Parallel Asynchronous Matrix Completion

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Joint work with

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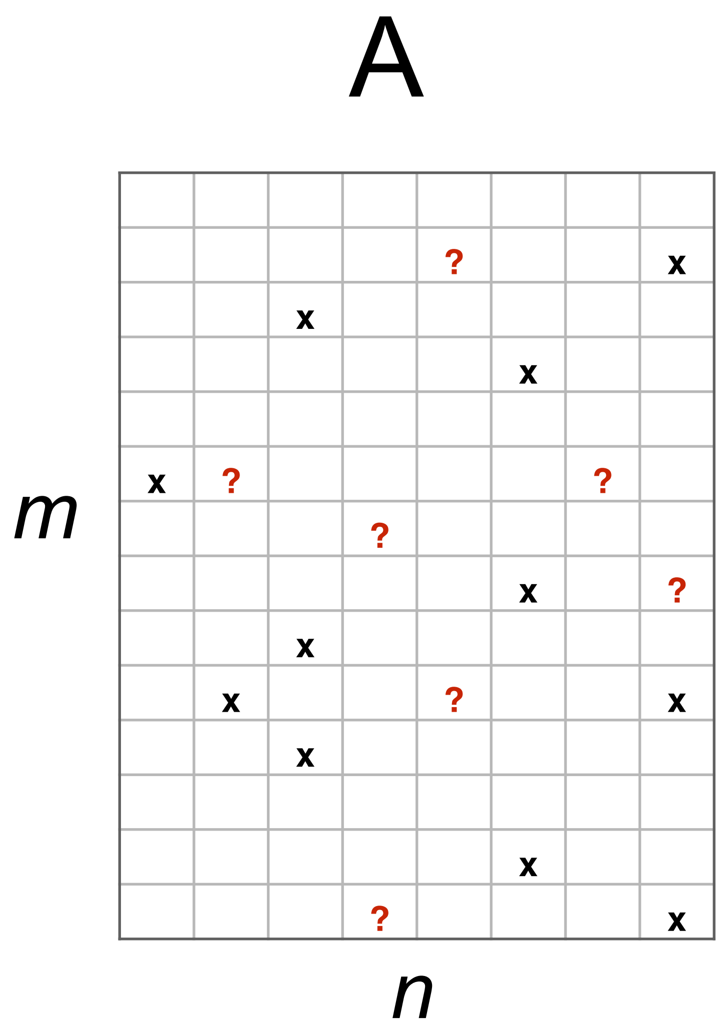
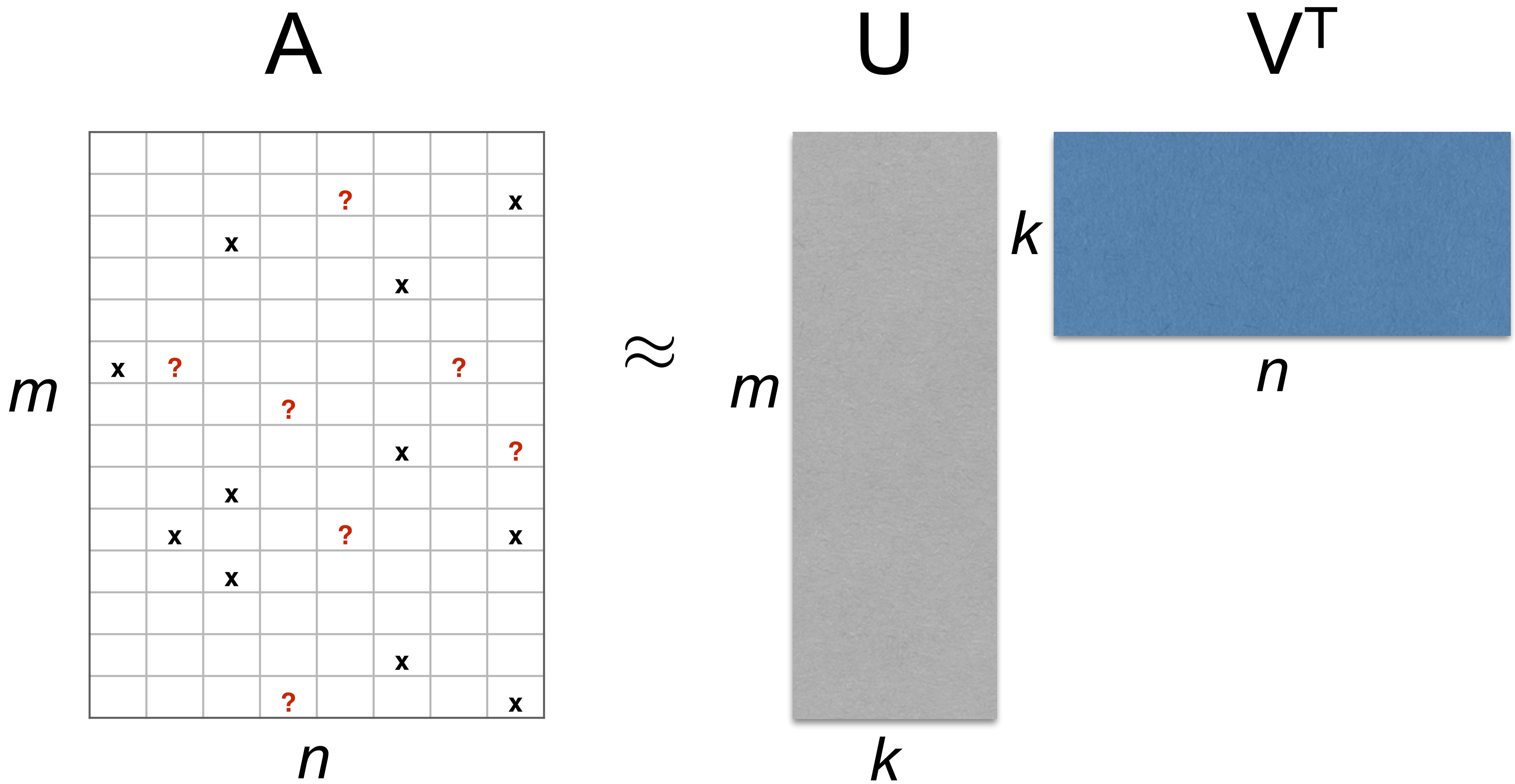
The $1M Netflix Prize [2006]

$1M was won in 2009

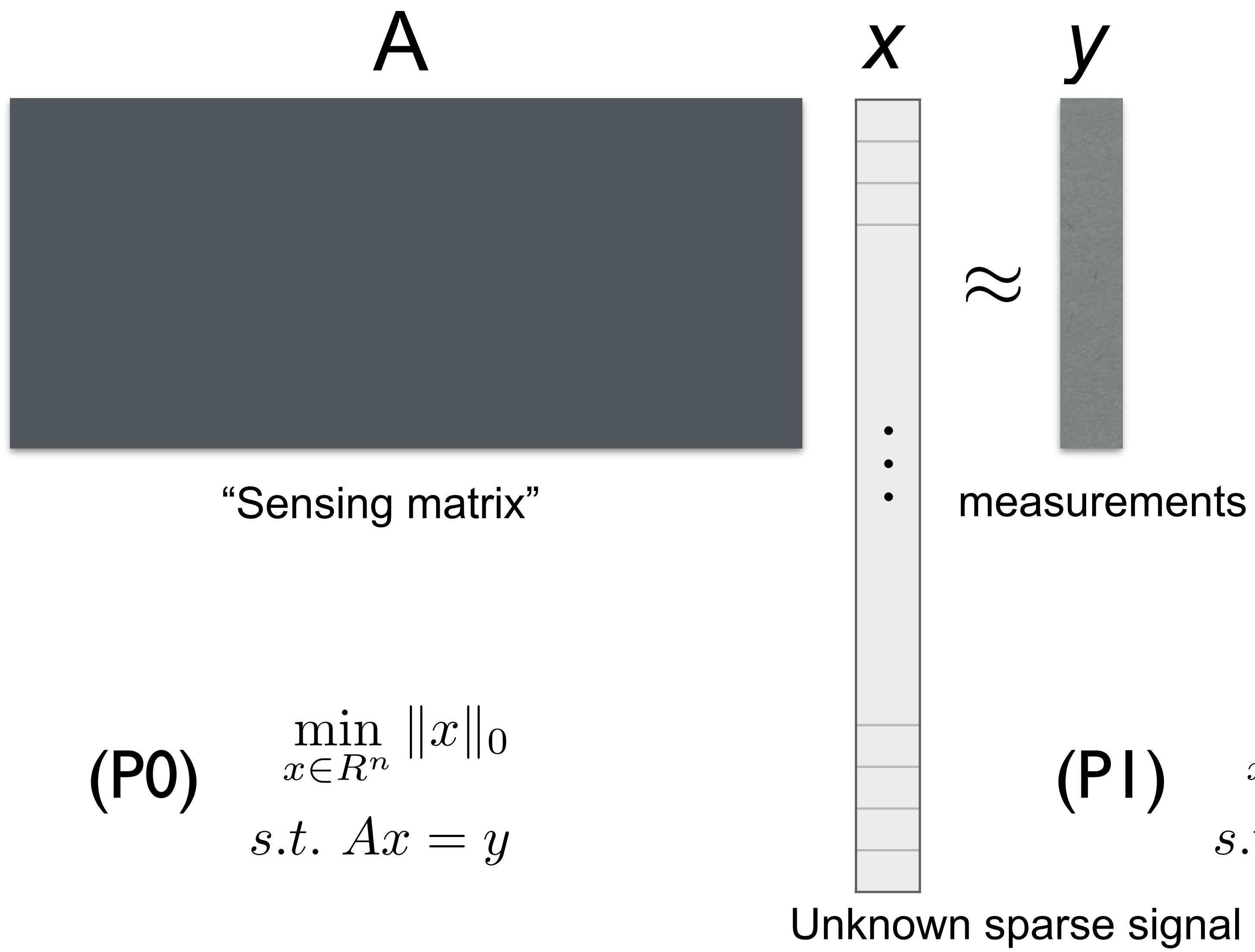
Missing Value Estimation



Low-rank Matrix Completion



Compressed Sensing



(P0) $\min_{x \in R^n} \|x\|_0$ $s.t.\ Ax = y$

(P1) $\min_{x \in R^n} \|x\|_1$ $s.t.\ Ax = y$

[Candes,Tao 2005] If *A* obeys the *restricted isometry property*:

$$(1-\delta_s)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1+\delta_s)\|x\|_2^2, \quad \forall x : \|x\|_0 = s$$

for some constant δ_s and the solution x_0 to problem (P0) is *s*-sparse, then x_0 is the unique solution to the convex relaxation (P1) and can be *exactly* recovered by solving a linear program.

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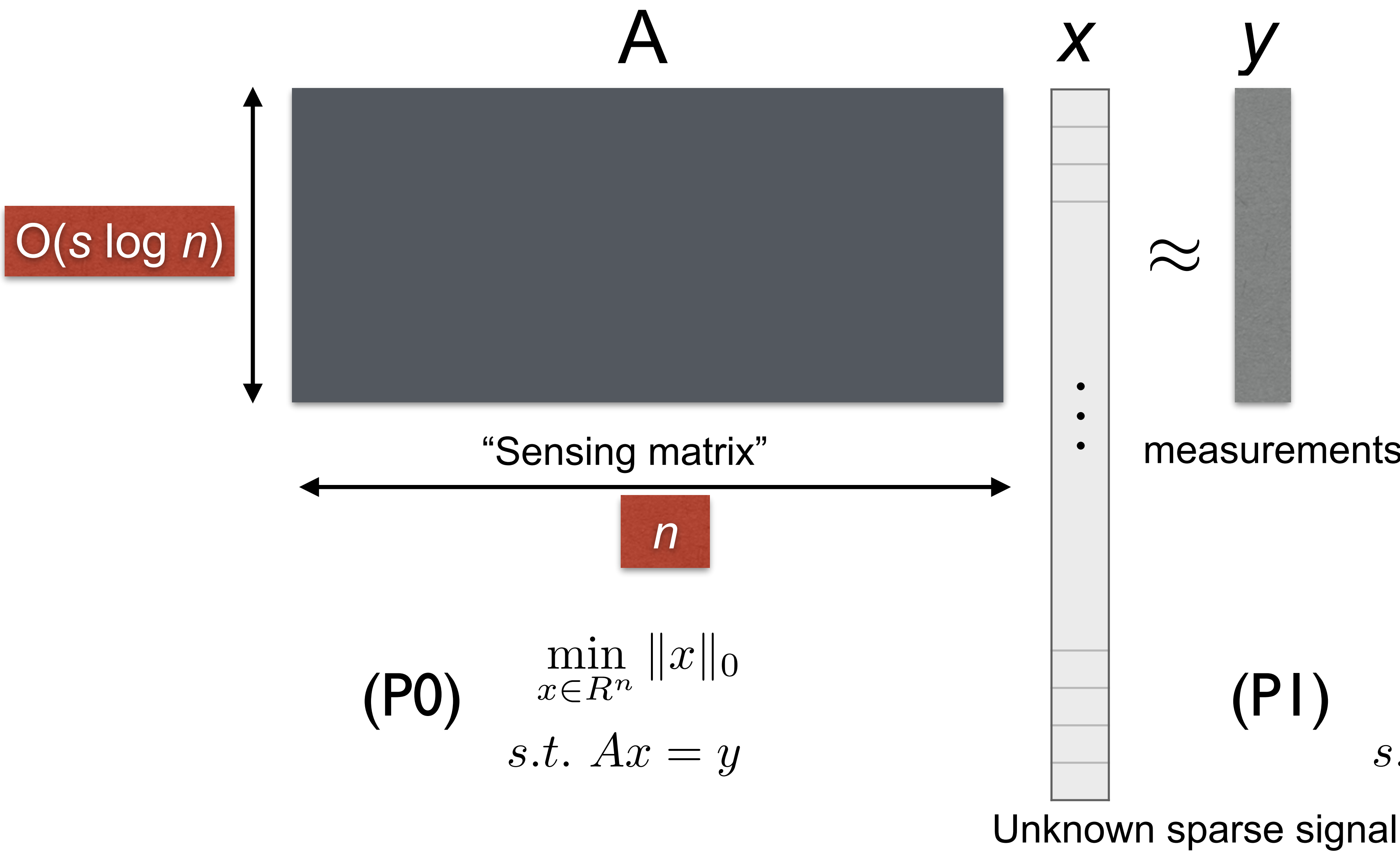

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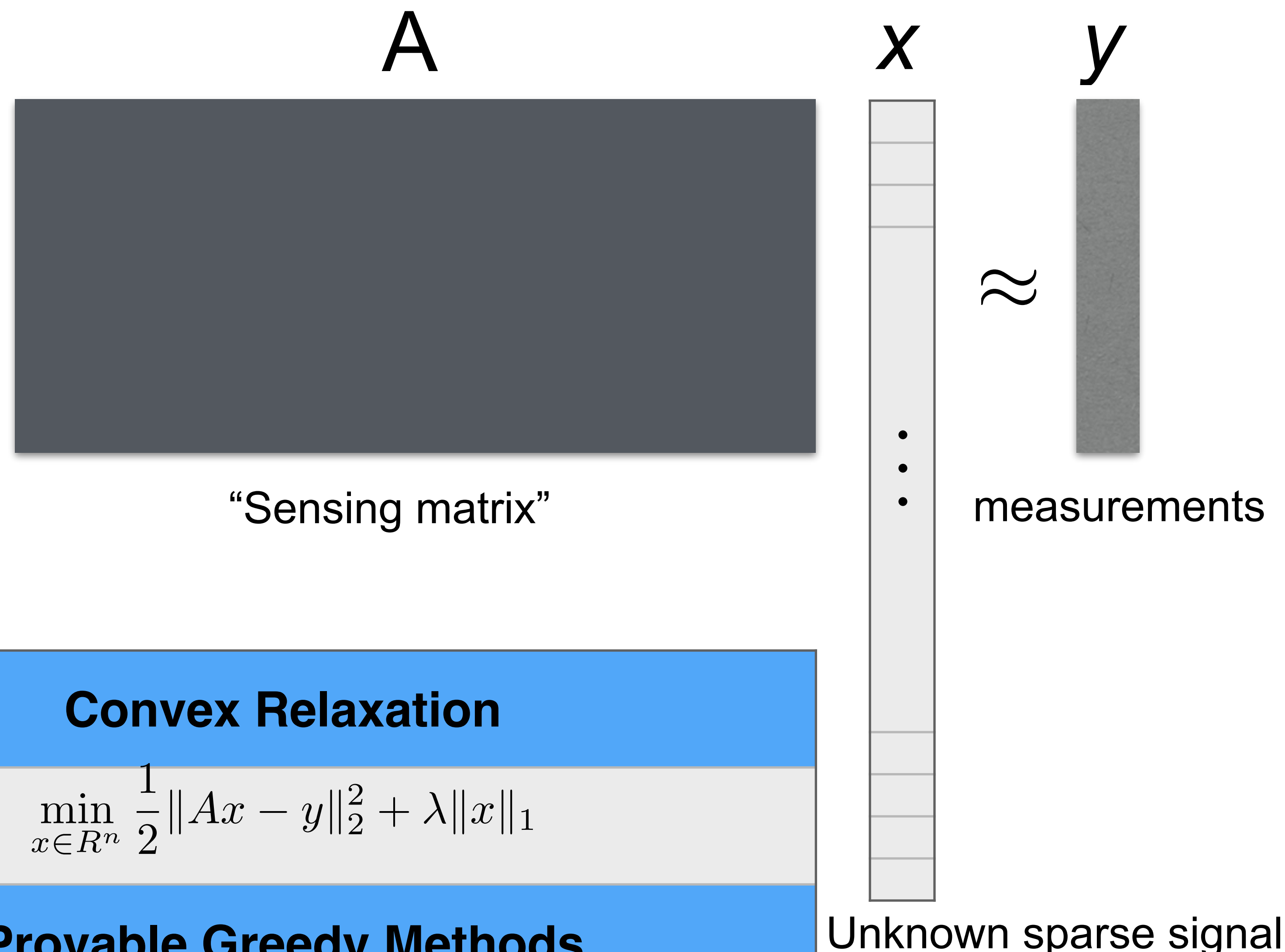


