Designing Parallel PDR

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Abstract—Property Directed Reachability (PDR) is an efficient model checking technique. However, the intrinsic high computational complexity prevents PDR from meeting the challenges of real world verification. To address this problem, this paper introduces the parallel algorithm P3 based on: 1) partitioning of the input problem, 2) exchanging of learned reachability information, and 3) using algorithm portfolios. The generic nature of the proposed techniques makes them immediately suitable for software verification. This paper investigates the benefits of these techniques while taken individually and when combined together, implemented using distributed computing environment on top of the SMT-based software model checker SPACER. In our experiments over SV-COMP benchmarks we observe up to an order of magnitude speedup with respect to the sequential implementation with twice as many instances solved within a timeout.

I. INTRODUCTION

Applying model checking to realistic, complex systems is highly non-trivial in part due to the computational complexity of the underlying decision problem. This paper studies how parallel computing can help in scaling model checking to such problems. We concentrate on parallelizing the execution of the Property directed reachability (PDR/IC3) algorithm [1] using algorithm portfolios and approaches based on divide-and-conquer together with sharing of information learned in the model checking process. In particular, we study how a computational cluster can speed up model checking of software represented as sets of Constrained Horn Clauses (CHC) over Satisfiability modulo theories (SMT) constraints.

The PDR/IC3 algorithm (PDR in brief) is a relatively recent procedure that, given a transition system and a safety property, computes a safe inductive invariant or finds a counterexample for the safety of the system. During the computation, the algorithm maintains an increasingly long sequence of frames $F_i$ representing symbolically safe over-approximations of states reachable from the initial state of the transition system in at most $i$ steps. The frames are constructed as sets of PDR-lemmas that are computed to block spurious counterexamples. The algorithm terminates either by finding a concrete counterexample or by showing that the set of states described by $F_{i+1}$ is a subset of the set of states described by its immediate predecessor frame $F_i$. In this case, the frame $F_{i+1}$ is a safe inductive invariant for the program. Constructing the frame sequence is a heuristic process which can employ several different strategies within the PDR algorithm.

In this paper we introduce, to the best of our knowledge, the first divide-and-conquer technique for PDR (called partitioning throughout the paper), combine it with a portfolio of PDR solvers running different strategies, and allow the solvers to share PDR-lemmas. We combine these techniques in our new algorithm called P3 (Parallelly Performed PDR).

The P3 algorithm enables efficient parallel model-checking using PDR with the aim of improving the current state-of-the-art of parallel software verification. The approach is based on three concepts that expand the sequential PDR algorithm as follows: (i) P3 applies a portfolio of sequential PDR implementations parameterized to compute the frame sequence in different ways through different search strategies and by randomizing the search heuristic of the underlying SMT solver; (ii) P3 implements the novel partitioning approach introduced here by using the transition function of the program to compute, based on the negation of the safety property, its pre-images which are then distributed over the solvers in the portfolio as new safety properties; and (iii) the PDR implementations in the portfolio may share the PDR lemmas stored in the frames among each other. To the best of our knowledge, P3 enables parallel PDR for software verification for the first time.

We implemented the P3 algorithm using the sequential SPACER model checker [12] as a basis. We performed a thorough experimental analysis processing over 1000 instances from the Software Verification Competition 2016 (SV-COMP) with different configurations. The experiments were run in a computational cluster of 60 CPU cores. Our results show that a combination of divide-and-conquer, lemma sharing, and algorithm portfolio is capable of solving twice as many instances within our timeout of 1000 seconds and provides up to an order of magnitude speedup compared to the best sequential SPACER configuration. Our experimentations furthermore reveals that the choice of the heuristic for selecting which clauses to share is important, and that especially the partitioning technique benefits the most from it. The results are interesting also in the sense that they report the first experiments on running PDR on a computing cluster targeting in particular software verification with SMT-based Constrained Horn Clauses.

The paper is structured as follows. We compare with related work in Section [11] and in Section [111] we review the basics of the PDR algorithm. In Section [V] we introduce our notion of distributed PDR and give details about our new parallelisation strategies. In Section [V] we describe our implementation used for empirical evaluation presented in Section [VI]. Finally, we conclude the paper in Section [VII].

