely used in other model checkers, e.g. the Kratos software model checker [13], like program slicing, expression propagation and bisimulation minimization, as well as Steensgaard's pointer analysis [14] to rewrite simple pointer expressions. The bit-precise memory model (similar to the one of CBMC [15]) supports limited pointer operations, including array-element and record-field addressing. After reaching a fixpoint of these optimizations, we construct the program's control flow graph, labeled with instructions of a guarded command language similar to that of [16]. This allows for efficient construction of weakest preconditions [16], [17]. The results of our preprocessing are in line with the CFAs produced by Kratos and only differ by one or two locations. While the presented approach is fully theory unaware and can be used for infinite-domain theories such as linear real arithmetic (LRA) we use the finite-domain theory of bit vectors. Our tool supports the Z3 and MathSAT SMT solvers; all following benchmarks were executed using the Z3 solver. To minimize overhead and reduce unnecessary pushing attempts we implemented the efficient pushing strategy of [18]. For storing frames efficiently we implemented the delta encoding approach of [9] which, in our experiments, reduced memory consumption as well as runtime significantly. For some benchmarks the reduction

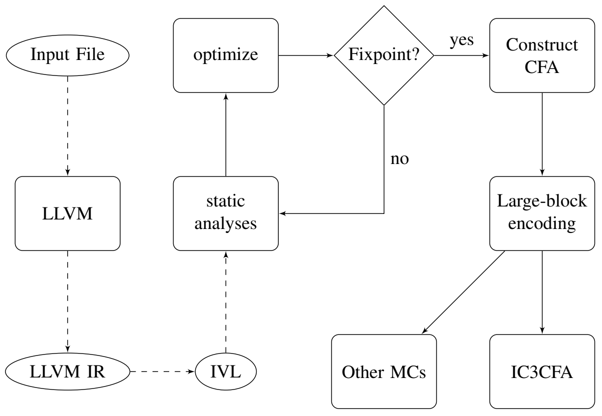


Fig. 3. Tool flow

in runtime was almost 20 times. Generalization of computed preimages is not implemented, yet.

Evaluation

For evaluation of our algorithm and to compare it to others we used a subset[1] of 28 programs from the set of benchmarks used in [2], originating from different domains such as device drivers, communication protocols, SystemC designs and textbook algorithms, some of which are contained in the set of benchmarks of the software verification competition (http://sv-comp.sosy-lab.org/2015/). One out of four programs contains a bug.

All presented results can be reproduced by our tool via the web-interface at http://www-i2.informatik.rwth-aachen.de/mctools/vplc/fmcad15/ .

All experiments have been executed on a cluster using a single core per instance, running at 2.1 GHz with a memory limit of 4GB per file and a timeout of 1200 seconds.

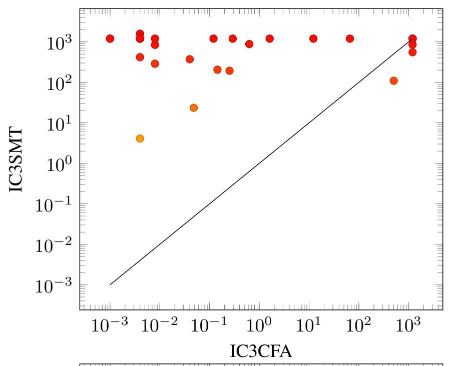
We briefly compare our implementation to the IC3SMT, Tree-IC3 and Tree-IC3-ITP algorithms of [2]. Scatter plots in Fig. 4 give a graphical idea of how IC3CFA compares time-wise against each of these algorithms while the appended table gives statistics of the overall performance of the four algorithms. The second column highlights the number of solved instances of the 28 benchmark programs. The third column shows the time in seconds for solved instances only, while the last column also takes timeouts into account.

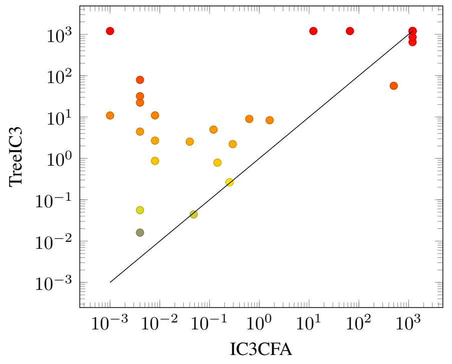
IC3SMT, the implementation of the straight-forward lifting of IC3 to SMT as described in the beginning of Sec. IV is clearly the one that performs worst from all four. This comparison is in line with the observations from [2] and shows that IC3CFA benefits from using the explicit representation of control flow over the implicit representation using a dedicated pc variable. IC3CFA is able to solve nine more instances than IC3SMT and does so in almost 9% of the time.