Convex Relaxation
$$\min_{x \in R^n} \frac{1}{2}\|Ax - y\|_2^2 + \lambda \|x\|_1$$
Provable Greedy Methods
[Pati,Rezaiifar,Krishnaprasad1993] *Orthogonal Matching Pursuit* (OMP)
[Needell, Tropp 2009] *Compressive Sampling Matching Pursuit* (CoSaMP)
[Jain,Tewari,Dhillon 2011] *Orthogonal Matching Pursuit with Replacement* (OMPR)

Linear Sensing of Low-Rank Matrices

(P0) $$\min_{X \in \mathbb{R}^{m \times n}} \ \text{rank}\,(X) \quad s.t.\ \mathcal{A}(X) = b$$

(P1) $$\min_{X \in \mathbb{R}^{m \times n}} \ \|X\|_* \quad s.t.\ \mathcal{A}(X) = b$$

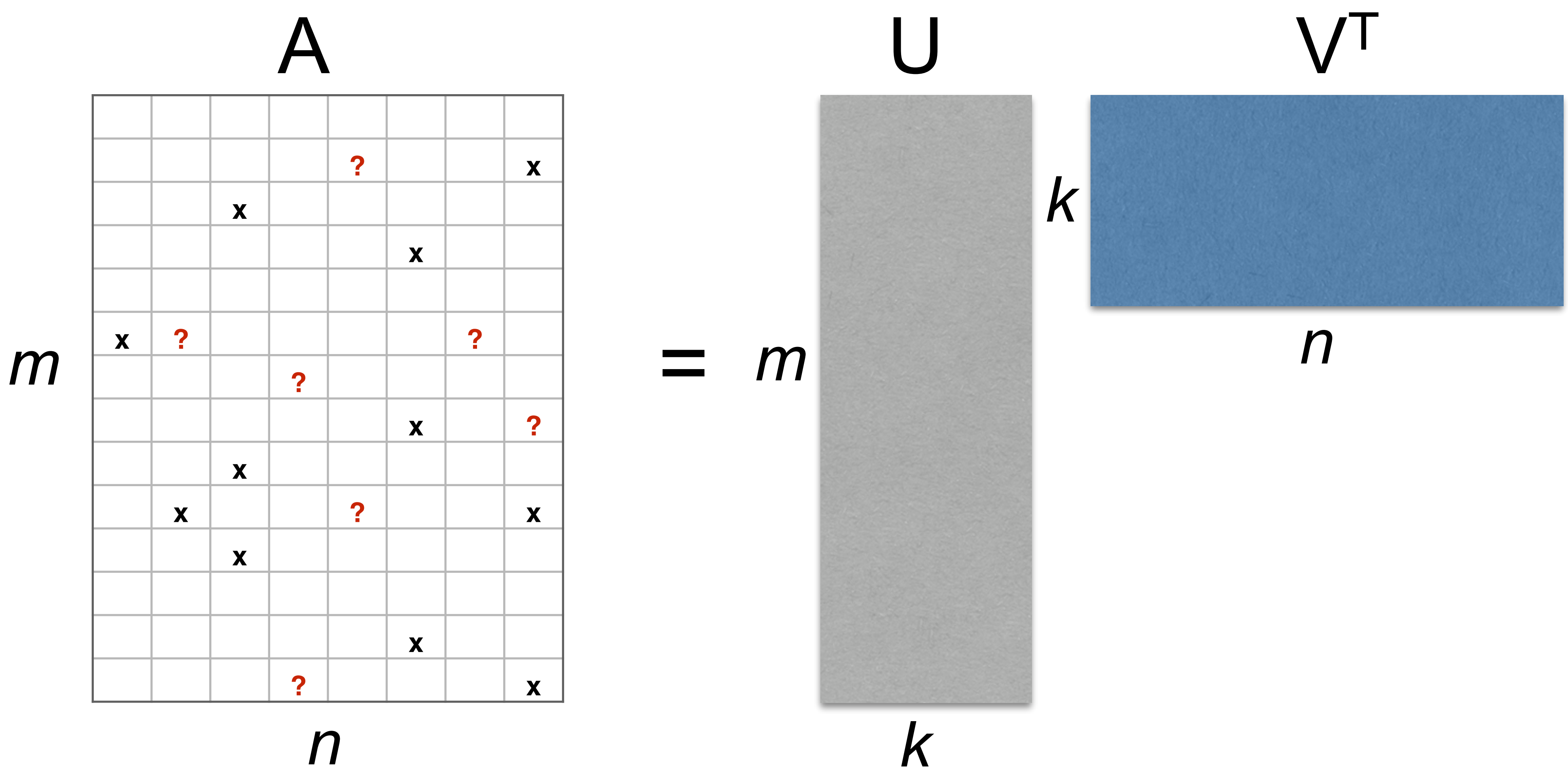
- Compressed sensing guarantees can be extended to low-rank matrix recovery problem
- Recovery depends on a *generalized Restricted Isometry Property* for linear maps:

[Recht,Fazel,Parrilo 2010] If $\mathcal{A}$ obeys the *restricted isometry property*:

$$(1 - \delta_r(\mathcal{A}))\|X\|_F \leq \|\mathcal{A}(X)\| \leq (1 + \delta_r(\mathcal{A}))\|X\|_F, \quad \forall X : \text{rank}(X) \leq r$$

for some constant δ_r and the solution X_0 to (P0) has rank r, then X_0 is the unique solution to the convex relaxation (P1).

Low-rank Matrix Completion

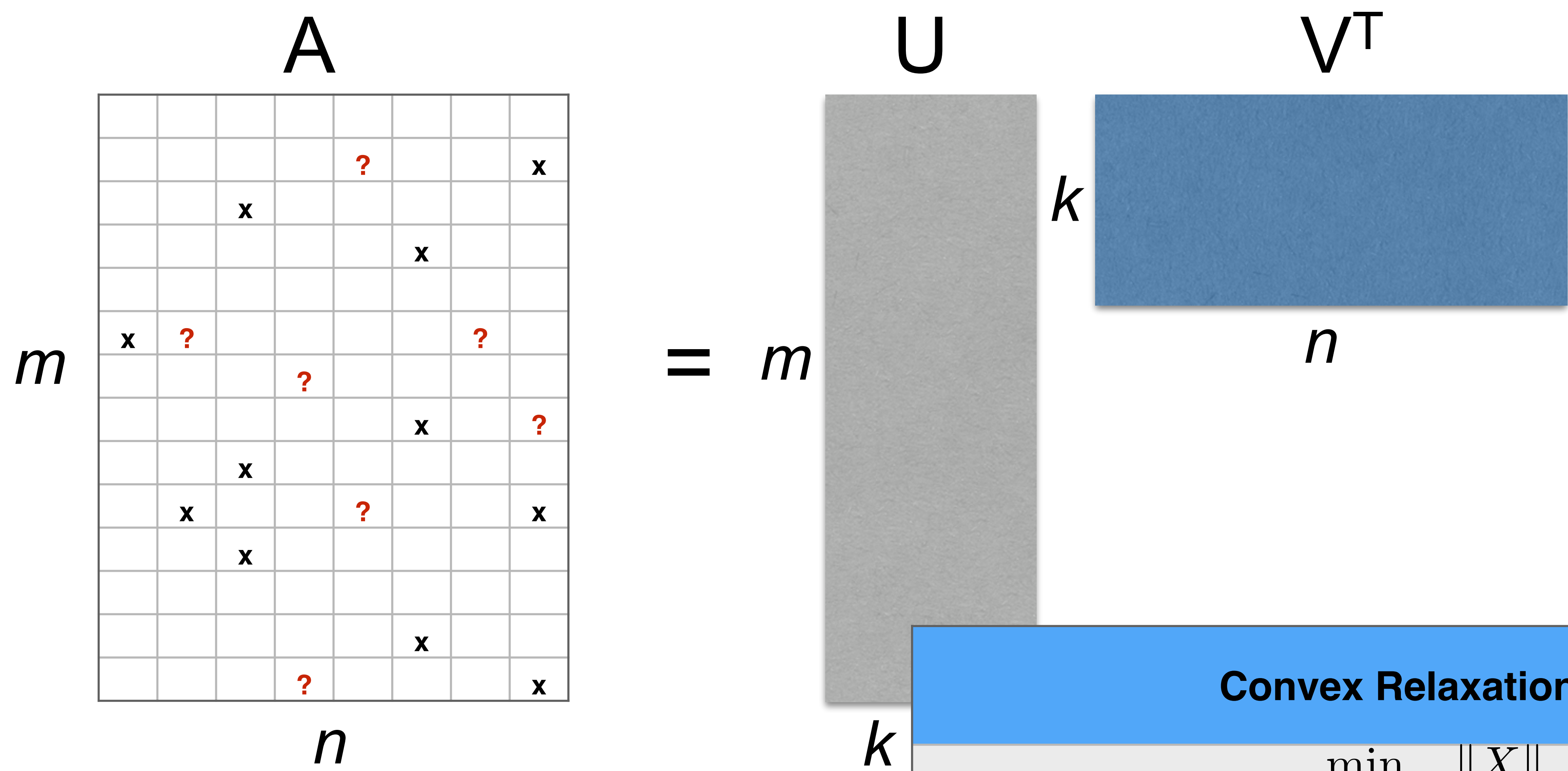


[Candes,Recht 2008] Nuclear norm minimization perfectly recovers most low-rank *n x n* matrices of rank *r*, if $O(n^{1.2} r \log n)$ entries (sampled uniformly at random) are observed.

[Recht 2009] Near-optimal sample complexity: $O(\max\{\mu_1, \mu_0\} nr \log^2 2n)$

[Gross 2009] A novel analysis yielding sample complexity $O(nr\nu \log^2 n)$

Low-rank Matrix Completion



Convex Relaxation

$$\min_{X \in \mathbb{R}^{m \times n}} \|X\|_*$$

$$\text{s.t.} \;\; X_{ij} = M_{ij}, (i,j) \in \Omega$$

Provable Greedy Methods

[Jain,Meka,Dhillon 2009]
Singular Value Projection (SVP)
[Jain,Netrapalli,Sanghavi 2013]
Alternating Least Squares (ALS)

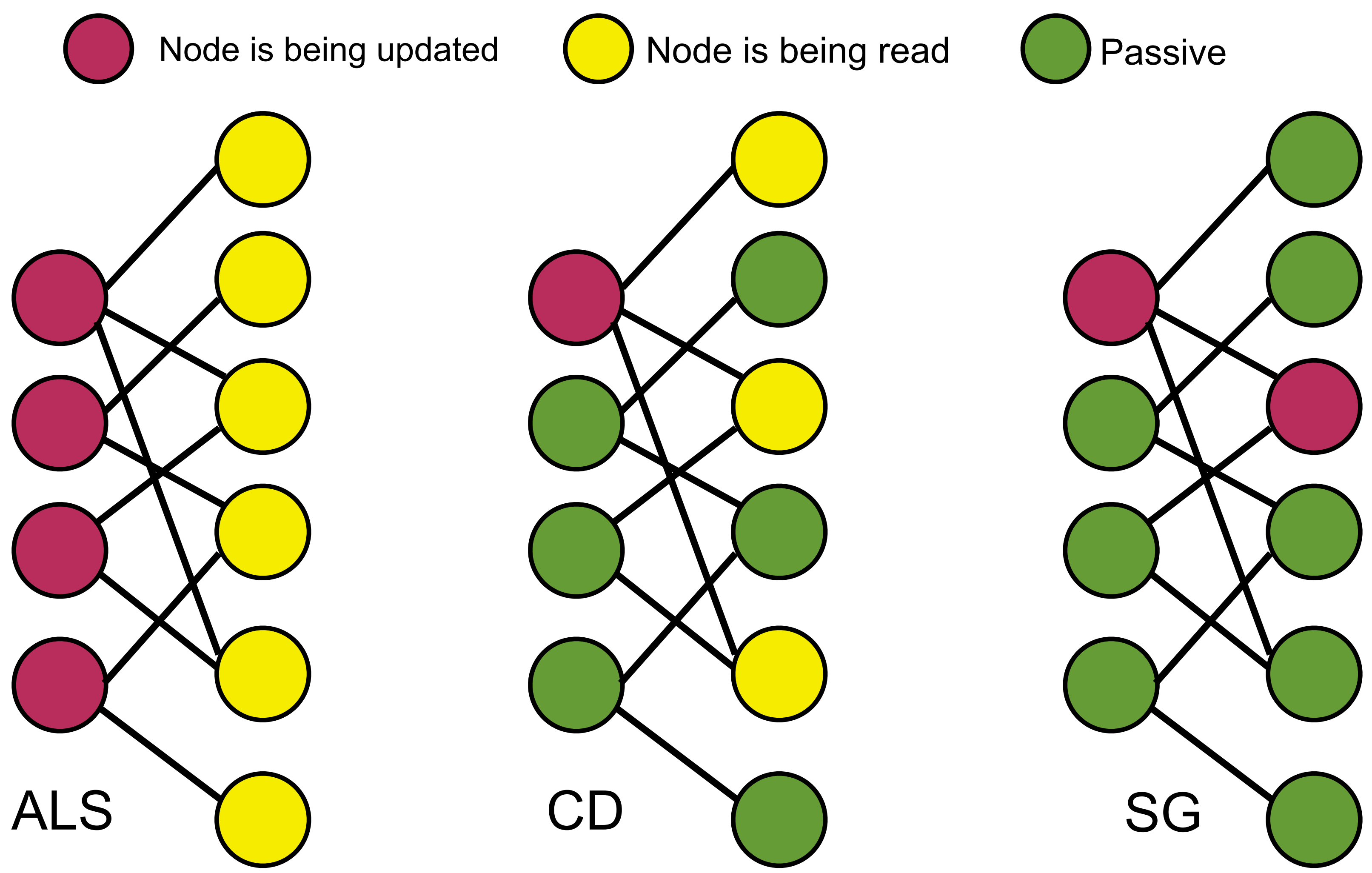
Low-rank Matrix Completion

$$\min_{U \in \mathbb{R}^{m \times k}, V \in \mathbb{R}^{n \times k}} \|W \odot (A - UV^T)\|_F^2 + \lambda(\|U\|_F^2 + \|V\|_F^2)$$

- How do we solve in practice?
- Typical time complexity for iterative procedure: $O(|\Omega|k^2 + (m+n)k^3)$
- Domains where recommender systems are used — Millions of rows and columns, and billions of observed samples
- Big Data — How do we scale to very large matrices?