II. RELATED WORK

The first attempt to parallelize PDR is mentioned in the original PDR paper [1], where the experimented parallel setting is based on sharing all the frames among different
computing threads on the same machine. This work is further improved in [3] where they study different parallel approaches, all of them focused on multi-threaded portfolios and PDR-lemma sharing, addressing propositional PDR.

Both [1, 3] are limited to propositional PDR, making them suitable mainly for hardware verification. In contrast to [1, 3], we propose and thoroughly evaluate different lemma sharing strategies by differentiating between k-invariants and \( \infty \)-invariants inside the frames. We introduce the novel partitioning technique for PDR, together with a concise algorithm capable of combining all these techniques in a sound manner. Moreover our implementation is based on the more scalable distributed computing, thus exploiting the much bigger computational power offered by cloud-computing environments compared to single-machine threads.

A first investigation of PDR in the setting of software verification is done in [4]. A different approach based on CHC for software verification is presented in [12]. We extend this work with the aim of improving the current state-of-the-art of parallel software verification.

There is a substantial amount of work on parallel SMT solving that can help software verification model checking techniques in general. The parallelization tree framework for combining divide-and-conquer and portfolio directly on SMT techniques in general. The parallelization tree framework for solving that can help software verification model checking in a certain number of steps. The algorithm presents these overapproximations as PDR-lemmas, formulas over variables \( X \) describing reachability information learned by PDR.

Definition 2 (Relatively Inductive and Invariant Lemmas). Given initial states represented by \( Init \), transition relation \( Tr \) and a set of formulas \( F \), a PDR-lemma \( \varphi \) is inductive relative to \( F \) if and only if

\[
Init \implies \varphi \land F \implies \varphi'.
\]

Whenever \( \varphi \) is inductive relative to true, we say that \( \varphi \) is an inductive lemma. A PDR-lemma \( \varphi \) is an invariant lemma if it is true in all the reachable states, i.e., \( post^n_{Tr}(F) \implies \varphi \). Every inductive PDR-lemma is invariant, but the converse is not true in general.

An instance \( S \) is safe if there exists a safe inductive invariant \( Inv(X) \) such that \( Inv(X) \implies \neg Bad(X) \). \( S \) is unsafe if there exists an \( n \in \mathbb{N} \) such that \( post^n_{Tr}(Init) \land Bad \) is satisfiable. For an unsafe \( S \), a satisfying assignment for

\[
Init(X^{[0]}) \land Bad(X^{[n]}) \land \bigwedge_{i=0}^{n-1} Tr(X^{[i]}, X^{[i+1]})
\]

is called a feasible counterexample. The satisfying assignment corresponds to a sequence of states where the first state satisfies \( Init \), each consecutive pair of states satisfies \( Tr \), and the final state satisfies \( Bad \), and can, therefore, be considered as an evidence for a programming error.

B. Property Directed Reachability (PDR)

In this section, we give a high-level overview of IC3/PDR algorithm. We refer the reader to [11, 3, 3, 6, 12] for the details of the original algorithm and its extensions to SMT.

Definition 3 (PDR Trace). Given an instance of the safety problem \( S \), a PDR trace for \( S \) is a sequence of frames \( F = \langle F_0, F_1, \ldots, F_N, \ldots \rangle \) such that each frame \( F_i \in F \) is a set...
of PDR-lemmas. Furthermore, the trace satisfies the following properties for \( i \geq 0 \):

\[
F_0 \equiv \text{Init} \\
F_i \land Tr \implies F'_i \\
F_i \implies F_{i+1} \\
i < N \implies (F_i \implies \neg \text{Bad})
\]

(1) (2) (3) (4)

Intuitively, each frame \( F_i \in \mathcal{F} \) over-approximates all the states reachable in at most \( i \) steps of the transition relation \( Tr \) from \( \text{Init} \). Moreover, the trace proves that \( S \) is safe up to \( N - 1 \) steps of \( Tr \) from \( \text{Init} \).

