Practical SAT Solving: Look-ahead Techniques

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SAT solving: DPLL

Davis Putnam Logemann Loveland [DP60,DLL62]

- Simplify (Unit Propagation)
- Split the formula
  - Variable Selection Heuristics
  - Direction heuristics

DPLL: Example

$$\mathcal{F}_{\mathrm{DPLL}} := (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_3) \wedge (\neg x_1 \vee \neg x_3)$$

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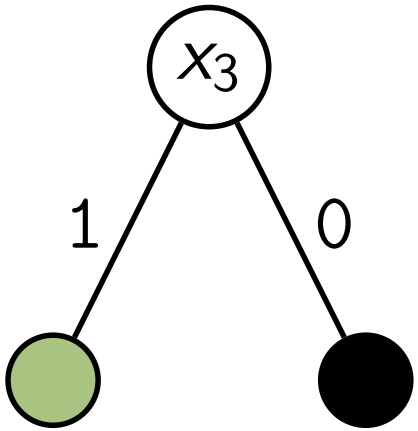

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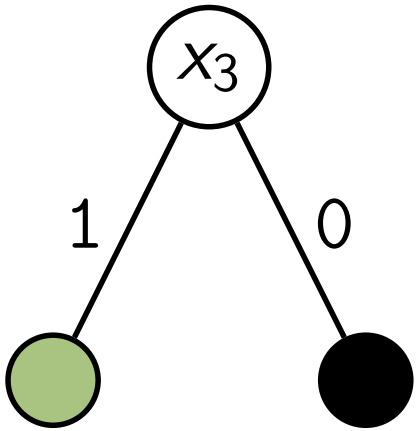

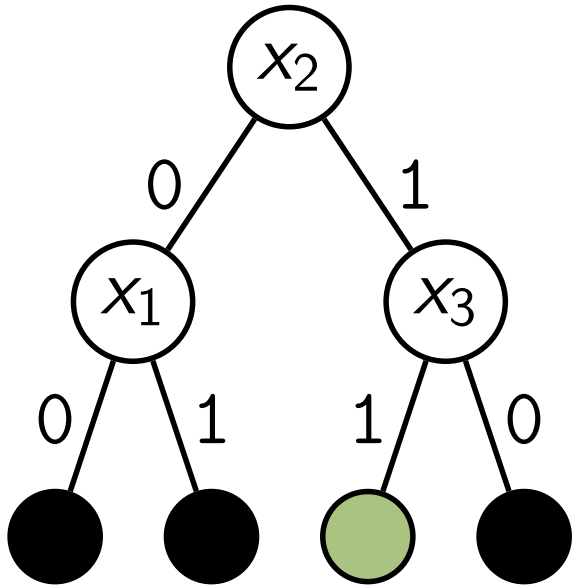

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DPLL with selection of (effective) decision variables by *look-aheads* on variables

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Look-ahead:

- Assign a variable to a truth value
- Simplify the formula
- Measure the reduction
- Learn if possible
- Backtrack

Look-ahead: Example

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \wedge (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

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- Examples: march, OKsolver, kcnfs

Look-ahead: Reduction heuristics

- Number of satisfied clauses

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- Number of implied variables