Matrix Completion Algorithms

- Alternating Least Squares (ALS): Fix W and solve for H (and vice versa)
- Coordinate Descent (CD): Choose a direction (1 through k) and update the corresponding feature in W and H
- Stochastic Gradient (SG): Sample an observation (i,j) at random and update the corresponding latent factors in W and H



Stochastic Gradient

- Well-suited for data arriving in an incremental fashion
- Continually update model parameters as data arrives
- Consider:

$$\min_{w \in \mathcal{W}} F(w) = E_Z[f(w;Z)]$$

- Idea: At each step *t* , obtain a gradient estimate at current parameter w_t such that it is "unbiased":

$$E_Z[\hat{g}_t] = \nabla_w F(w_t)$$

- Update $w_{t+1} = \Pi_{\mathcal{W}}(w_t - \eta_t \hat{g}_t)$

[Nemirovski 2009] If F is strongly convex and smooth w.r.t. *w,* and $E[\|\hat{g}_t\|^2]$ is bounded, then:

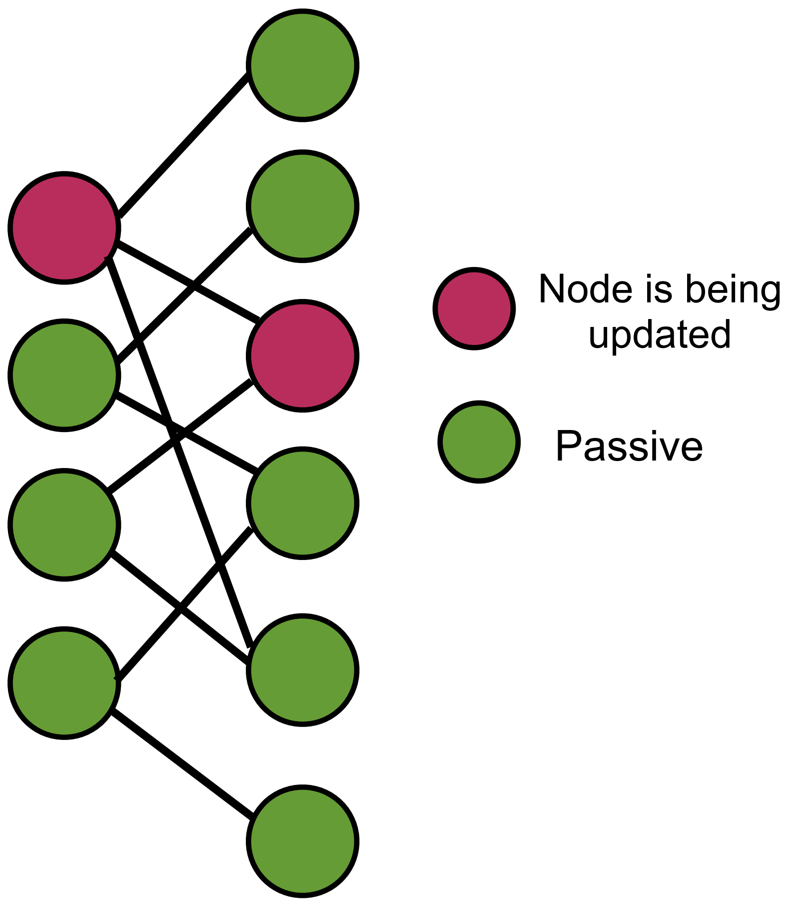
$$E[F(w_t) - F(w^*)] = O\left(\frac{1}{t}\right)$$

Stochastic Gradient: Matrix Completion

- Sample random rating (*i,j*) and update corresponding latent factors:

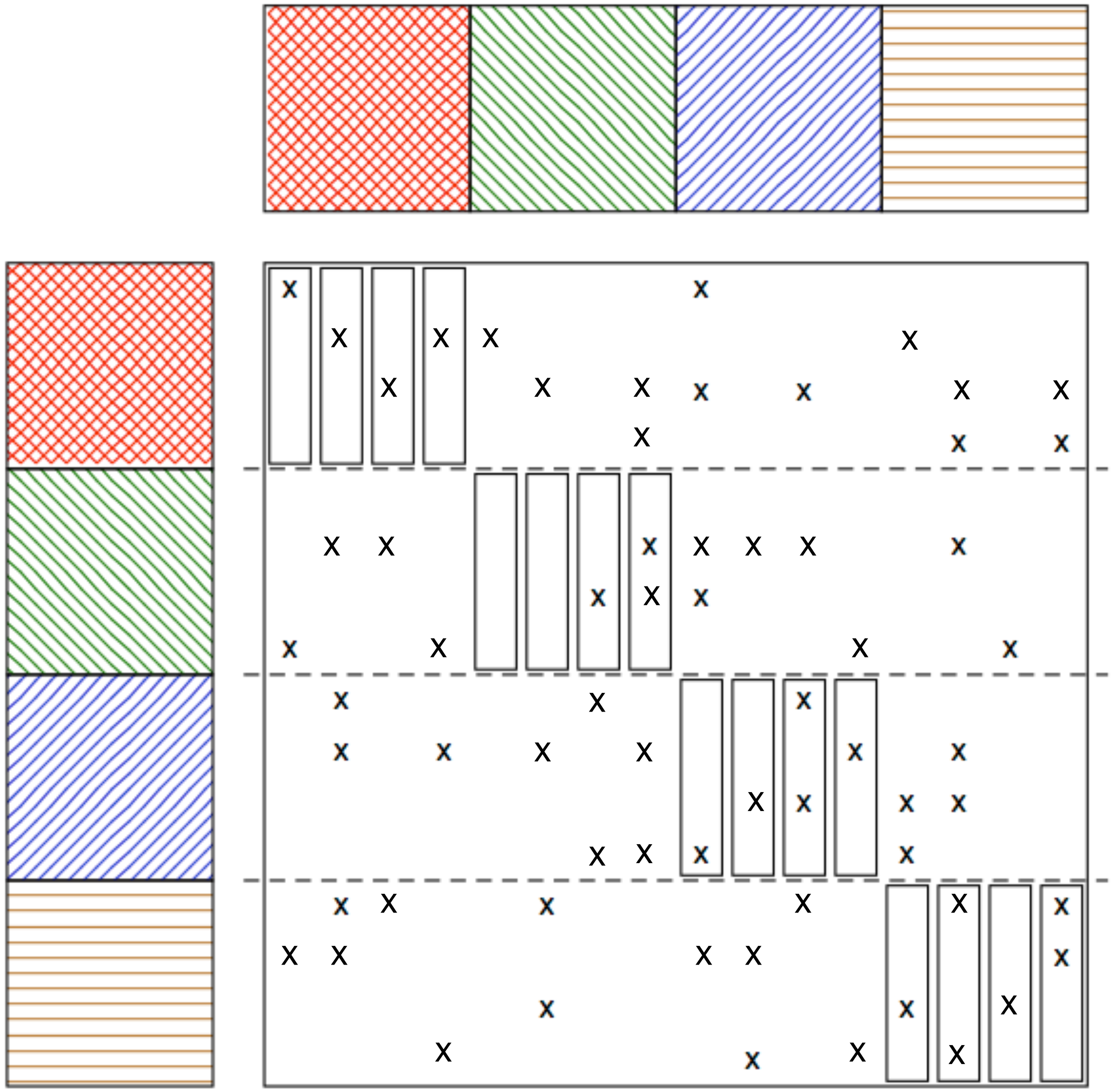
$$\mathbf{u}_i \leftarrow \mathbf{u}_i - \eta(A_{ij} - \langle \mathbf{u}_i, \mathbf{v}_j \rangle \mathbf{v}_j + \lambda \mathbf{u}_i)$$
$$\mathbf{v}_j \leftarrow \mathbf{v}_j - \eta(A_{ij} - \langle \mathbf{u}_i, \mathbf{v}_j \rangle \mathbf{u}_i + \lambda \mathbf{v}_j)$$

- Time per update $O(k)$
- Effective for very large-scale problems



Distributed Stochastic Gradient Descent

- Decoupled updates — easy to parallelize [Gemulla et al 2011]
- But communication & computation are done in sequence



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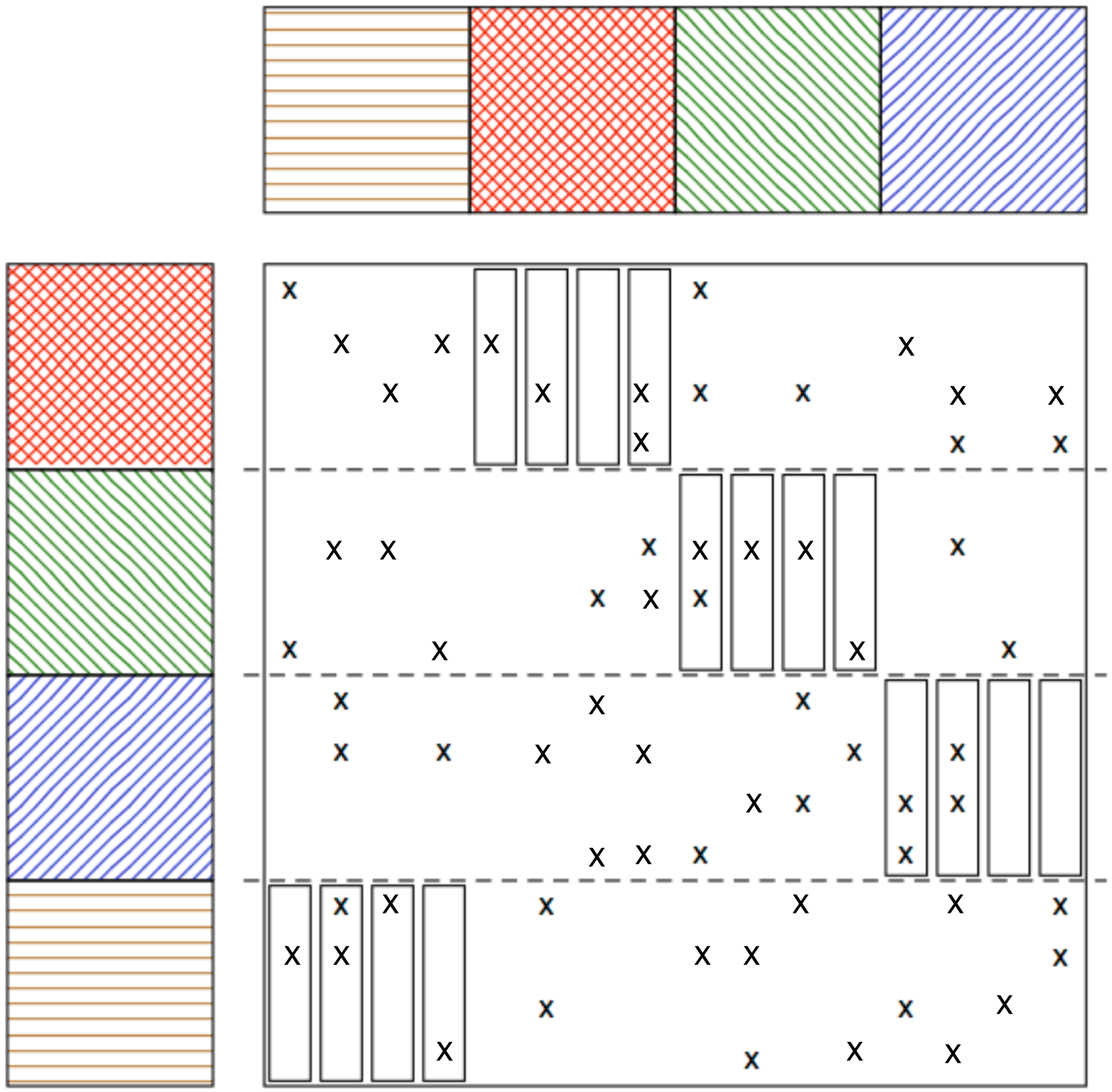

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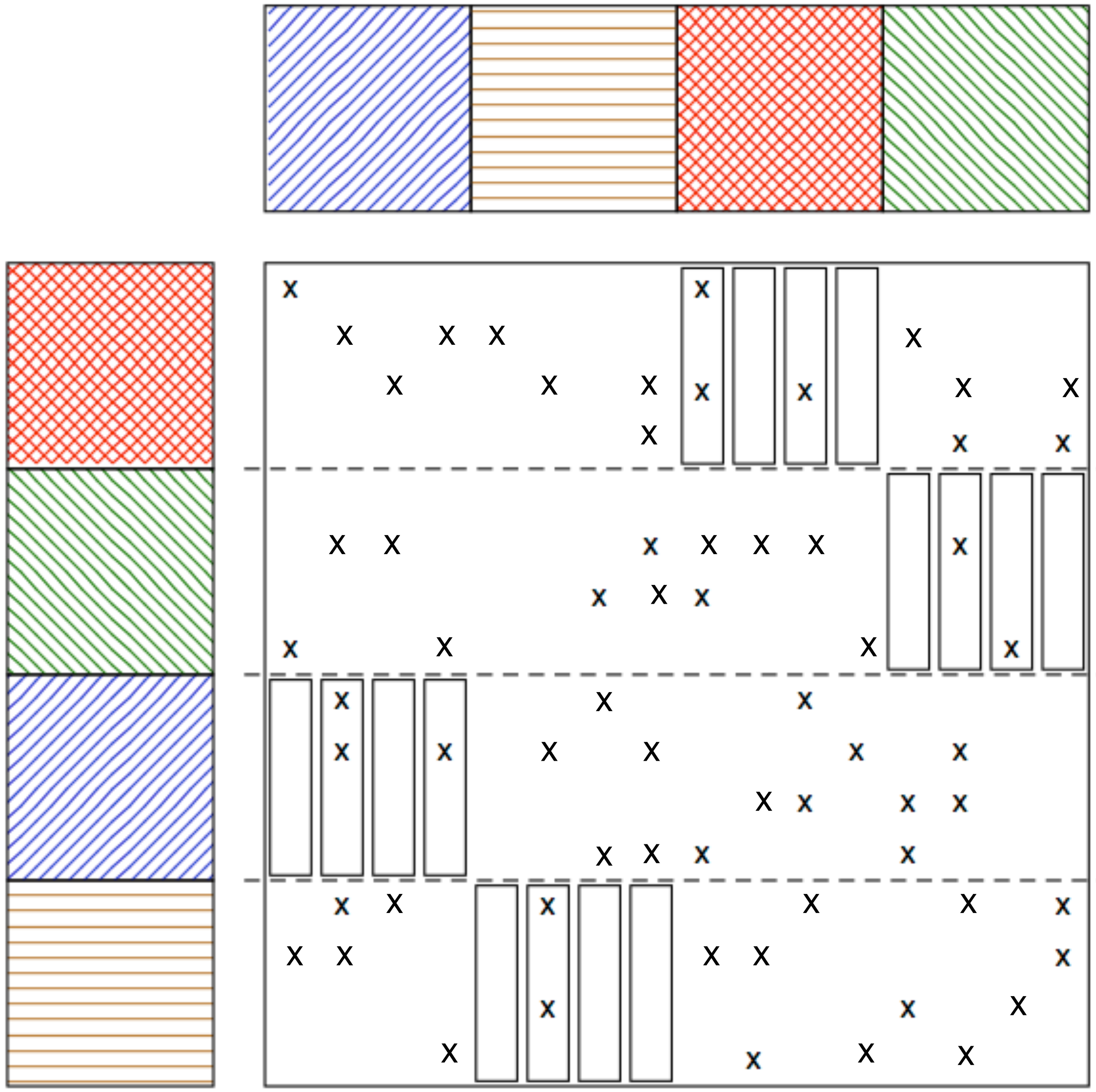

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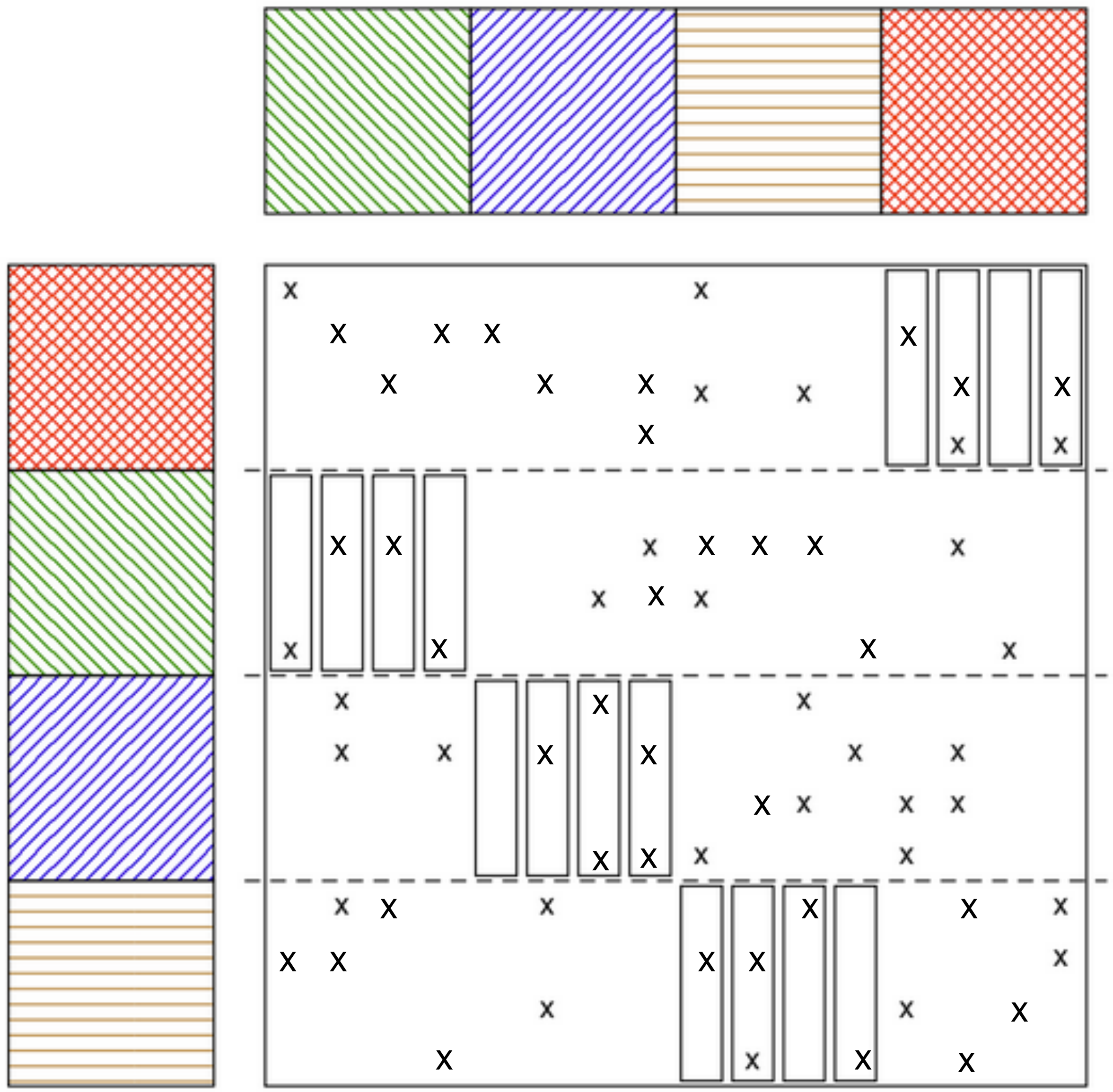

Our solution: NOMAD

- **Goal**: Keep CPU & network simultaneously busy.