PDR uses the trace to compute an increasing bound of steps from \( \text{Init} \) up to which \( S \) is safe. The algorithm works by iteratively adding an initially empty frame \( F_N \) at the end of the trace. PDR then tries to either prove safety of \( F_N \) by strengthening it, or to find a feasible counterexample based on it.

**Definition 4 (Proof Obligation).** Given an instance \( S \) of the safety problem and a PDR trace \( \mathcal{F} \) for \( S \), a proof obligation is the pair \( \langle \sigma, i \rangle \) where \( \sigma \) is a conjunction of predicates over state variables and \( i \leq N \). In addition, the proof obligation satisfies the following:

- \( \sigma \land F_i \) is satisfiable, and
- for all models \( m \) such that \( m \models \sigma \), post\( _{\mathcal{T}_r} \)(\( m \)) \land \text{Bad} is satisfiable.

The conjunction \( \sigma \) represents a set of the states consistent with a frame \( F_i \in \mathcal{F} \), containing states that can reach \text{Bad} with a feasible path.

Given an instance \( S \), PDR computes a trace \( \mathcal{F} \) of increasing length for \( S \) until either a fixed point is found for \( Tr \) or the algorithm determines a feasible counterexample. In the process, PDR constructs candidate counterexamples, proof obligations, that are stored in an obligation queue \( Q \). The proof obligations \( \langle \sigma, i \rangle \) are propagated towards the initial state by computing their pre-image with respect to \( Tr \), resulting in \( \sigma^- \), which is then inserted into \( Q \). If a counterexample candidate is not feasible, a proof obligation will at some point be blocked by a frame. This happens if for a proof obligation \( \langle \sigma, i \rangle \) it holds that \( F_{i-1} \land Tr \land \sigma^- \) is unsatisfiable. The clause \( \neg \sigma \) is then simplified to a PDR-lemma and inserted to \( F_i \).

PDR proves \( S \) safe if:

\[ \exists i < N \cdot F_{i+1} \implies F_i \]

This simplifies Equation (2) to \( F_i \land Tr \implies F'_i \). Thus together with Equations (1) and (4) \( F_i \) is proved to be both a fixed point for \( Tr \) and a safe inductive invariant for \( S \).

The way PDR computes the fixed point leaves room for some flexibility in how the lemmas are organized. In particular, \[6\] suggests to separate the inductive lemmas to a distinct frame \( F_\infty \). Hence the frame \( F_\infty \) is initially empty and always consists of those lemmas \( \varphi \in \bigcup F_i \) inductive relative to \( F_\infty \). Thus, \( S \) is safe when

\[ F_\infty \implies \neg \text{Bad}. \]

PDR proves \( S \) unsafe whenever a proof obligation \( \langle \sigma, 0 \rangle \) is added to the obligations queue. By Definition (4) \( \sigma \) represents a set of states in \( \text{Init} \) from which there is a feasible path leading to a state in \text{Bad}.

**Definition 5 (PDR Configurations).** Given an instance of the safety problem \( S \), a PDR configuration is the quadruple \( C = (N, \mathcal{F}, F_\infty, Q) \) where:

- \( N \in \mathbb{N} \)
- \( \mathcal{F} = \langle F_0, \ldots, F_N, \ldots \rangle \) is a trace of \( S \),
- \( F_\infty \) is the inductive frame of \( \mathcal{F} \),
- \( Q = \{ \langle \sigma, i \rangle, \ldots \} \) is the obligation queue, where \( i \leq N \).

The initial configuration of PDR for \( S \) is \( C_0 = (1, \langle \text{Init}, \emptyset, \ldots \rangle, 0, \emptyset) \).

Given a PDR configuration \( C \) of a safety problem \( S \), each of the following operation performed on \( C \) updates its components resulting in a new configuration \( C' \).