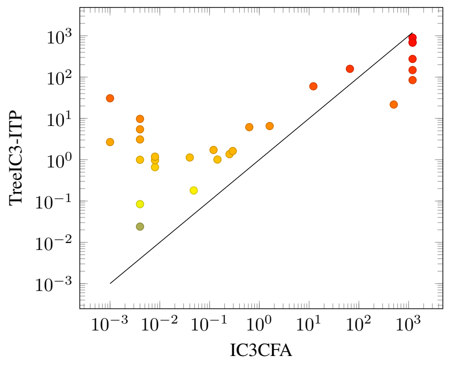
The comparison of IC3CFA with the Tree-IC3 implementation shows the effect of constructing frames for control flow locations, rather than unrolling the ART, as both algorithms are based on a very similar representation of control flow automata and both use weakest preconditions to construct the preimage of a state. IC3CFA is able to solve one more instance than Tree-IC3 in a third of the time.

Besides these two direct comparisons we also evaluated our implementation against the Tree-IC3-ITP implementation that almost resembles Tree-IC3 with the only difference in the computation of preimages, where Tree-IC3-ITP uses interpolants. This comparisons shows the advantage of using interpolants over weakest preconditions, as Tree-IC3-ITP is able to solve six more instances than IC3CFA.

[1]Currently we do not support assignment of nondeterministic values inside a loop, due to limitations in our current memory model.







Algorithm	# solve	solve t	total t
IC3SMT	13/28	6328s	24328s
Tree-IC3	21/28	1752s	10152s
Tree-IC-ITP	28/28	3107s	3107s
IC3CFA	22/28	584s	7784s

Fig. 4. Comparison of Tree-IC3 and IC3CFA

Results

From the results in Fig. 4 we can come to three conclusions. First, as already discovered in [2], the IC3SMT approach, while easy to lift, is very inefficient and is not competitive to control flow based techniques. Second, while the TreeIC3 approach is, in very rare cases, slightly more efficient on very small examples, the absence of overhead in ART unrolling makes our approach much faster on medium to large scale examples. Third, the effect of how predecessors are constructed, i.e. weakest preconditions against interpolants, is another important factor that has to be considered when evaluating the performance of IC3 based software model checking algorithms.

VI. Conclusion

In this paper we have presented an approach to lift the application domain of the inductive, incremental verification algorithm IC3 from hardware to software model checking. In resemblance to other adaptations of IC3 to software verification, it supports the handling of infinite-state systems by generalising SAT to SMT solving. Its distinguishing feature, however, is the explicit consideration of a program's control flow by maintaining location-specific frames for storing approximate reachability information. In comparison to more implicit approaches such as Tree-IC3, which is based on an unrolling of the ART, this allows to apply a stronger form of relative inductiveness than obtained in comparable algorithms. The gain in efficiency is demonstrated by experimental evaluations against different implementations of existing, control flow based IC3 algorithms. Due to the novelty of IC3 there is a wide range of future research on this field. One particularly interesting perspective that we intend to investigate are the potentialities of using generalization as a way to overapproximate the exact preimages that we use up to now.

References

[1] Bradley, A.R.: SAT-based model checking without unrolling. In Jhala, R., Schmidt, D.A., eds.: Verification, Model Checking, and Abstract Interpretation - 12th International Conference, VMCAI 2011, Austin, TX, USA, January 23-25, 2011. Proceedings. Volume 6538 of Lecture Notes in Computer Science., Springer (2011) 70–87

[2] Cimatti, A., Griggio, A.: Software model checking via IC3. In Madhusudan, P., Seshia, S.A., eds.: Computer Aided Verification - 24th International Conference, CAV 2012, Berkeley, CA, USA, July 7-13, 2012 Proceedings. Volume 7358 of Lecture Notes in Computer Science., Springer (2012) 277–293

[3] Cimatti, A., Griggio, A., Mover, S., Tonetta, S.: IC3 modulo theories via implicit predicate abstraction. In Ábrahám, E., Havelund, K., eds.: Tools and Algorithms for the Construction and Analysis of Systems - 20th International Conference, TACAS 2014, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2014, Grenoble, France, April 5-13, 2014. Proceedings. Volume 8413 of Lecture Notes in Computer Science., Springer (2014) 46–61