> Non-locking
> stOchastic
> Multi-machine algorithm for
> Asynchronous &
> Decentralized matrix factorization

- Stochastic gradient update rule: only a small set of variables involved
- *Nomadic token passing*: avoid conflict without explicit remote locking
- Fully asynchronous computation!

H. Yun, H.-F. Yu, C.-J. Hsieh, S. V. N. Vishwanathan, I. S. Dhillon. NOMAD: Non-locking, stOchastic Multi-machine algorithm for Asynchronous and Decentralized matrix completion. To appear in *Proceedings of the VLDB Endowment*, 2014.

Nomadic Token Passing

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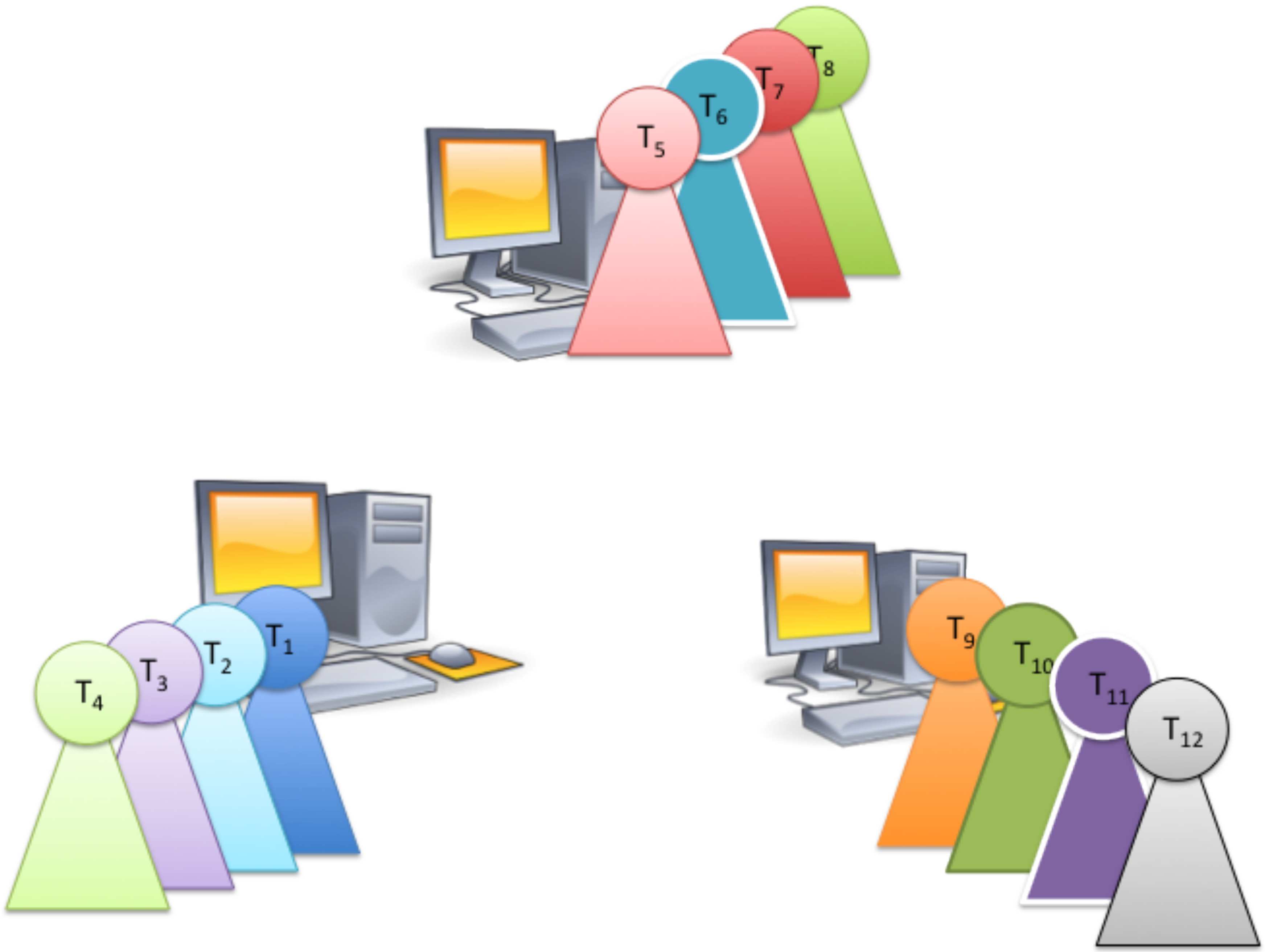

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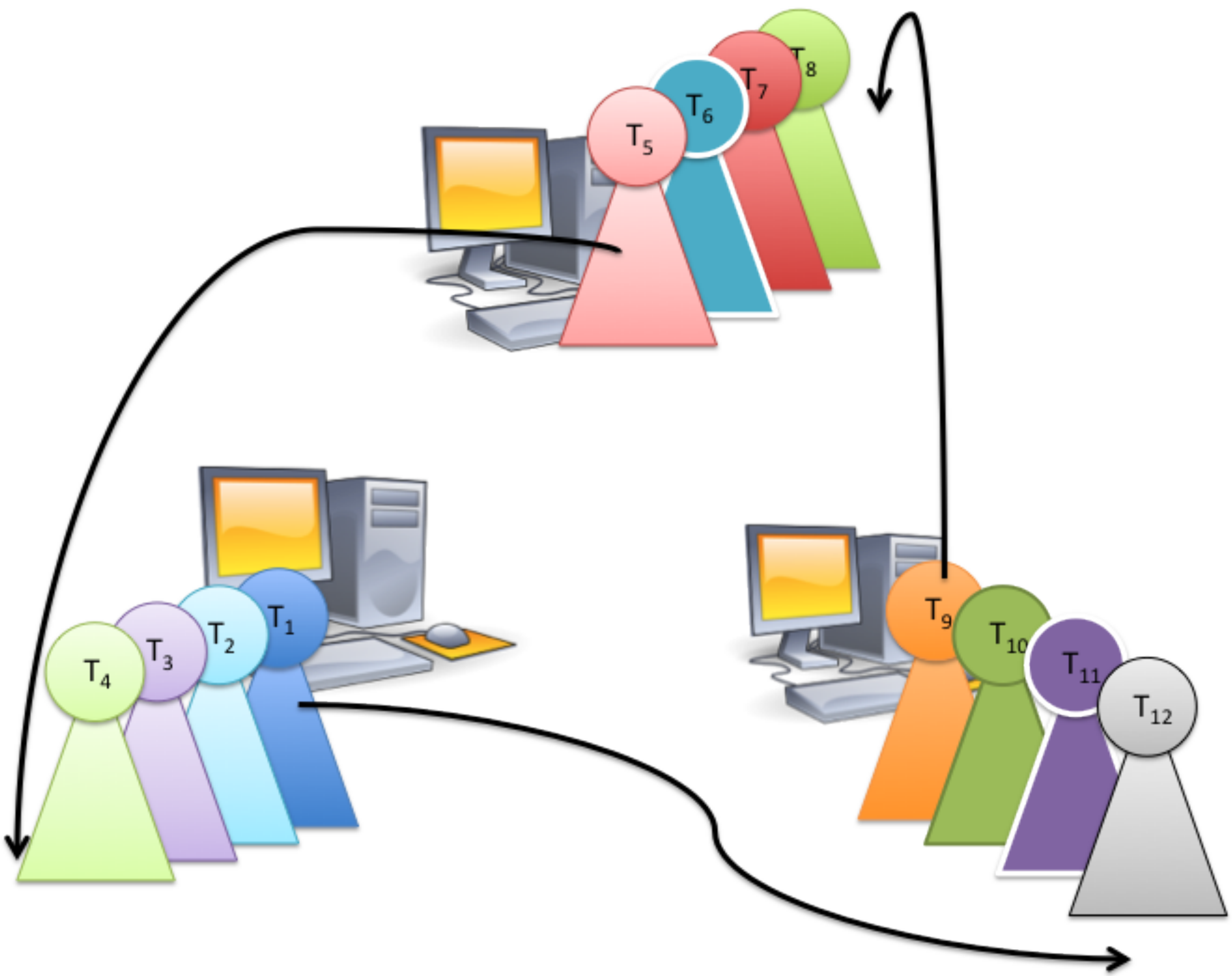

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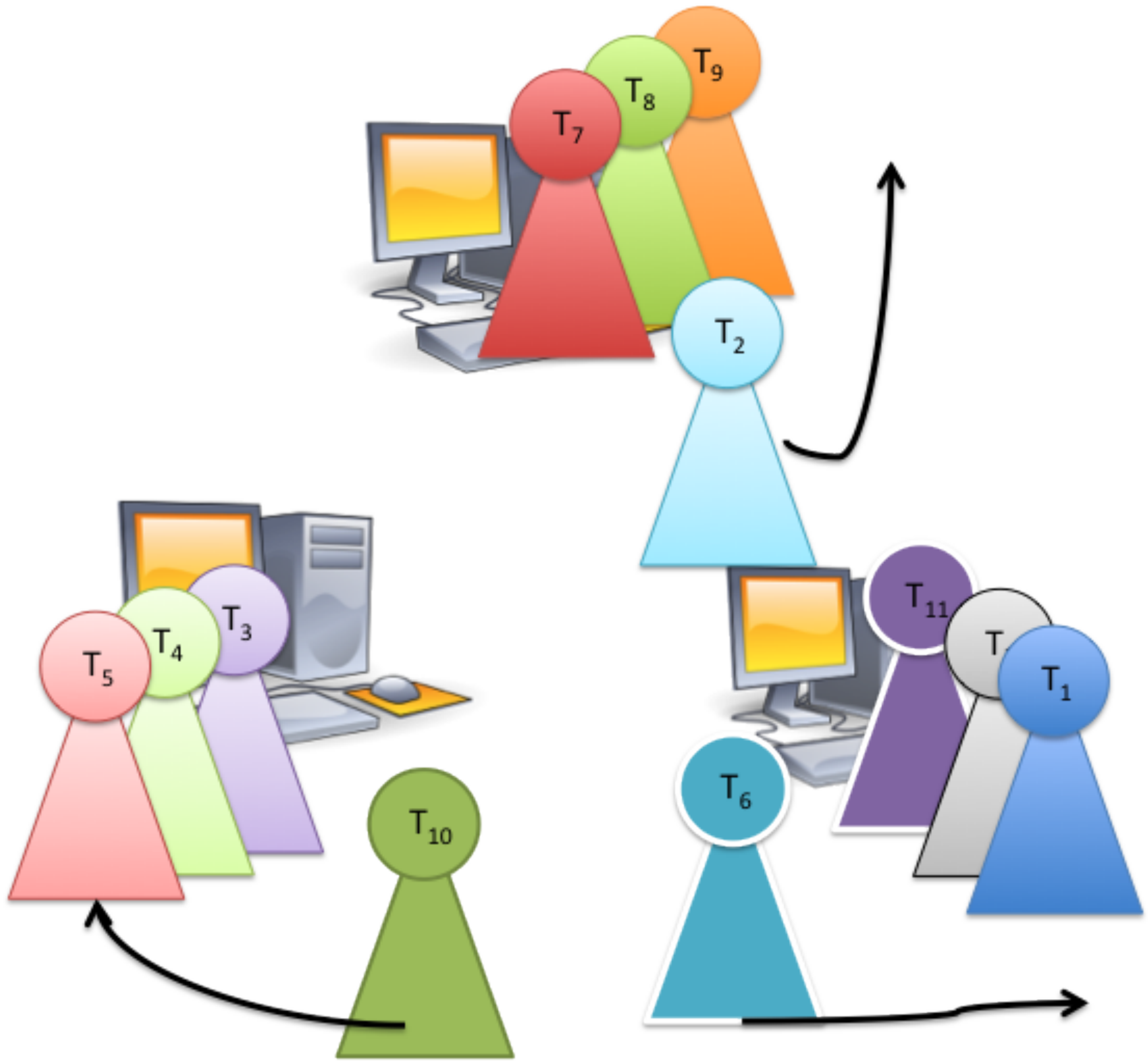
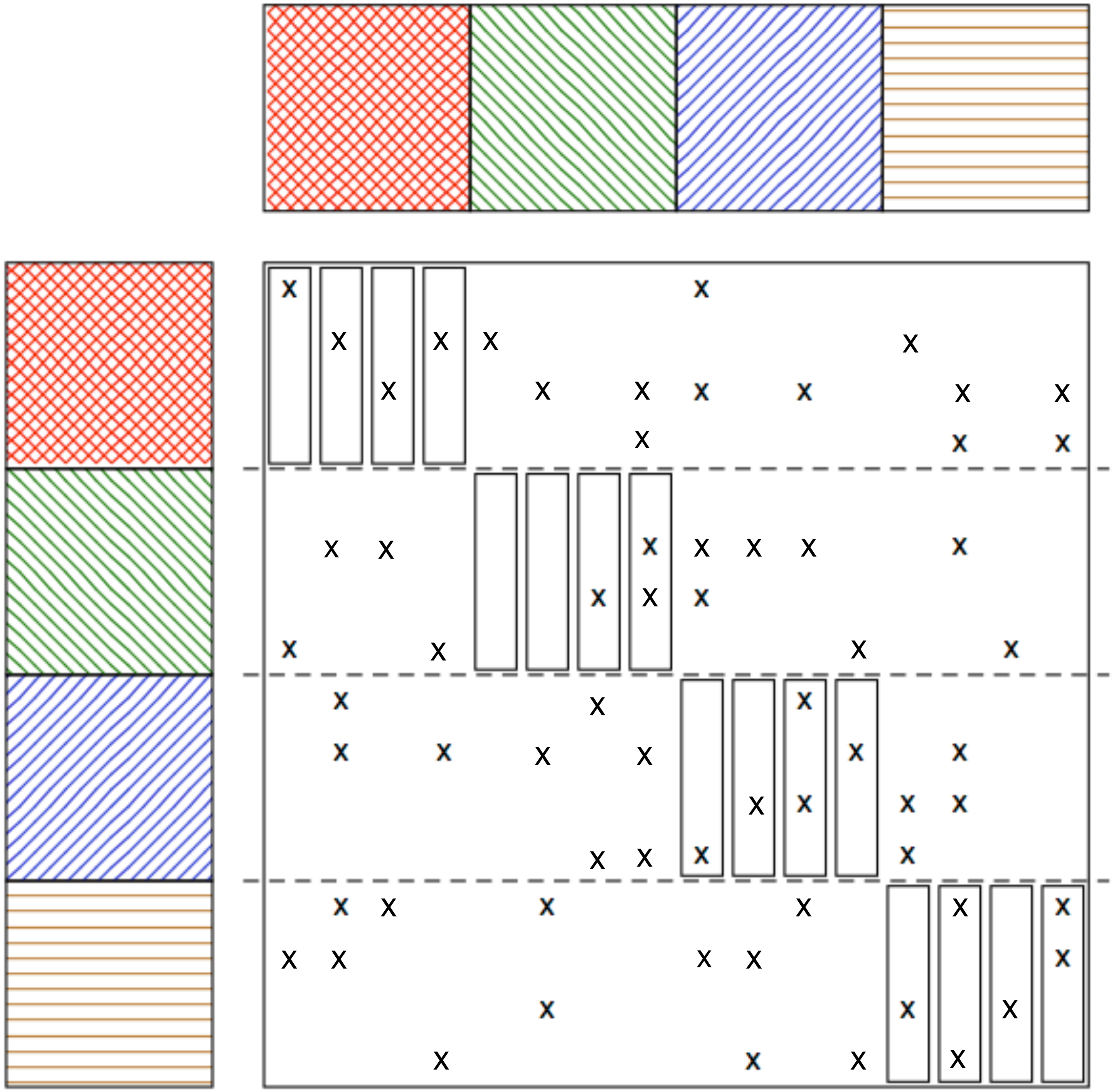