- **Candidate.** If there exists \( \sigma \) such that \( \sigma \implies F_N \land \text{Bad} \), then the proof obligation \( \langle \sigma, N \rangle \) is added to \( Q \).
- **Predecessor.** Given \( \langle \sigma, i \rangle \in Q \) with \( i > 0 \), if there exists a conjunction of predicates \( \delta \) such that for all consistent \( m \) such that \( m \models \delta, m \land Tr \models \sigma^\prime \) holds, then \( \langle \delta, i - 1 \rangle \) is added to \( Q \).
- **Blocking.** Given \( \langle \sigma, i \rangle \in Q \) with \( i \geq 1 \), if PRedecessor is not applicable then remove \( \langle \sigma, i \rangle \) from \( Q \) and add the PDR-lemma \( \varphi \) to all \( F_j \) with \( 1 \leq j \leq i \) such that

\[ \text{Init} \implies \varphi \land \varphi \implies \neg \sigma \land F_{i-1} \land Tr \implies \varphi^\prime \]

- **Unfold.** If \( Q = \emptyset \) and Candidate is not applicable then \( N := N + 1 \).
- **Inductive.** Given a subset of lemmas \( \varphi \subseteq F_i \) with \( 0 \leq i < N \) s.t. \( \varphi \land F_\infty \land Tr \implies \varphi^\prime \), add \( \varphi \) to \( F_\infty \) (and to each \( F_i \)).

In addition PDR has the following two rules that guarantee the termination of the algorithm when always taken when they are applicable:

- **Safe.** If \( F_\infty \implies \neg \text{Bad} \) report safe.
- **Unsafe.** If any \( \langle \sigma, 0 \rangle \in Q \) report unsafe and generate a counterexample.

In PDR with theories, and, therefore, in our implementation, the operation Predecessor employs Model-Based Projection \[12\] to ensure termination, while Blocking uses interpolation \[8\] to build the lemma.

**Definition 6 (PDR Strategy).** Given an instance \( S \) of the safety problem and a PDR configuration \( C \), a PDR strategy \( T_S \) is a function that maps \( C \) to one of the possible PDR operations applicable for \( C \), based on \( S \).

A PDR execution is a sequence of configurations \( \langle C_0, C_1, \ldots, C_T \rangle \) such that for every \( i \in \{1, \ldots, T\} \), \( C_i \) is
the result of the operation $T_S(C_{i-1})$ on $C_{i-1}$, $C_0$ is the initial PDR configuration for $S$, i.e., $(1, \langle \text{Init}, \emptyset, \ldots \rangle, \emptyset, \emptyset)$, and $T_S(C_T) \in \{\text{Safe, Unsafe}\}$.

### IV. The P3 Algorithm

In this section we introduce the P3 (Parallelly Performed PDR) algorithm for parallel model-checking with PDR.

P3 implements three parallelisation techniques for PDR: portfolio, partitioning, and lemma sharing. These techniques can be combined in order to exploit each one strengths.

Portfolio takes advantage by using different PDR strategies while partitioning focuses the search by constraining the problem. In general, partitioning means dividing a problem into several sub-problems. The PDR partitioning technique introduced in this paper partitions the problem by restricting the paths leading to the bad states.

Finally, lemma sharing provides each solver with useful information arising from search diversification, and possibly not derivable locally. The intuition is that PDR-lemmas express what is learned by each PDR execution. Since different executions employs different strategies, PDR-lemmas that are easily found by one strategy can be difficult to find by another.

In the rest of the section, we formalize portfolio, partitioning, and lemma sharing strategies and provide details of P3.

### A. Portfolio

The most naïve parallel technique is a portfolio – concurrent and independent execution of multiple sequential PDR strategies on the same problem. A portfolio terminates as soon as one of its instances terminates successfully.

We define a notion of a distributed PDR configuration to model a PDR portfolio with any combination of lemma sharing and partitioning.

**Definition 7** (Distributed PDR Configuration). Given an instance $S$ of the safety problem, a distributed PDR configuration is a set of tuples:

$$ D^n = \{ (T^i, C^i) \} $$

where for each $i \in \{1, \ldots, n\}, n \in \mathbb{N}$:

- $T^i$ is a PDR strategy from Definition 6
- $C^i = (N_i, F_i = \langle F_0^i, F_1^i, \ldots, F_{N_i}^i, \ldots, F_\infty^i, Q_i^i \rangle)$ is a PDR configuration from Definition 5

A distributed PDR configuration expresses a PDR portfolio when for every $i \in \{1, \ldots, |D|\}, T^i$ is a strategy for the input problem $S$. That is, every strategy evolves the corresponding PDR-configuration independently, performing asynchronous and arbitrary choices based on $S$.