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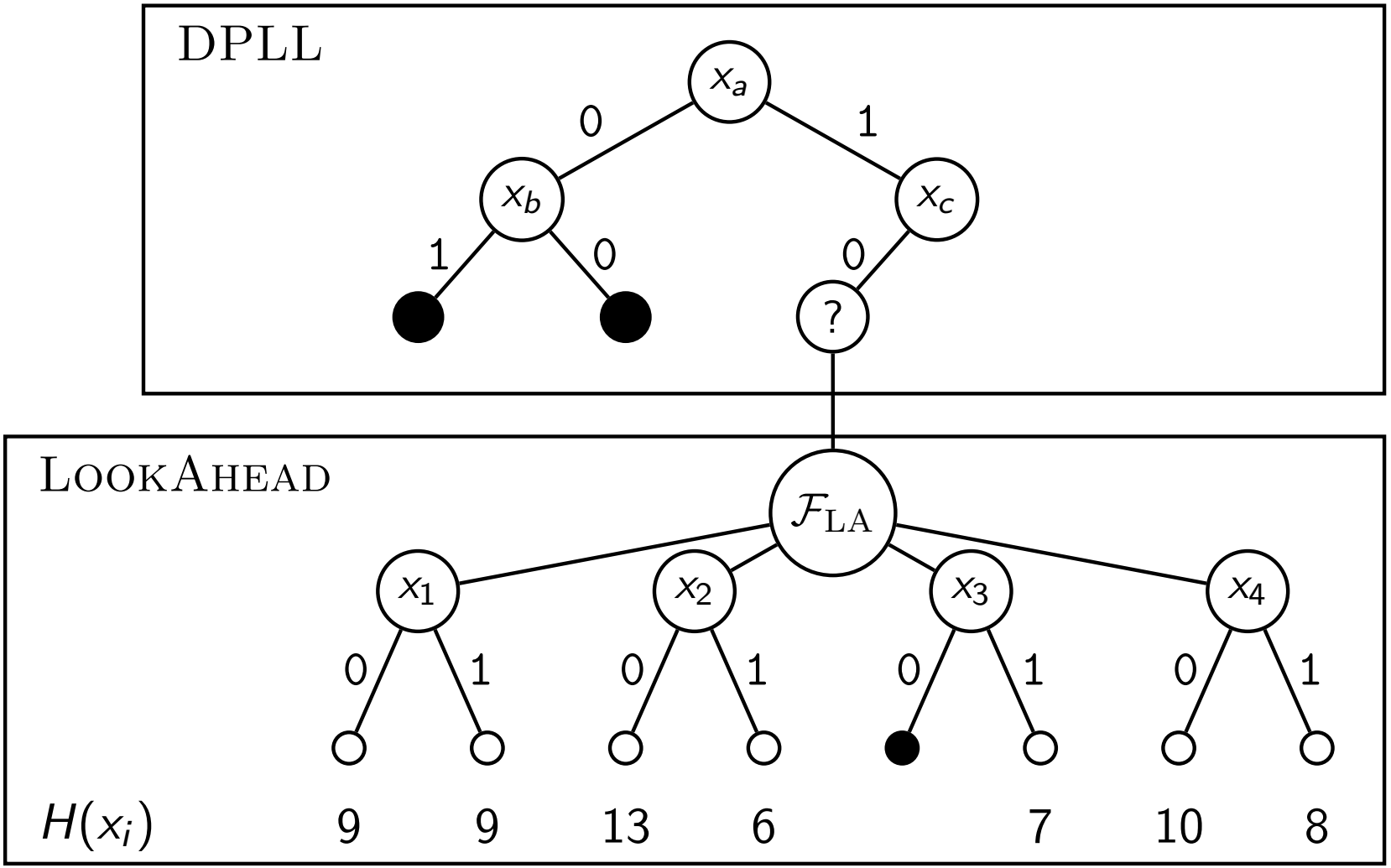
- Number of satisfied clauses
- Number of implied variables
- New (reduced, not satisfied) clauses
  - Smaller clauses more important
  - Weights based on occurring

Look-ahead: Architecture



Look-ahead: Pseudo-code

1: $\mathcal{F} := \textsc{Simplify}\ (\mathcal{F})$
2: **If** $\mathcal{F}$ is empty **then return** `satisfiable`
3: **If** $\emptyset \in \mathcal{F}$ **then return** `unsatisfiable`
4: $\langle \mathcal{F};\ l_{\text{decision}} \rangle := \textsc{LookAhead}(\ \mathcal{F}\ )$
5: **if** DPLL$(\ \mathcal{F}(l_{\text{decision}} \leftarrow 1)) =$ `satisfiable` **then**
6: **return** `satisfiable`
7: **end if**
8: **return** DPLL$(\ \mathcal{F}(l_{\text{decision}} \leftarrow 0))$

Look-ahead: Local learning

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(local) constraint resolvents
a.k.a. hyper binary resolvents:
$(x_2 \vee x_3)$ and $(x_2 \vee \neg x_6)$

Look-ahead: Necessary assignments

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An *1-autarky* is a partial assignment that satisfies all touched clauses except one

Look-ahead: Autarky detection

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \wedge (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

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Could reduce computational cost on UNSAT

Look-ahead: Autarky or Conflict on 2-SAT Formulae

Lookahead techniques can solve 2-SAT formulae in polynomial time. Each lookahead on l results:

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2. in a conflict: forcing l to be false

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SAT game

by Olivier Roussel

`http://www.cs.utexas.edu/~marijn/game/2SAT`

Look-ahead: 1-Autarky learning

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \wedge (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