Nomadic Token Passing

NOMAD Illustration



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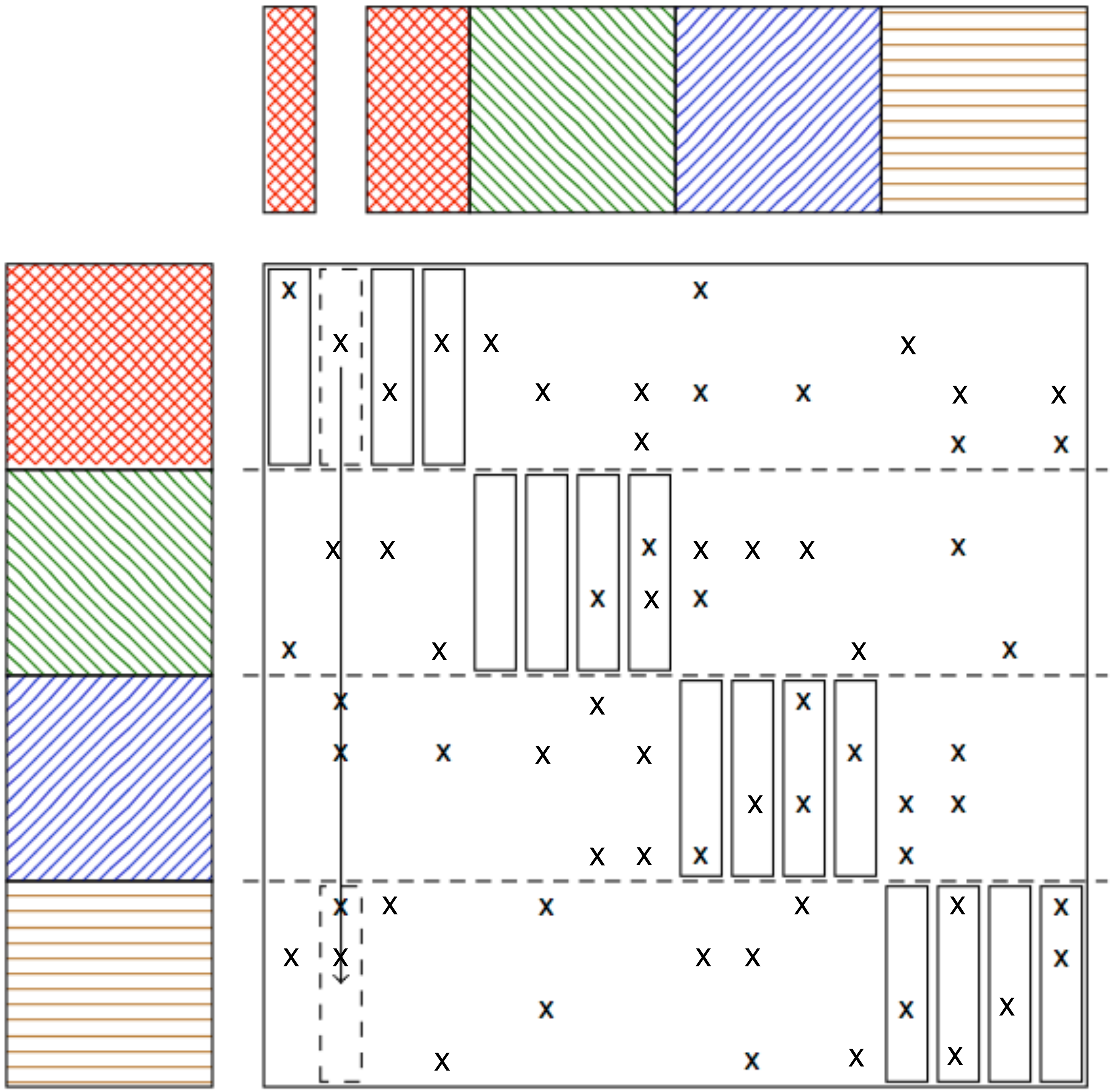

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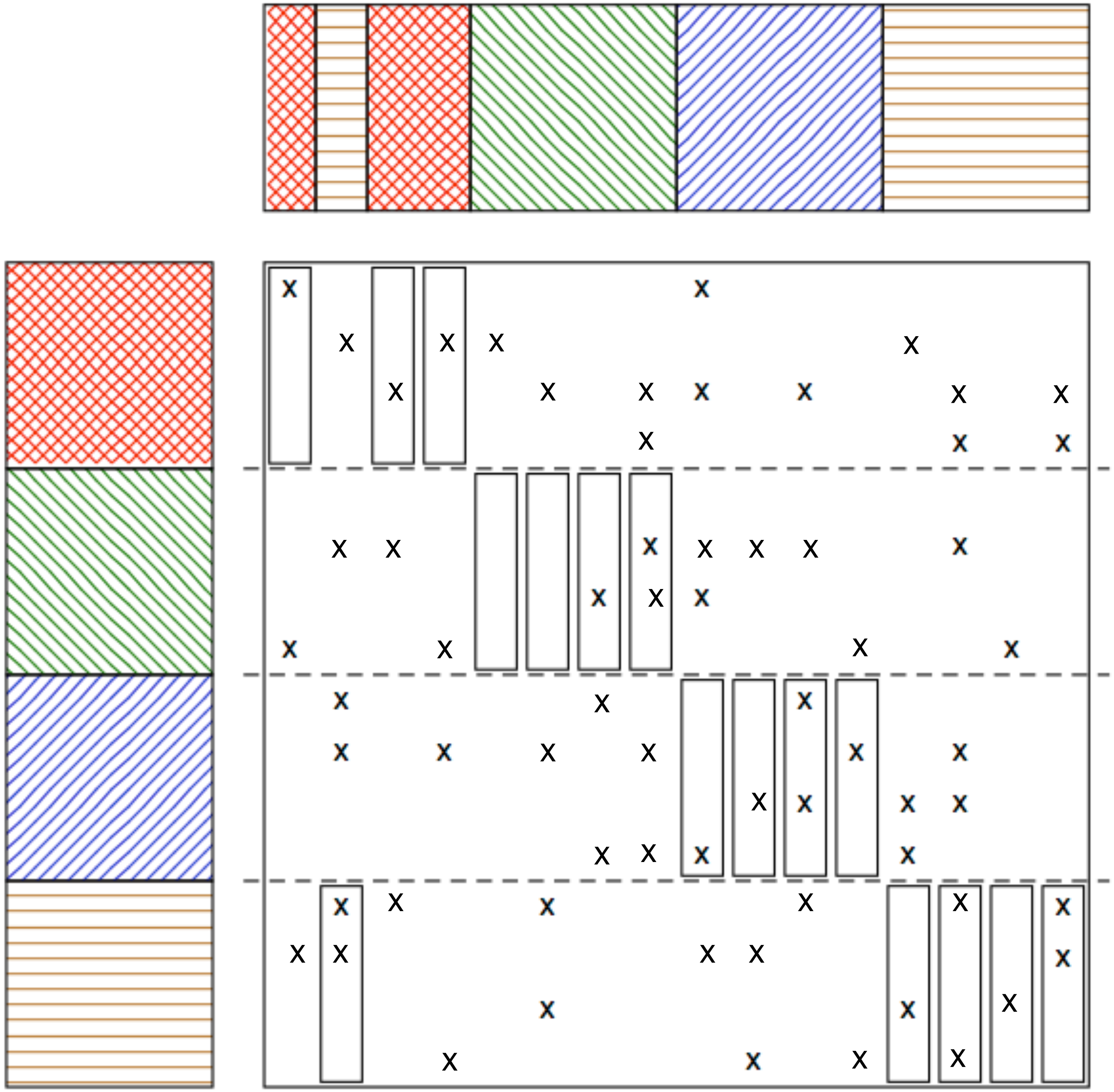

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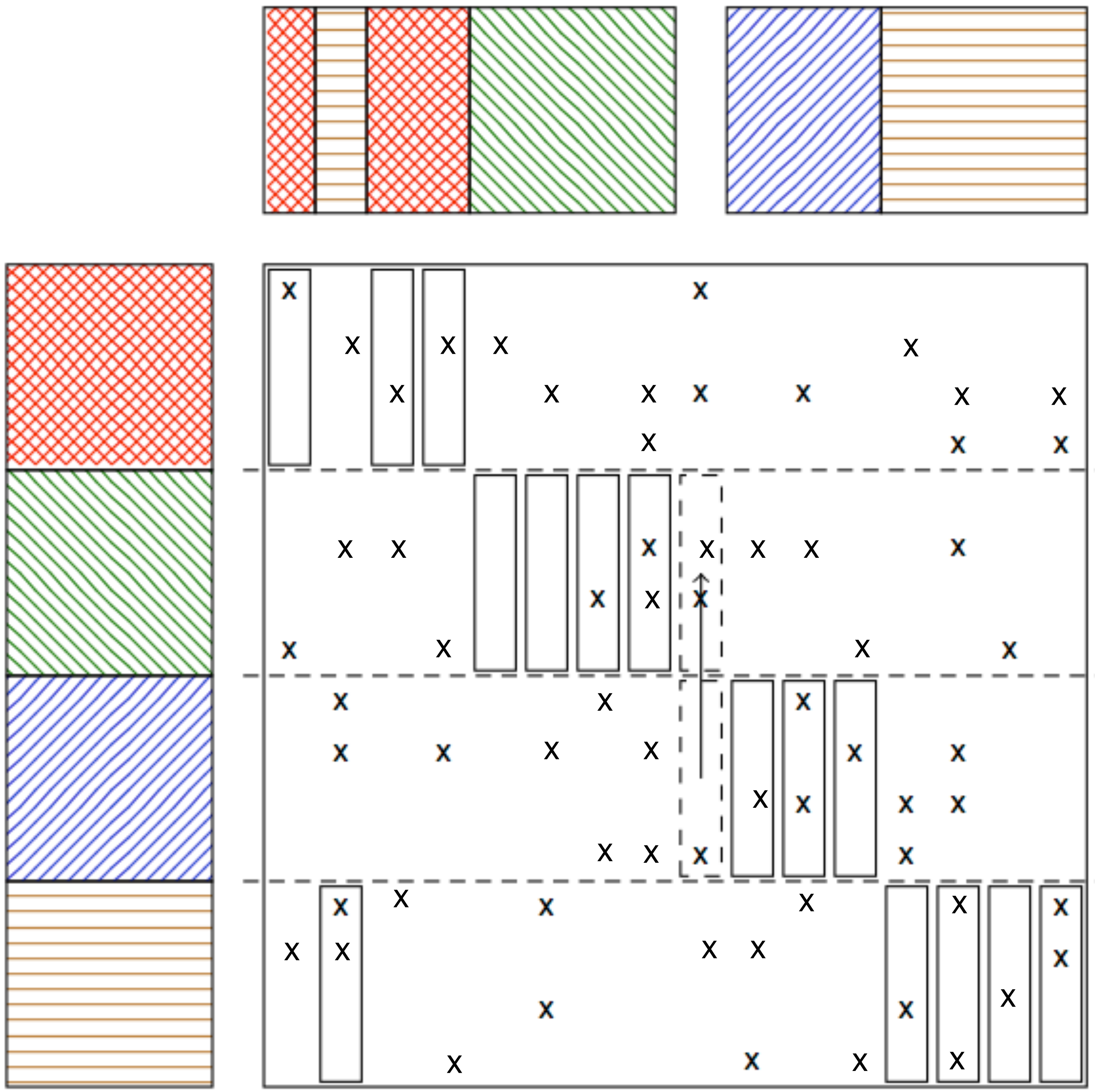

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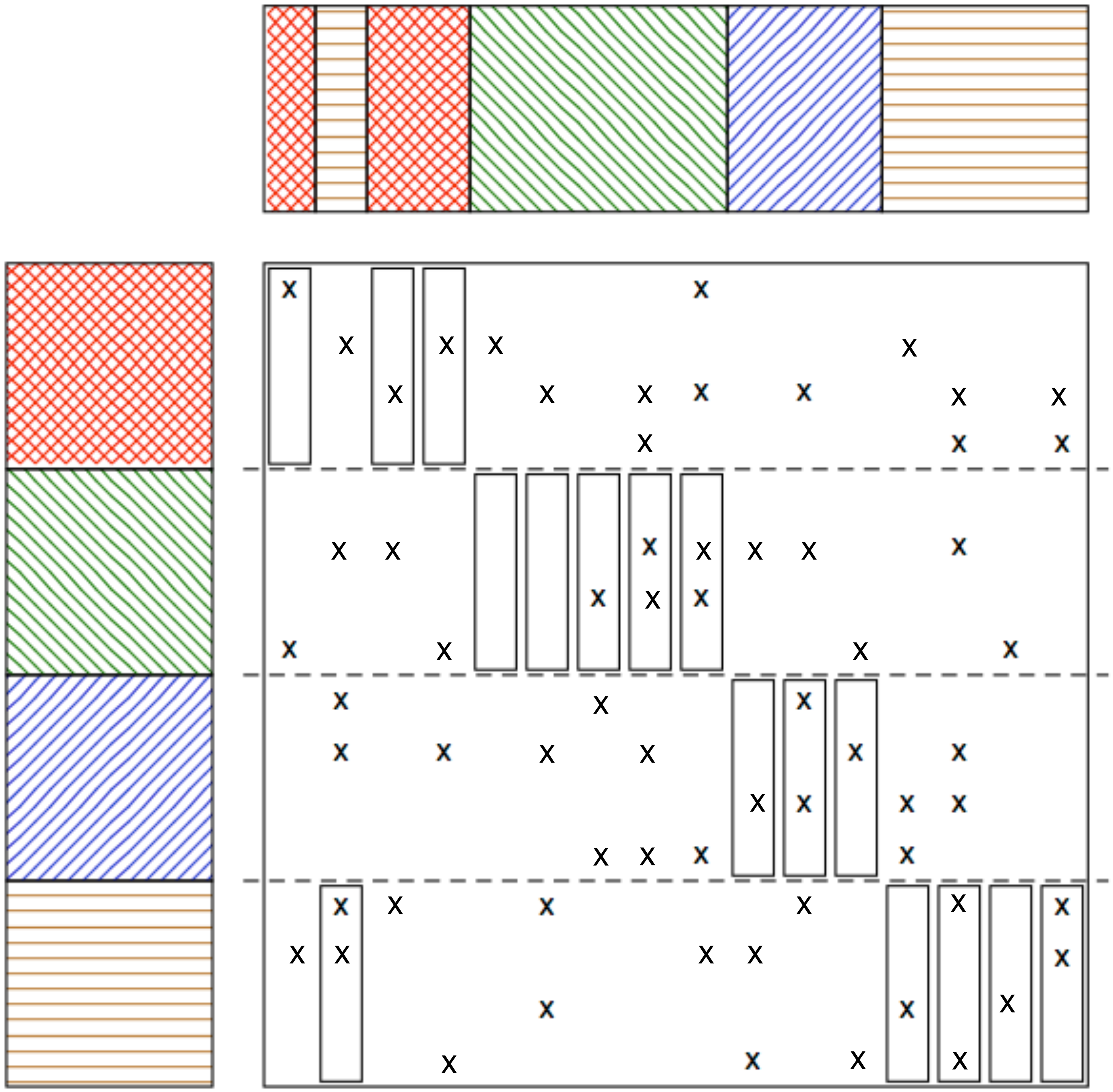