A PDR portfolio $D$ terminates when there exists $T_S^i(C^i) \in \{\text{Safe, Unsafe}\}$ for some $i \in \{1, \ldots, |D|\}$. Termination and soundness of this setting follows trivially from PDR because every execution is independent.

### B. Partitioning

In this section, we define partitioning strategy and argue for its soundness.

**Definition 8.** Given a safety problem $S$, partition($S$) is a set of problems $\{S_{p_1}, \ldots, S_{p_n}\}$, where each instance $S_{p_i} = \langle \text{Init}(X), \text{Tr}(X, X'), p_i(X) \rangle$ is called a partition of $S$, and such that

$$ \bigvee_{i} p_i \iff \exists X' \cdot \text{Tr}(X, X') \land \text{Bad}(X') $$

From Definition 8 it follows that $S$ is safe if and only if all of its partitions are safe. A distributed PDR configuration $D$ expresses partitioning if for each partition $S_p \in \text{partition}(S)$ there is a pair $(T_S^i, C^i) \in D$. The result of a distributed configuration with partitioning is Unsafe if there exists $T_S^i(C^i) = \text{Unsafe}$, and the result is Safe if for each $S_p \in \text{partition}(S)$ there exist $T_S^i(C^i) = \text{Safe}$.

The soundness is by construction: a counterexample for $S_p$ is also valid for $S$, while the safety of all $S_p$ ensures the safety of $S$ because every state leading to Bad in one step is expressed in a partition.

### C. Lemma Sharing

In this section, we give the formal definition of lemma sharing and argue for its soundness in a distributed portfolio setting.

**Definition 9** ((k-)invariant). A PDR-lemma $\psi$ is k-invariant if it is true in all the states reachable in k steps or less, i.e., post$_k^\psi$ (Init) $\Rightarrow \psi$. If $\varphi$ is invariant, then it is k-invariant for any $k \in \mathbb{N}$.

Following Definition 9, each frame $F_k$, $k \in \mathbb{N}$, is a set of k-invariants for $S$, while $F_\infty$ is a set of invariants for $S$.

**Theorem 1** (Lemma Sharing). Given a distributed PDR configuration $D$ for an instance $S$ of the safety problem, the PDR-lemma $\psi \in F_k^i, k \in \mathbb{N}$ is a k-invariant for $S$ and the operation of adding $\psi$ to any $F_i^l$ with $i \neq j$ and $l \leq k$ keeps $C^j$ a valid PDR configuration for $S$.

The same holds for $\varphi \in F_\infty^j$ when added to any $F_l^i, i \neq j$.

**Proof.** The proof follows from Definition 2. Each $\varphi \in F_k^i$ is a k-invariant if $k \in \mathbb{N}$, or an invariant if $k = \infty$ and can be used to soundly refine a different abstraction of states reachable in up to $k$ steps.

Similarly, sharing invariants is sound and makes every $F_\infty^i$ an invariant for $S$. Thus, $S$ is safe whenever any $F_\infty^i$ implies $\neg \text{Bad}$. \qed

### D. Parallelly Performed PDR

The P3 algorithm is shown in Algorithm 1. It combines portfolio, lemma sharing, and partitioning. The algorithm works as follows. Until there are available computing resources, the procedure Entrust randomly selects a partition not yet solved, creates a new strategy and allocates the necessary resources in order to run PDR.

The procedure Exclude at line 12 is taken exactly once for each partition in $P$ whenever a corresponding sequential PDR instance terminates. This happens finitely many times because there are finitely many partitions. Therefore, the algorithm
Algorithm 1: The P3 algorithm.

V. IMPLEMENTATION

We implemented our parallel PDR algorithm using the tool SMTSERVICE [13], a framework for parallel and distributed solving already used in [14] for distributed SMT. A Graphical User Interface [2] is also available for analysing the parallel executions. We upgraded it to also support distributed PDR. SMTSERVICE is a client-server based framework. The server implements Algorithm 1. The client uses the SMT-based PDR model checker SPACER [12]. We modified SPACER in order to expose the API for lemma sharing. The overview of the architecture is shown in Figure 1.