[4] Graf, S., Saidi, H.: Construction of abstract state graphs with PVS. In Grumberg, O., ed.: Computer Aided Verification, 9th International Conference, CAV '97, Haifa, Israel, June 22-25, 1997, Proceedings. Volume 1254 of Lecture Notes in Computer Science., Springer (1997) 72–83

[5] Tonetta, S.: Abstract model checking without computing the abstraction. In Cavalcanti, A., Dams, D., eds.: FM 2009: Formal Methods, Second World Congress, Eindhoven, The Netherlands, November 2-6, 2009. Proceedings. Volume 5850 of Lecture Notes in Computer Science., Springer (2009) 89–105

[6] Hoder, K., Bjørner, N.: Generalized property directed reachability. In Cimatti, A., Sebastiani, R., eds.: Theory and Applications of Satisfiability Testing - SAT 2012 - 15th International Conference, Trento, Italy, June 17-20, 2012. Proceedings. Volume 7317 of Lecture Notes in Computer Science., Springer (2012) 157–171

[7] Clarke, E.M., Grumberg, O., Peled, D.: Model Checking. MIT Press (2001)

[8] Henzinger, T.A., Jhala, R., Majumdar, R., Sutre, G.: Lazy abstraction. In Launchbury, J., Mitchell, J.C., eds.: Conference Record of POPL 2002: The 29th SIGPLAN-SIGACT Symposium on Principles of Programming Languages, Portland, OR, USA, January 16-18, 2002, ACM (2002) 58–70

[9] Eén, N., Mishchenko, A., Brayton, R.K.: Efficient implementation of property directed reachability. In Bjesse, P., Slobodová, A., eds.: International Conference on Formal Methods in Computer-Aided Design, FMCAD '11, Austin, TX, USA, October 30 - November 02, 2011, FMCAD Inc. (2011) 125–134

[10] Bradley, A.R., Manna, Z.: Checking safety by inductive generalization of counterexamples to induction. In Baumgartner, J., Sheeran, M., eds.: Formal Methods in Computer-Aided Design, 7th International Conference, FMCAD 2007, Austin, Texas, USA, November 11-14, 2007, Proceedings, IEEE (2007) 173–180

[11] Lattner, C., Adve, V.S.: LLVM: A compilation framework for lifelong program analysis & transformation. In: 2nd IEEE / ACM International Symposium on Code Generation and Optimization (CGO 2004), 20-24 March 2004, San Jose, CA, USA, IEEE Computer Society (2004) 75–88

[12] Albarghouthi, A., Li, Y., Gurfinkel, A., Chechik, M.: UFO: A framework for abstraction- and interpolation-based software verification. In: Computer Aided Verification - 24th International Conference, CAV 2012, Berkeley, CA, USA, July 7-13, 2012 Proceedings. Volume 7358 of Lecture Notes in Computer Science., Springer (2012) 672–678

[13] Cimatti, A., Griggio, A., Micheli, A., Narasamdya, I., Roveri, M.: Kratos - a software model checker for SystemC. In Gopalakrishnan, G., Qadeer, S., eds.: Computer Aided Verification - 23rd International Conference, CAV 2011, Snowbird, UT, USA, July 14-20, 2011. Proceedings. Volume 6806 of Lecture Notes in Computer Science., Springer (2011) 310–316

[14] Steensgaard, B.: Points-to analysis in almost linear time. In Boehm, H., Jr., G.L.S., eds.: Conference Record of POPL'96: The 23rd ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, Papers Presented at the Symposium, St. Petersburg Beach, Florida, USA, January 21-24, 1996, ACM Press (1996) 32–41

[15] Kroening, D., Tautschnig, M.: CBMC - C bounded model checker - (competition contribution). In Ábrahám, E., Havelund, K., eds.: Tools and Algorithms for the Construction and Analysis of Systems - 20th International Conference, TACAS 2014, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2014, Grenoble, France, April 5-13, 2014. Proceedings. Volume 8413 of Lecture Notes in Computer Science., Springer (2014) 389–391

[16] Flanagan, C., Saxe, J.B.: Avoiding exponential explosion: generating compact verification conditions. In Hankin, C., Schmidt, D., eds.: Conference Record of POPL 2001: The 28th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, London, UK, January 17-19, 2001,, ACM (2001) 193–205

[17] Leino, K.R.M.: Efficient weakest preconditions. Information Processing Letters **93**(6) (2005) 281–288

[18] Suda, M.: Triggered clause pushing for IC3. CoRR **abs/1307.4966** (2013)