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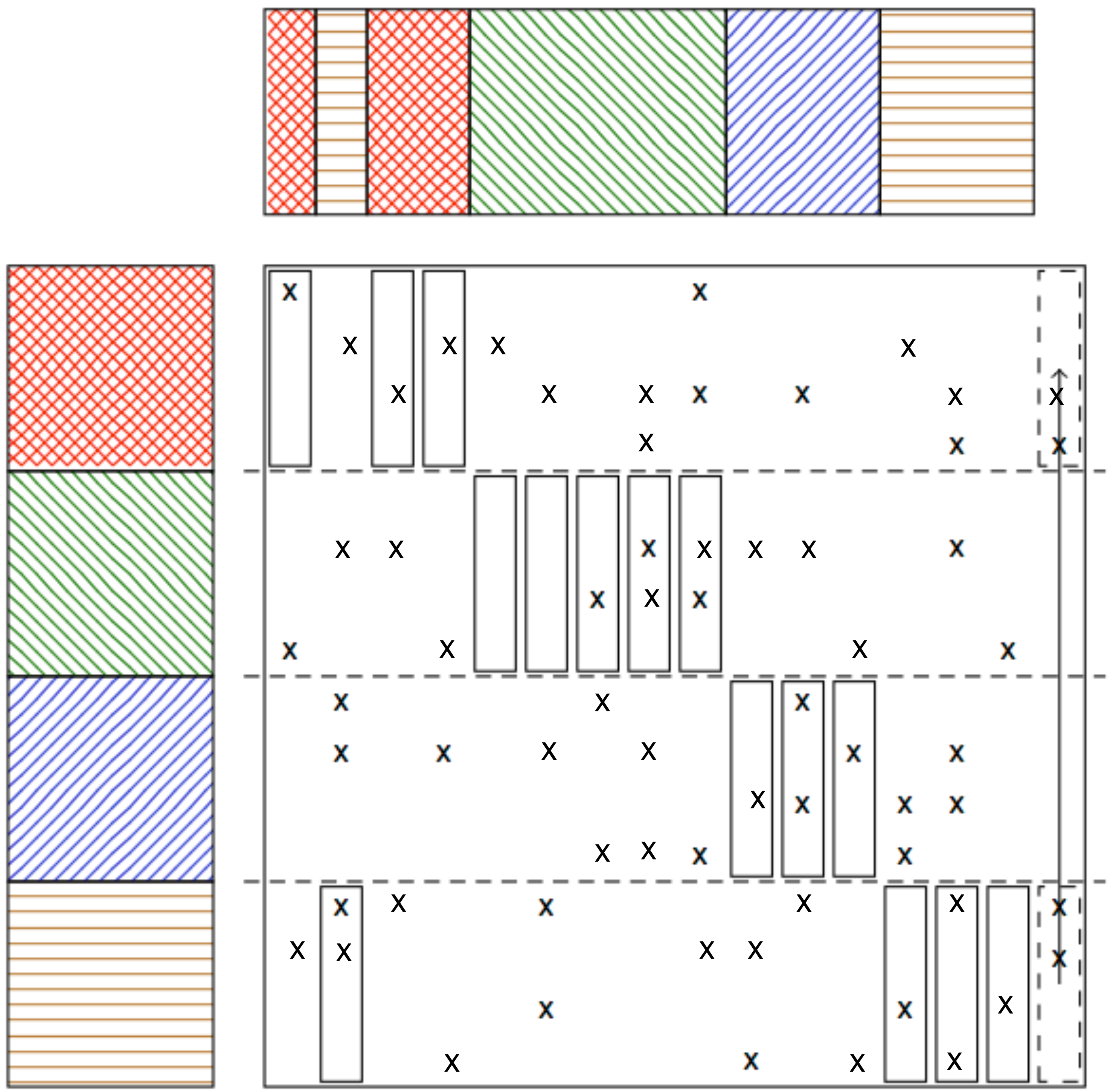

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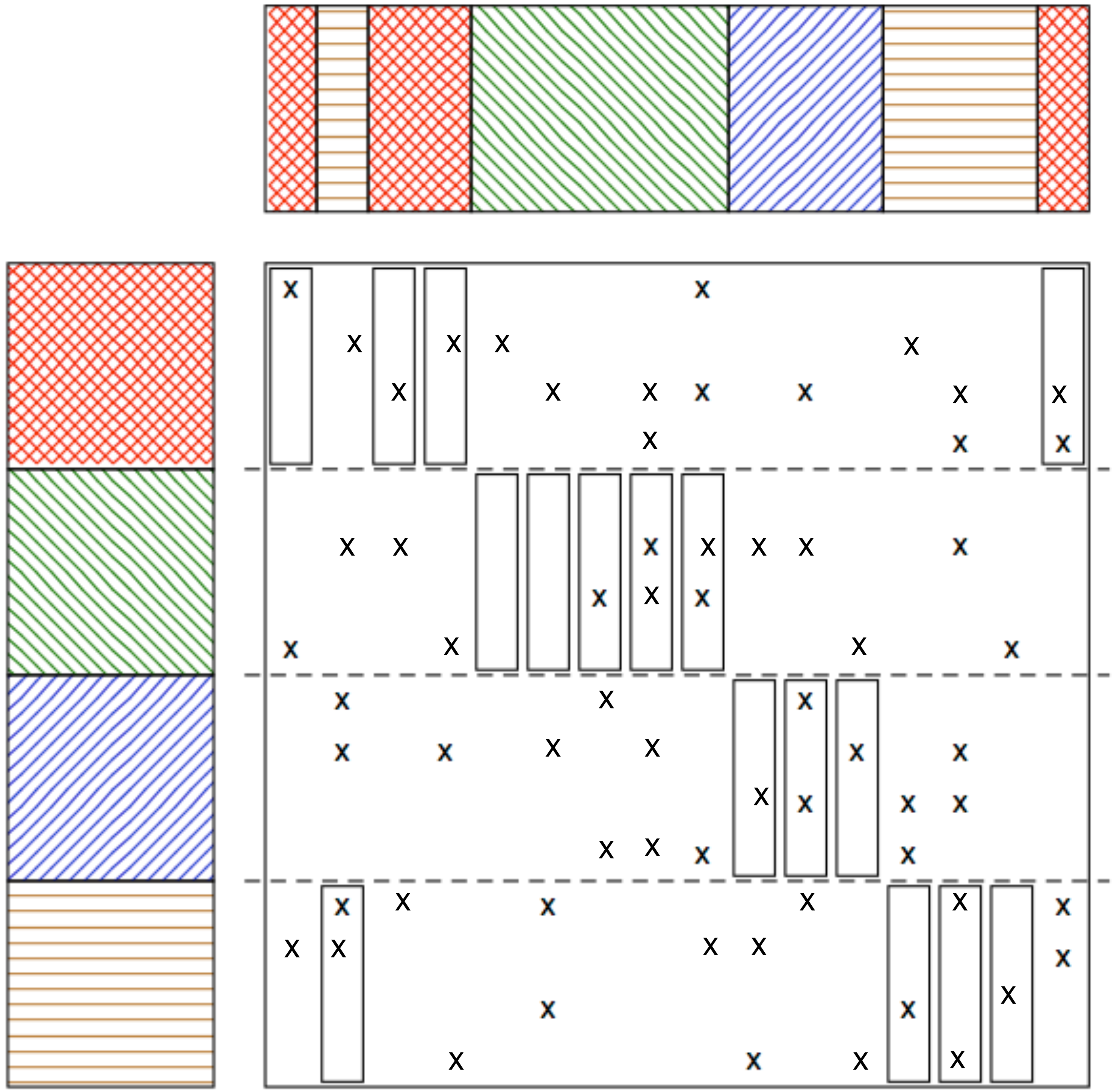

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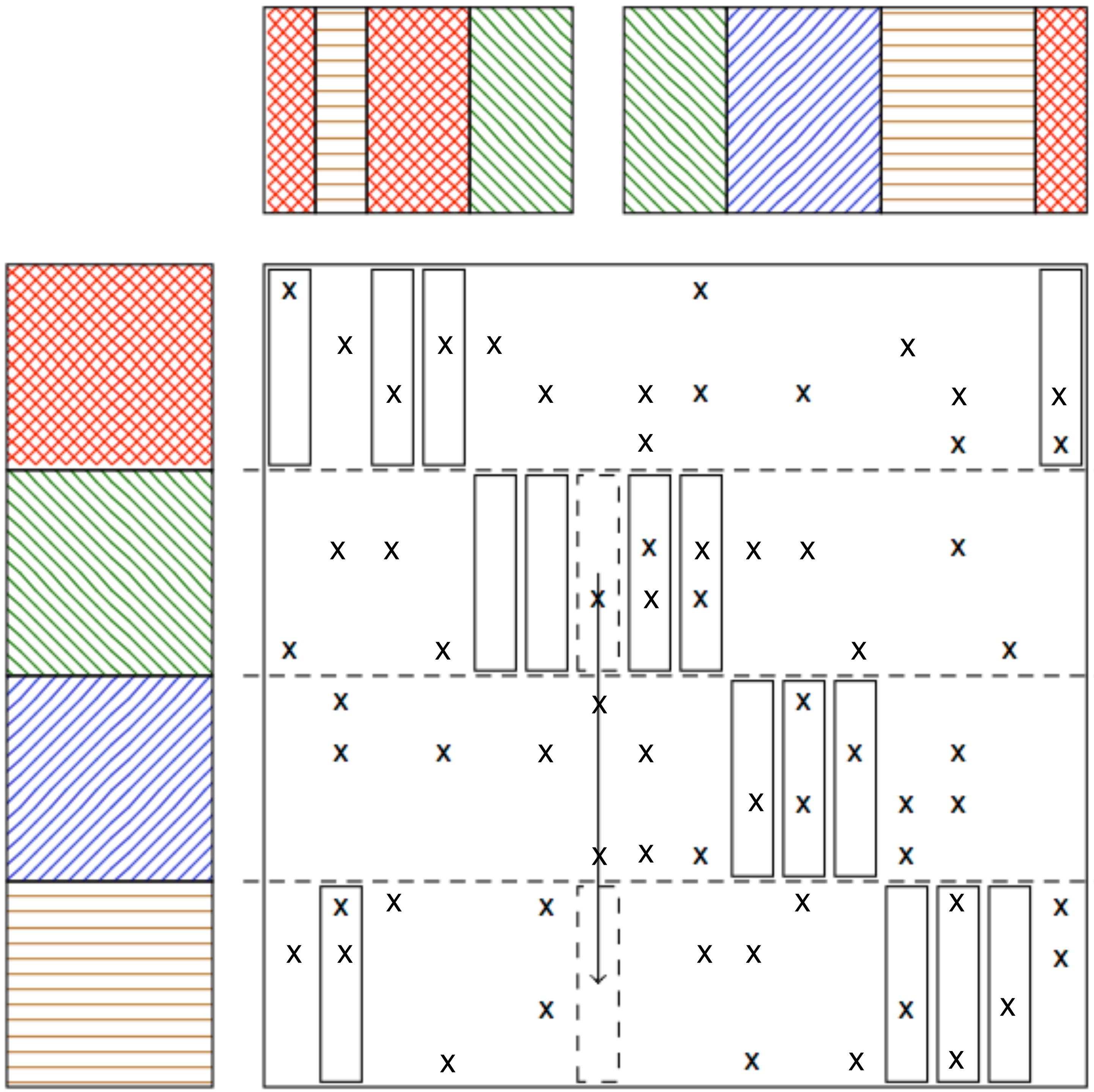

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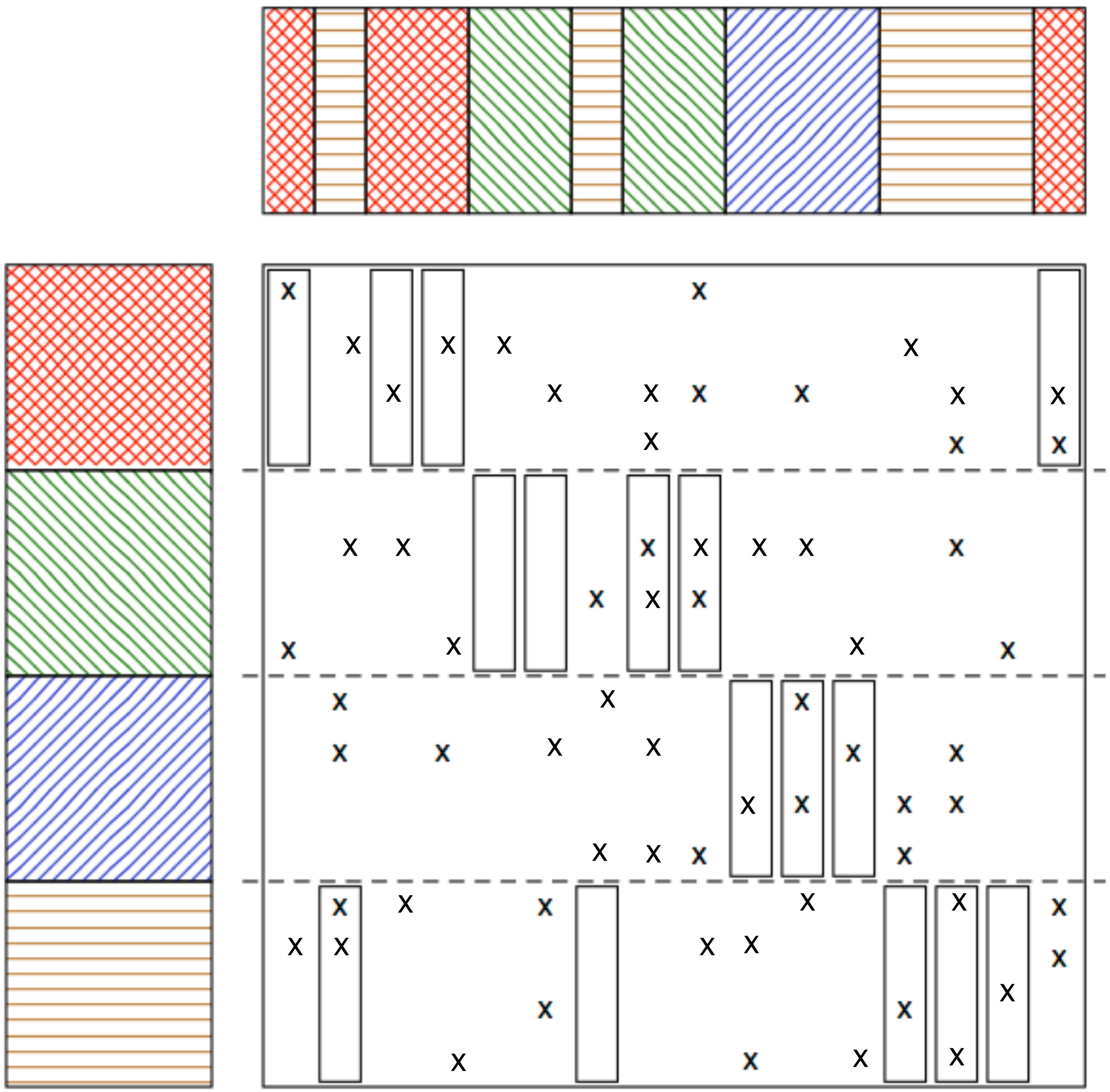