The server is written in PYTHON and its behaviour can be controlled through a configuration file. The server is in charge of pre-processing the input instances, managing clients connections and solving tasks and collecting statistics sent by the client solvers. The server stores its information in an SQLite3 database called Logs. The database is also used to analyse the steps of the parallel solving process. Instances to be solved are provided to the server at run time through the control socket. The control socket allows either the user or a tool (e.g., a model checker) to interact with the server.

The partitioning component creates the partitions. The partitioning process is guided by the CHC syntactic structure of the input instances. When partitioning is disabled, the partitioner creates a single partition corresponding to the bad states.

The scheduler keeps the list of all the instances and their partitions and manages connected clients. The scheduler solves one instance at a time. The partitions of the current instance are evenly distributed among the connected clients. Once a partition is proven unsatisfiable, the client working on it is provided with a remaining unsolved partition. SPACER implements three strategies, SPACER(DEF), SPACER(IC3), SPACER(GPDR), which differ in how they manage the queue of proof obligations. The scheduler proceeds in a best-effort way assigning each partition to 3 solvers, each configured with one strategy. If there are still solvers available then the same procedure is repeated using a different random seed in the underlying SMT solver of the client. Client failures and connection of new clients are handled in a sound way so that computational power may be added or removed on request.

The Lemma Database is implemented as a separate server. Each client periodically pushes learned lemmas to the Lemma Database accordingly to the sharing strategy provided by the server. The pull of the lemmas is also periodic and each client only pulls lemmas that are yet unknown for that client.

We implemented 4 lemma sharing strategies. The server reads from the configuration file which is the desired lemma sharing strategy and forwards it to the solver together with the instance. The procedure Lemma Sharing at line 5 in Algorithm 1 describes the strategy ∗-invariants, namely the exchanging of every learned PDR-lemma. Sharing only PDR-lemmas from $F_\infty$ (∞-invariants) is done by fixing $n = \infty$, while sharing only $k$-invariants is achieved by constraining $n \in \mathbb{N}$. Finally, lemma sharing is disabled when the procedure Lemma Sharing is never taken. Once an instance is solved its lemmas are removed to reduce memory consumption. In the case of partitioning combined with lemma sharing, our implementation shares lemmas only among solvers working on the same partition.
VI. EXPERIMENTS

This section presents an extensive experimental evaluation of the P3 algorithm on instances from the Software Verification Competition. We measure separately the performance of the algorithm on instances that are known to be easy and hard for the SPACER model checker, and study the performance of combinations of lemma sharing, portfolio, and partitioning. We also experiment on different lemma sharing heuristics on these settings.

All the reported experiments are executed on a cluster where each computing node is equipped with 2×Intel E5-2650 v3 CPU, 64 GB of RAM and Intel 40Gbps Infiniband network adapter. The parallel experiments are executed using 60 solvers on 6 computing nodes, with the server running on a separate node. The timeout is set to 1000 seconds (wall-clock time) on all the experiments.

The benchmark set used in our evaluations is constructed by SEA horn [7], a fully automated analysis framework for LLVM-based languages. Given as input the source file, SEA horn constructs the triplet \( \langle \text{Init}(X), \text{Tr}(X, X'), \text{Bad}(X) \rangle \) expressed in SMT-LIB v2 like language and representing the input safety problem ready to be provided to SPACER. Our benchmark set is based on 1,802 C problems taken from the SV-COMP 2016 Device Drivers Linux 64-bit (LDV) categories available at https://github.com/sosy-lab/sv-benchmarks/tree/master/c and preprocessed by SEA horn.

We first evaluate the benchmarks sequentially using the different strategies available in SPACER: IC3, GPDR and DEF. We call these settings sequential. Those benchmarks solved in less than one second are removed from the set and we experiment over the remaining 562 benchmarks.

Based on these evaluations we create two different benchmark sets of easy and hard instances respectively:

- **less500**: benchmarks solved in less than 500 seconds by at least one strategy. It contains 251 benchmarks.
- **more500**: benchmarks solved in more than 500 or timed out by at least one strategy. It contains 325 benchmarks.