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$$\varphi = \{x_2{=}0\}$$

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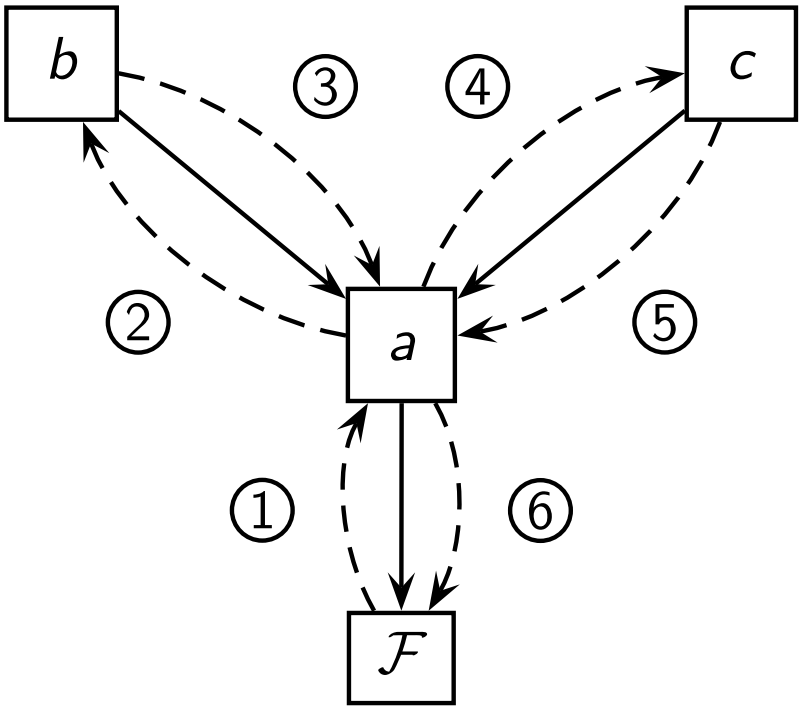
(local) 1-autarky resolvents:
$(\neg x_2 \vee \neg x_4)$ and $(\neg x_2 \vee \neg x_5)$

Tree-based Look-ahead

Given a formula F which includes the clauses $(a \vee \bar{b})$ and $(a \vee \bar{c})$, tree-based look-ahead can reduce the costs of look-aheads.



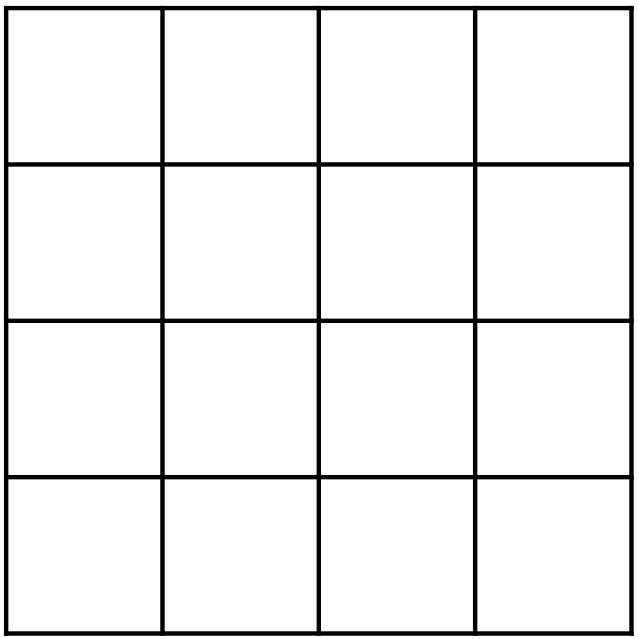
implication

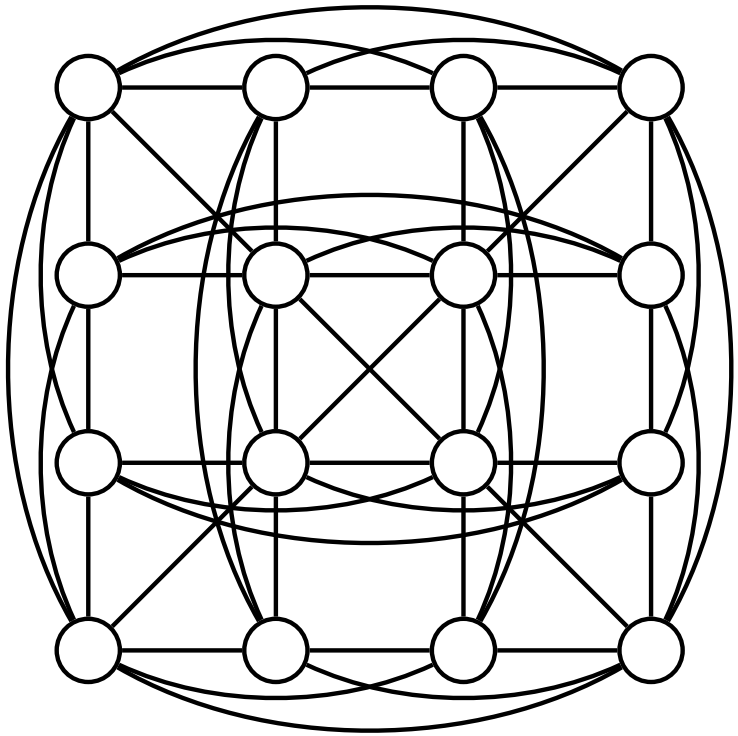
action

1. propagate a
2. propagate b
3. backtrack b
4. propagate c
5. backtrack c
6. backtrack a

Homework puzzle

Populate the matrix with the following numbers:
$10, 40, 50, 12, 21, 31, 35, 43, 46, 56, 74, 75, 83, 87, 89, 98$
such that no digit repeats in any row, column or diagonal.





Practical SAT Solving: Look-ahead Techniques

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