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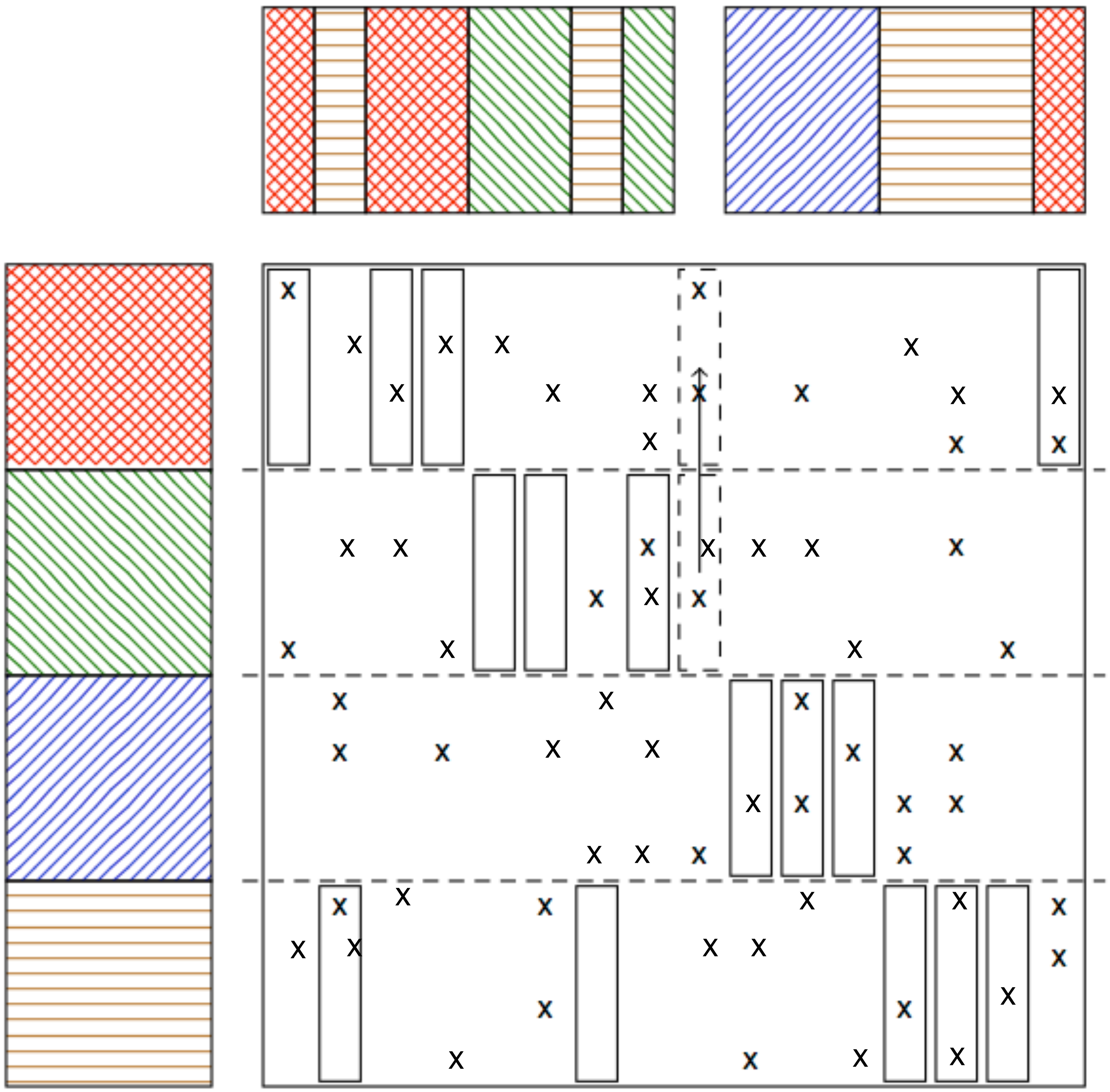

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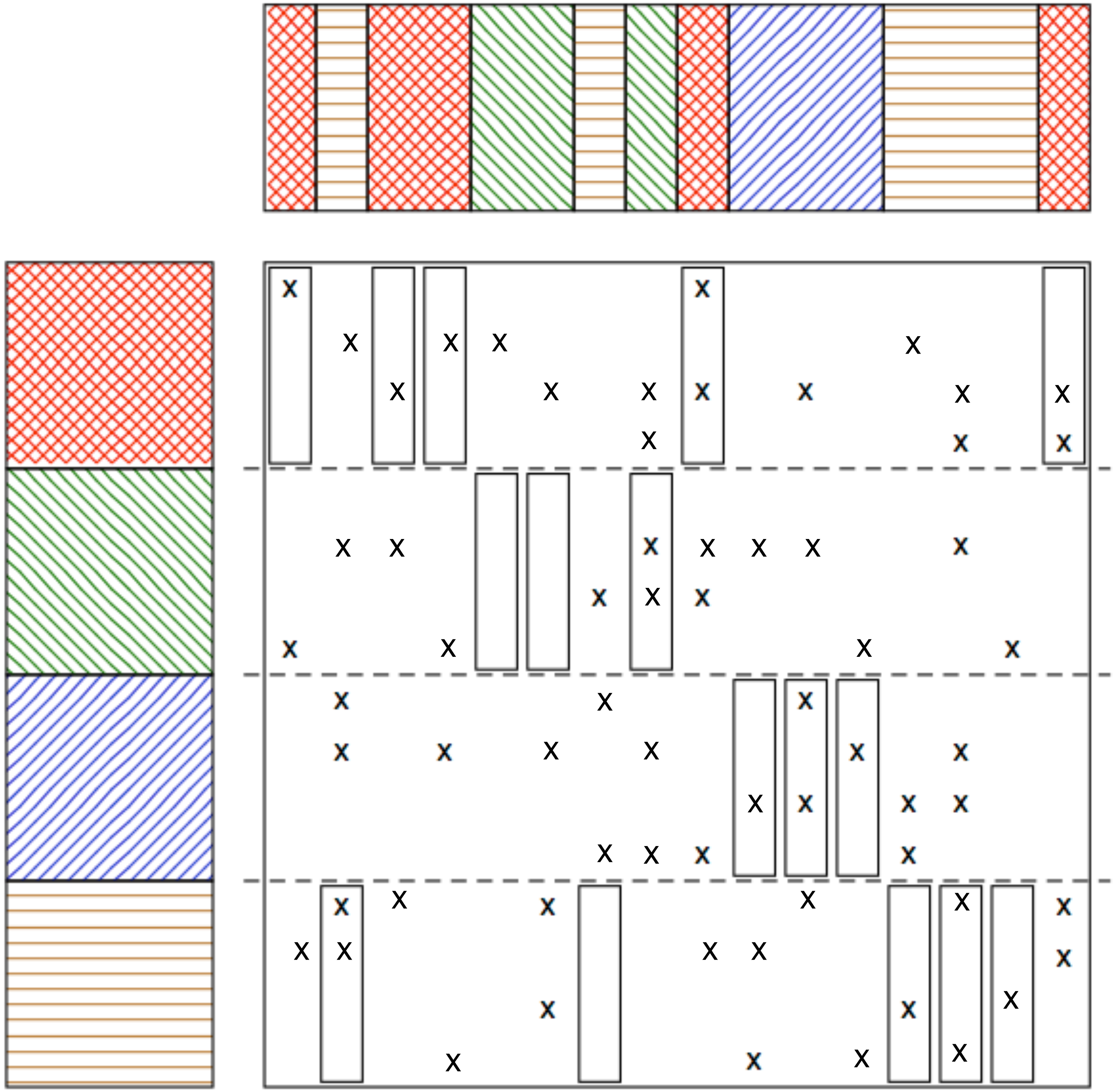

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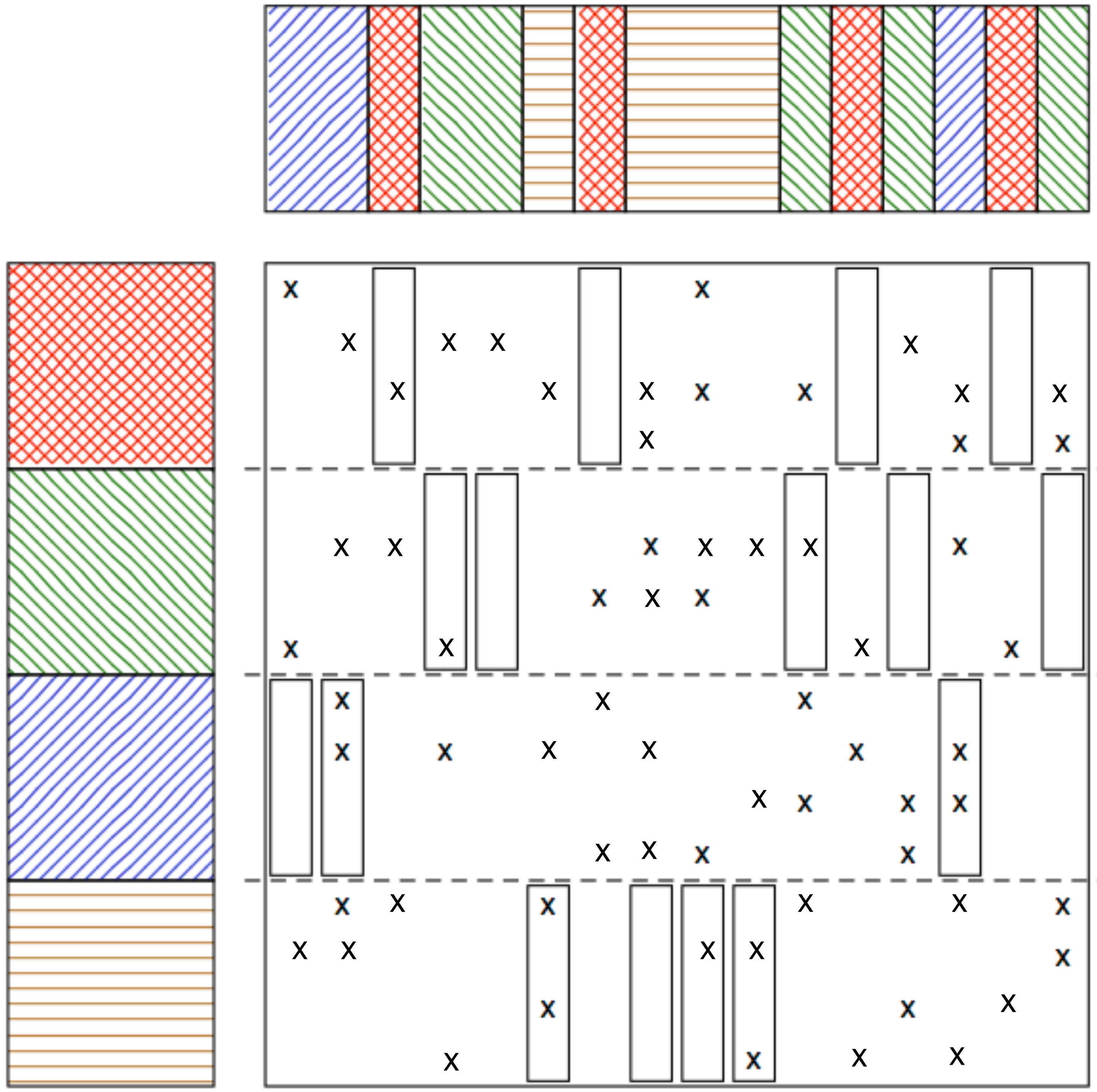
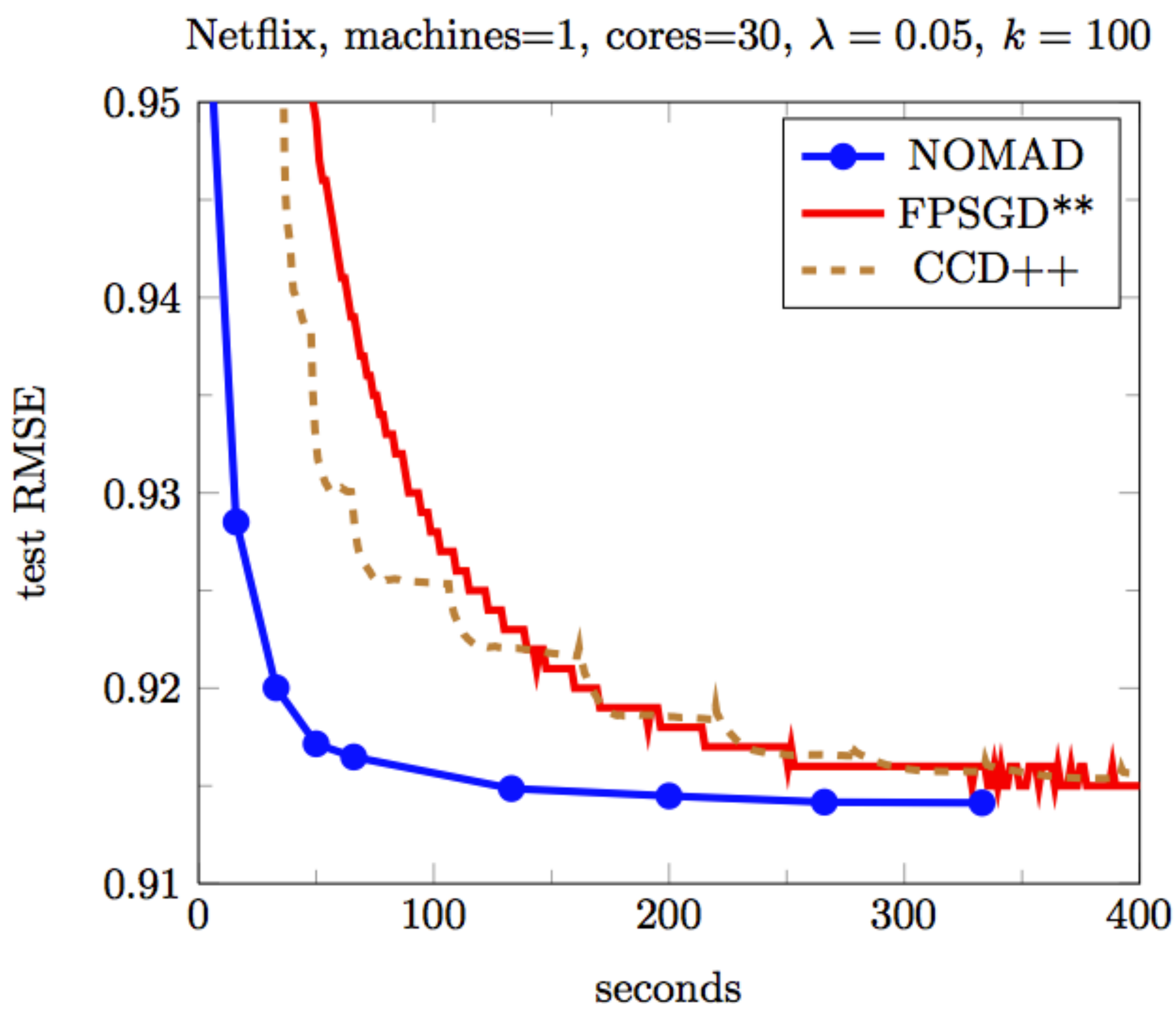
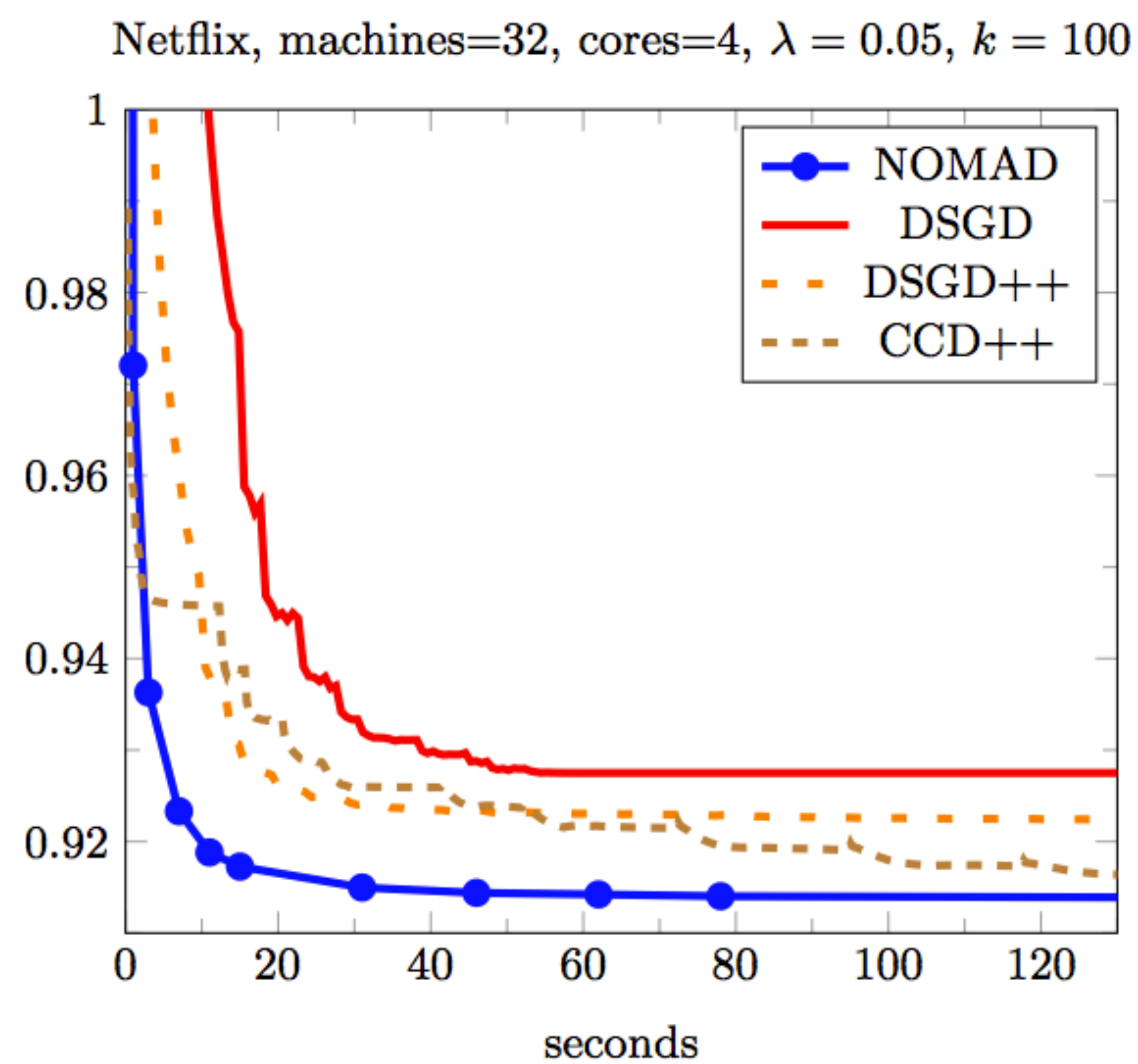