These benchmark sets partially overlap by having 14 benchmarks in common.

Table II shows an overall evaluation over all the experiments we carried out. For each technique, we report the number of instances proven reachable, unreachable, and those unsolved, for both the experimental sets. The table is partitioned into 4 parts. Going from top to bottom: part 1 contains the results from sequential executions; part 2 contains the result for lemma sharing strategies with pure portfolio; part 3 contains results with partitioning; and part 4 presents the results of the virtual best solver that uses the best configuration for each problem. This virtual best is achievable by running in isolation a portfolio of the 8 combinations, using 8×60 CPUs. Notably, every parallel technique outperforms sequential execution.

For the more500 set the partitioning-based techniques perform the best. For the less500 set, especially for reachable instances, portfolio-based technique is the best, matching the virtual best solver.

In Table II, we show average time speedups between sequential executions and the respective best parallel techniques for both sets, over benchmarks that did not time out. The columns 60 CPU show the performance of the best technique: \(∞\)-invariants for less500 and partitioning\(+k\)-invariants for more500. An overview of performance over all the techniques is shown in Figure 2. We present the runtime performance for the three sequential strategies (IC3, GPDR and DEF) and all

### Table I

<table>
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<tr>
<th>Technique</th>
<th>less500</th>
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<th></th>
<th>more500</th>
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### Table II

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<th>Sequential strategy</th>
<th>less500</th>
<th></th>
<th></th>
<th>more500</th>
<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>60 CPU</td>
<td>virtual best</td>
<td></td>
<td>60 CPU</td>
<td>virtual best</td>
<td></td>
</tr>
<tr>
<td>SPACER(GPDR)</td>
<td>8×</td>
<td>10×</td>
<td></td>
<td>59×</td>
<td>91×</td>
<td></td>
</tr>
<tr>
<td>SPACER(IC3)</td>
<td>26×</td>
<td>32×</td>
<td></td>
<td>56×</td>
<td>88×</td>
<td></td>
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<tr>
<td>SPACER(DEF)</td>
<td>27×</td>
<td>33×</td>
<td></td>
<td>53×</td>
<td>83×</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2. Comparison among all tested techniques on the sets less500 (top) and more500 (bottom). $k$-invariants refers to sharing only lemmas from the trace, $\infty$-invariants refers to sharing just $F_\infty$, while $*$-invariants implements both. Finally, for each benchmark we report the virtual best runtime among all the tested techniques.

the parallel techniques tested in the cluster.

An overview for the set less500 is given in Figure 2 (top). Sharing $\infty$-invariants over portfolio is the best technique for this set, already performing similarly to the virtual best. In fact, $\infty$-invariants solves all the benchmarks with an average 30% slowdown with respect to the virtual best, and up to $27 \times$ faster than sequential, as reported in Table 1.

Figure 2 (bottom) shows a similar overview for more500. Regarding this set, partitioning techniques are the best choices. In fact, the best technique for this set is partitioning+$k$-invariants, followed by partitioning+$*$-invariants. However, 40 benchmarks are solved by only one of these two techniques, making them complementary rather than strictly better than the other.

A possible way to address the complementarity issue is by implementing a portfolio of multiple isolated parallel techniques, giving priority to the complementary ones. This approach is capable of gradually improving performance toward the virtual best results, where the best performance is reached with the highest CPU resource allocation.

Regarding more500, a portfolio of partitioning+$k$-invariants and partitioning+$*$-invariants is capable of solving 140 instances, 10 less than the virtual best and with an average 15% slowdown. Then, by adding $*$-invariants and $k$-invariants it is possible to solve 148 instances with 5% slowdown and half computing resources with respect to virtual best. The missing 2 instances are only solved by partitioning+$\infty$-invariants and $\infty$-invariants.

Regarding less500, a similar setting can only decrease solving time. This is because $\infty$-invariants already solved all the benchmark. Figure 3 shows the comparison between partitioning and portfolio with $*$-invariant sharing being quite complementary. Thus, a portfolio of these two techniques is capable of solving the entire less500 set using one fourth the CPU power requested by the virtual best, with a 14% slowdown.

