Local Search Techniques

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Local Search: Generic structure

Generic structure of local search SAT solvers

```
for i in 1 to MAX_TRIES do
    φ := random initial assignment
    for j in 1 to MAX_STEPS do
        if φ satisfies 𝓕 then
            return satisfiable
        end if
        φ := FLIP( φ )
    end for
end for
return unknown
```

Local Search: Global vs Local flips

Global flips

- Pro: Big improvements
- Neg: Probabilistic incomplete

Local flips

- Neg: Small improvements
- Pos: Probabilistic complete

Local Search: WalkSAT Flips

Select a random unsatisfied clause C

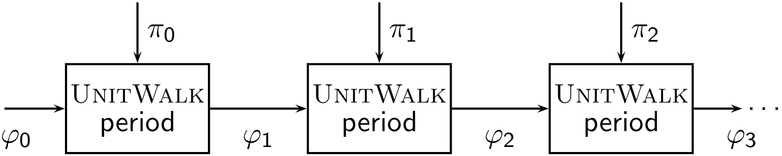
- Free flip
- Random flip
- Heuristic flip

Local Search: WalkSAT Code

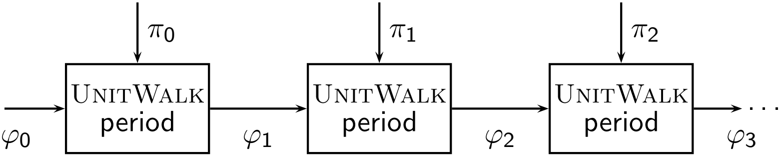
FLIP_WALKSAT(φ)

$C :=$ random falsified clause by $\varphi \circ \mathcal{F}$
if a variable $\in C$ can be flipped for free **then**
    flip in φ that variable
else
    flip in φ with p a random $x_i \in C$
    flip in φ with $1 - p$ the "optimal" $x_i \in C$
end if
return φ

The UnitWalk Algorithm



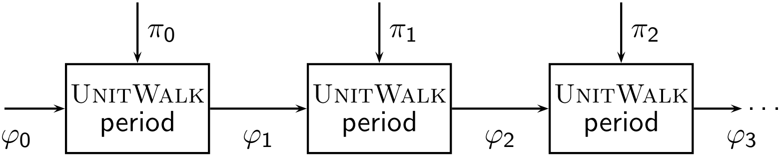
The UnitWalk Algorithm



The general idea of a UnitWalk period:

- Within each unsatisfied clause in $\varphi_i \circ \mathcal{F}$ the assignment to the least important variable (based on π_i) is flipped

The UnitWalk Algorithm



The general idea of a UnitWalk period:

- Within each unsatisfied clause in $\varphi_i \circ \mathcal{F}$ the assignment to the least important variable (based on π_i) is flipped

For example:

- $\mathcal{F} = (x \vee \neg y),\ \varphi_0 = \{x = 0, y = 1\},\ \pi_0 = \{y, x\}$

UnitWalk Period Example

$$\begin{aligned}
\mathcal{F}_{\text{example}} &:= (x_1 \vee x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge \\
&\quad (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee \neg x_4) \\
\varphi_{\text{master}} &:= \{x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0\} \\
\varphi_{\text{active}} &:= \{x_1 = *, x_2 = *, x_3 = *, x_4 = *\} \\
\pi &:= (x_2, x_1, x_4, x_3)
\end{aligned}$$

do

iterative propagate unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$

extend φ_{active} with most important free variable according to π

while φ_{active} contains $*$'s

UnitWalk Period Example

$$
\begin{aligned}
\mathcal{F}_{\text{example}} &:= (x_1 \vee x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge \\
&\quad (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee \neg x_4) \\
\varphi_{\text{master}} &:= \{x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0\} \\
\varphi_{\text{active}} &:= \{x_1 = *, x_2 = *, x_3 = *, x_4 = *\} \\
\pi &:= (x_2, x_1, x_4, x_3)
\end{aligned}
$$

do

$\rightarrow$ iterative propagate unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$

extend φ_{active} with most important free variable according to π

while φ_{active} contains $*$'s

Action:

- no unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$

UnitWalk Period Example

$$
\begin{aligned}
\mathcal{F}_{\text{example}} &:= (x_1 \vee x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge \\
&\quad (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee \neg x_4) \\
\varphi_{\text{master}} &:= \{x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0\} \\
\varphi_{\text{active}} &:= \{x_1 = *, x_2 = 1, x_3 = *, x_4 = *\} \\
\pi &:= (x_2, x_1, x_4, x_3)
\end{aligned}
$$

do

iterative propagate unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$

$\rightarrow$ extend φ_{active} with most important free variable according to π

while φ_{active} contains $*$'s

Action:

- extend φ_{active} with $x_2 := 1$ (the truth value in φ_{master})

UnitWalk Period Example

$$
\begin{aligned}
\mathcal{F}_{\text{example}} &:= (x_1 \vee x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge \\
&\quad\; (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee \neg x_4) \\
\varphi_{\text{master}} &:= \{x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0\} \\
\varphi_{\text{active}} &:= \{x_1 = *, x_2 = 1, x_3 = 0, x_4 = *\} \\
\pi &:= (x_2, x_1, x_4, x_3)
\end{aligned}
$$

do
→ iterative propagate unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$
extend φ_{active} with most important free variable according to π
while φ_{active} contains $*$'s

Action:

- detected unit clause $\neg x_3 \rightarrow x_3 := 0$

UnitWalk Period Example

$$
\begin{aligned}
\mathcal{F}_{\text{example}} &:= (x_1 \vee x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge \\
&\quad (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee \neg x_4) \\
\varphi_{\text{master}} &:= \{x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0\} \\
\varphi_{\text{active}} &:= \{x_1 = *, x_2 = 1, x_3 = 0, x_4 = 0\} \\
\pi &:= (x_2, x_1, x_4, x_3)
\end{aligned}
$$

do

→ iterative propagate unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$

extend φ_{active} with most important free variable according to π

while φ_{active} contains $*$'s

Action:

- detected unit clauses x_4 and $\neg x_4 \rightarrow$ conflict
- assign x_4 to truth value in $\varphi_{\text{master}} \rightarrow x_4 := 0$

UnitWalk Period Example

$$\begin{aligned}
\mathcal{F}_{\text{example}} &:= (x_1 \vee x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge \\
&\quad (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee \neg x_4) \\
\varphi_{\text{master}} &:= \{x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0\} \\
\varphi_{\text{active}} &:= \{x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 0\} \\
\pi &:= (x_2, x_1, x_4, x_3)
\end{aligned}$$

do

iterative propagate unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$

$\rightarrow$ extend φ_{active} with most important free variable according to π

while φ_{active} contains $*$'s

Action:

- extend φ_{active} with $x_1 := 0$ (the truth value in φ_{master})

UnitWalk Period Example

$$\begin{aligned}
\mathcal{F}_{\text{example}} &:= (x_1 \vee x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge \\
&\quad\; (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee \neg x_4) \\
\varphi_{\text{master}} &:= \{x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0\} \\
\varphi_{\text{active}} &:= \{x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 0\} \\
\pi &:= (x_2, x_1, x_4, x_3)
\end{aligned}$$

do

iterative propagate unit clauses in $\varphi_{\text{active}} \circ \mathcal{F}_{\text{example}}$

extend φ_{active} with most important free variable according to π

$\rightarrow$ **while** φ_{active} contains $*$'s

Action:

- end of period because all variables are assigned in φ_{active}

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