Results: Recommender Systems

Netflix dataset: 480,189 users, 17,770 movies, ~100M ratings



Multicore



Distributed

Results: Recommender Systems

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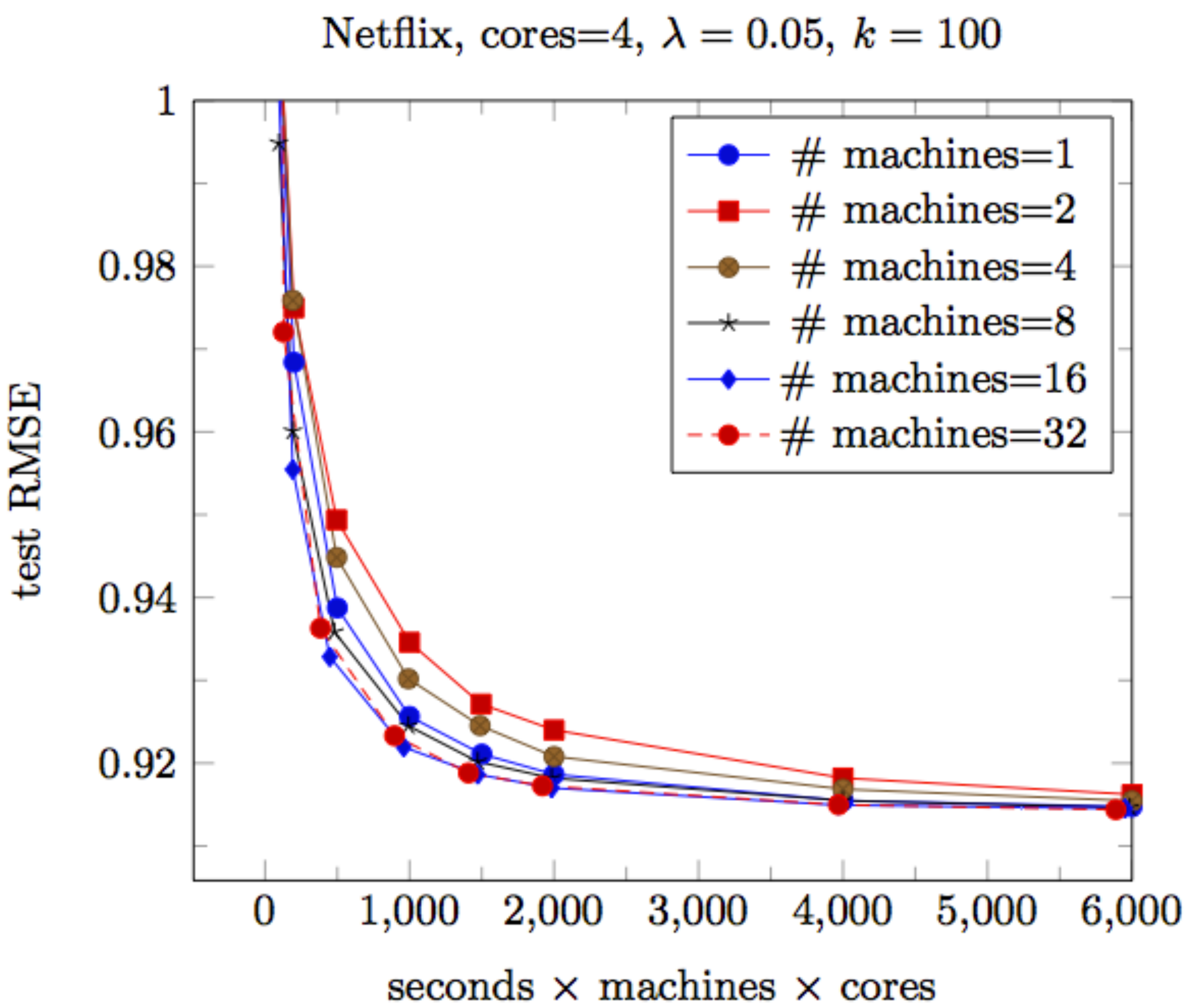
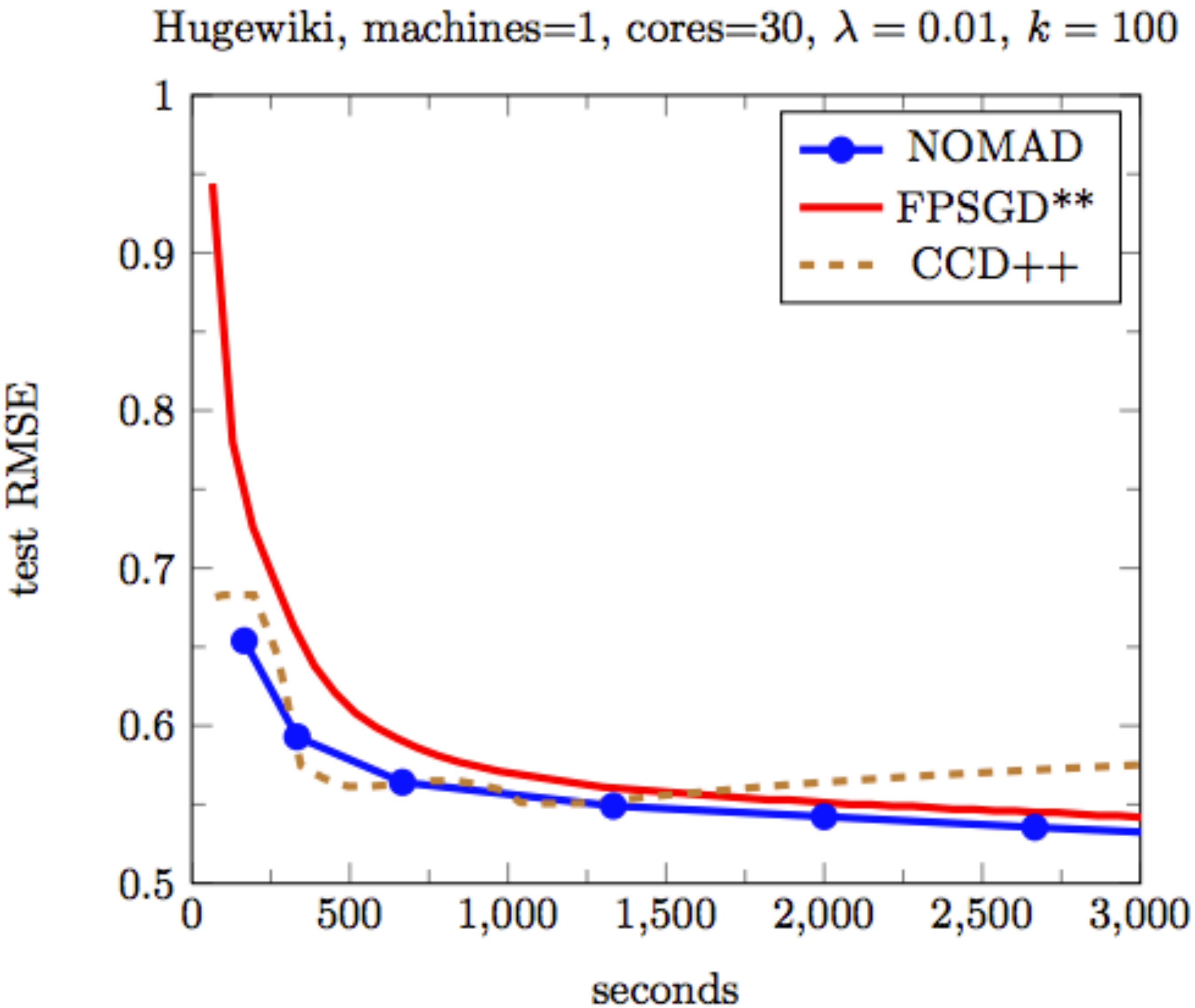
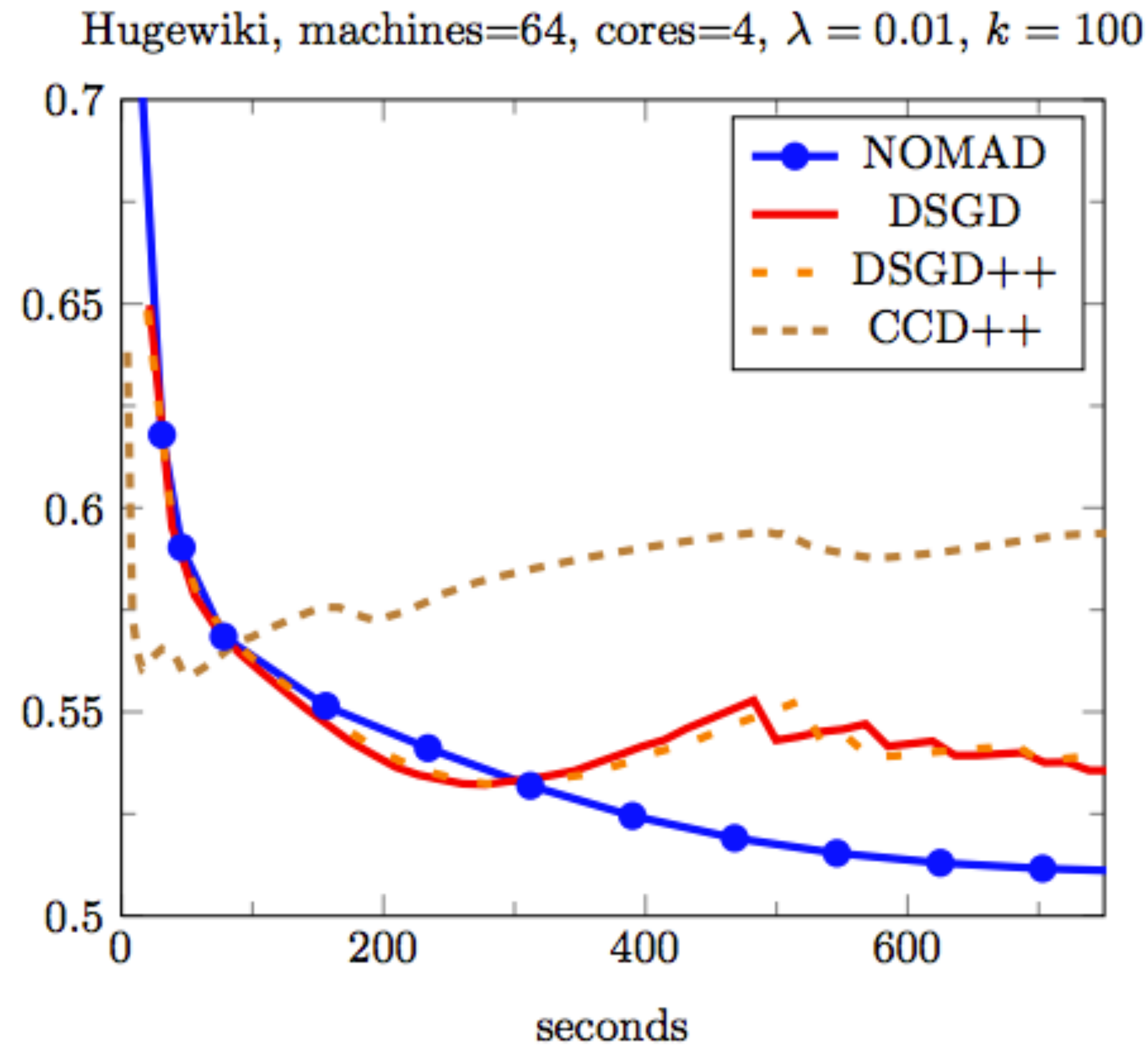


Results: Recommender Systems

HugeWiki dataset: 50,082,603 users, 39,780 movies, ~2.7B ratings



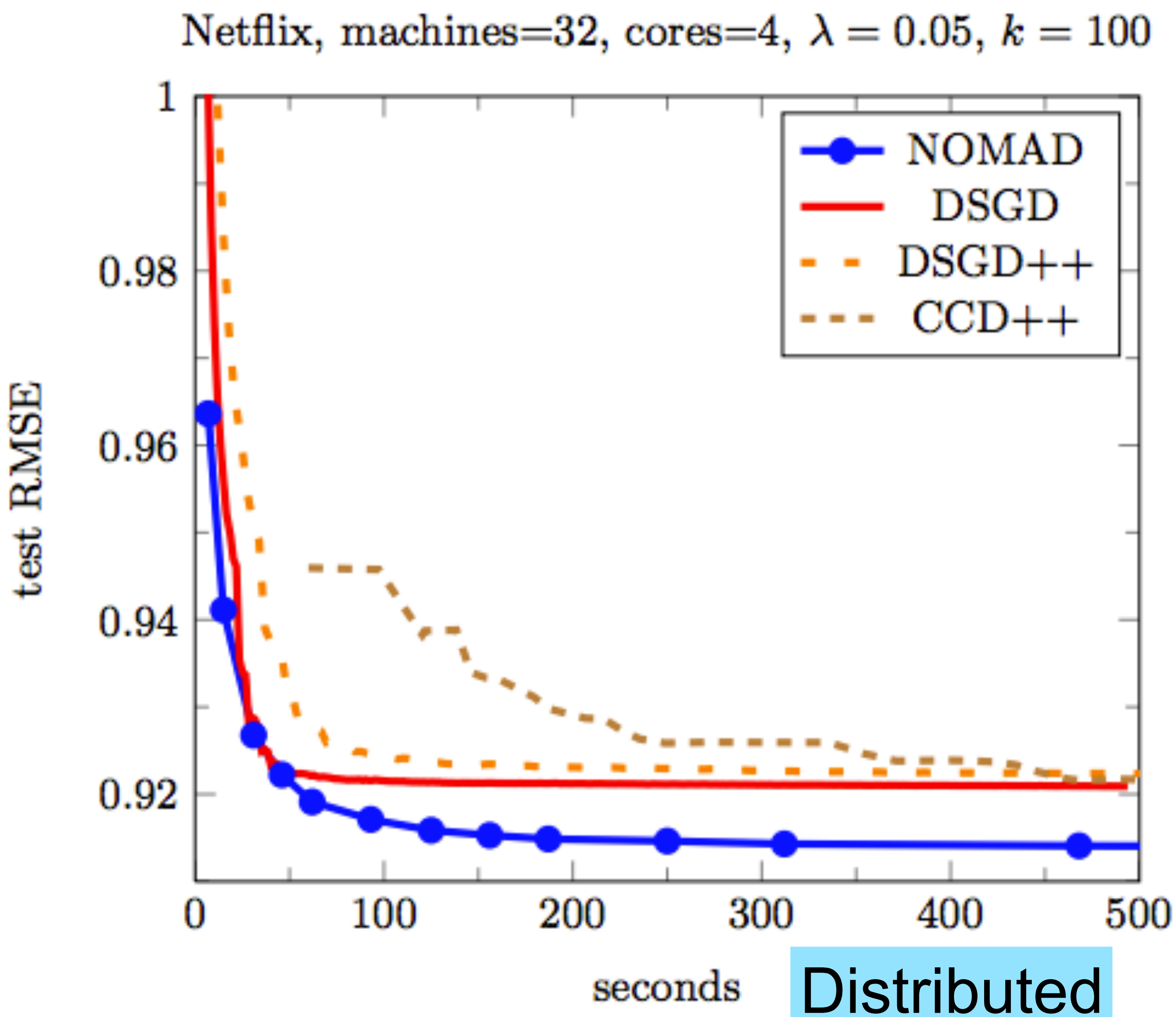
Multicore



Distributed

Results on Amazon Web Services

Netflix dataset: 480,189 users, 17,770 movies, ~100M ratings



Distributed

Conclusions

- Many “latent factor” models arise in data analysis (SVD,NMF,MC,…)
- NOMAD: Parallel Asynchronous Algorithm
  - Distributed, Lock-free & Serializable
  - Works well in practice
  - Convergence/recovery proof? Blocking? 2-D tiling? Communication Avoiding?
  - Can be applied to other latent-factor models, such as, “probabilistic topic modeling”
- Lots of demand for such scalable software in industry
  - No good solutions exist, so opportunities are great

References

- Yun, H., Yu, H., Hsieh, C., Vishwanathan S., & Dhillon, I (2014). *NOMAD: Non-locking, stOchastic Multi-machine algorithm for Asynchronous and Decentralized matrix completion*. To appear in Proceedings of the VLDB Endowment.
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