Another important result shown in Figure 3 that partitioning often outperforms pure portfolio on reachable instances. This is because $\infty$-invariants often outperforms pure portfolio on reachable instances. This is because $\infty$-invariants already solved all the benchmark. Figure 3 shows the comparison between partitioning and portfolio with $*$-invariant sharing being quite complementary. Thus, a portfolio of these two techniques is capable of solving the entire less500 set using one fourth the CPU power requested by the virtual best, with a 14% slowdown.

Table III shows results about lemma sharing. The columns $time$ show the average amount of time spent on lemma sharing push and pull, with respect to solving time. The columns $#lemmas$ show the average number of PDR-lemmas exchanged. The amount of time spent on PDR-lemmas push and pull is about 1% and 3% of solving time respectively for portfolio based techniques, and partitioning based technique. The average amount of PDR-lemmas generated is significantly lower than in parallel SAT and SMT [10], [11]. While heuristics for clause sharing within parallel SAT and SMT are required due to the high throughput, in this settings lemma sharing

<table>
<thead>
<tr>
<th>Parallel technique</th>
<th>less500</th>
<th>more500</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>time</td>
<td>#lemmas</td>
</tr>
<tr>
<td>portfolio +</td>
<td></td>
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</tr>
<tr>
<td>$\infty$-invariants</td>
<td>0.35%</td>
<td>141</td>
</tr>
<tr>
<td>$k$-invariants</td>
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<td>252</td>
</tr>
<tr>
<td>$*$-invariants</td>
<td>1.55%</td>
<td>243</td>
</tr>
<tr>
<td>partitioning +</td>
<td></td>
<td></td>
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<tr>
<td>$\infty$-invariants</td>
<td>1.46%</td>
<td>170</td>
</tr>
<tr>
<td>$k$-invariants</td>
<td>3.51%</td>
<td>140</td>
</tr>
<tr>
<td>$*$-invariants</td>
<td>3.27%</td>
<td>221</td>
</tr>
</tbody>
</table>
heuristics are less crucial.

Overall, our experimental evaluations show that parallel techniques are highly beneficial. In particular, parallelization with portfolio combined with sharing PDR-lemmas from \( F_\infty \) is the best choice for easier instances, while partitioning with sharing PDR-lemmas from the trace is the best choice for harder instances. This demonstrate that the choice of the lemma sharing strategy is important.

More experimental results are available at [http://verify.inf.usi.ch/content/p3-experimental-results-fmcad2017](http://verify.inf.usi.ch/content/p3-experimental-results-fmcad2017)

VII. CONCLUSIONS

This work introduces the first parallel approach of the IC3/PDR algorithm for software model checking based on constrained horn clauses and satisfiability modulo theories. The P3 algorithm is based on combining in a new and PDR-specific way algorithm portfolios, divide-and-conquer approaches, and sharing of information learned during the algorithm execution. We describe our algorithm and its parallel extensions in a unified framework that allows us to both reason about the correctness of the implementation, and study the effect of each component in relative isolation. In addition we identify two different types of PDR-lemmas, the \( k \)-invariants, and the \( \infty \)-invariants, and give the first results on constructing lemma sharing heuristics. To the best of our knowledge in particular the divide-and-conquer approach and the lemma sharing heuristics have not been previously used in the context of PDR. The techniques we propose improve the previously introduced powerful portfolio technique for PDR, and we believe that the contributions help in applying parallel and distributed computing both in hardware and software verification.

We implemented the P3 algorithm following the same principle of isolation between the different techniques. The implementation is based on the SPACER model checker and is adaptable to both distributed environments and multi-core computing through our SMTSERVICE framework. Our experimental results obtained by running P3 on a representative set of software verification benchmarks from the SV-COMP 2016 competition show that the parallel approach is vastly superior to sequential SPACER configurations, solving over hundred more instances within our timeout and providing on the average super-linear speed-ups. We also show that each of the new techniques work in isolation and that they have interesting interaction with the different clause-sharing heuristics.

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REFERENCES