NOMAD: A Distributed Framework for Latent Variable Models

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Distributed Machine Learning and Matrix Computations

Outline

- Challenges
- Matrix Completion
  - Stochastic Gradient Method
  - Existing Distributed Approaches
  - Our Solution: NOMAD-MF
- Latent Dirichlet Allocation (LDA)
  - Gibbs Sampling
  - Existing Distributed Solutions: AdLDA, Yahoo LDA
  - Our Solution: F+NOMAD-LDA

Large-scale Latent Variable Modeling

- Latent Variable Models: very useful in many applications
  - Latent models for recommender systems (e.g., MF)
  - Topic models for document corpus (e.g., LDA)
- Fast growth of data
  - Almost 2.5×10^{18} bytes of data added each day
  - 90% of the world's data today was generated in the past two year

Challenges

- *Algorithmic* as well as *hardware* level
  - Many effective algorithms involve fine-grain iterative computation $\Rightarrow$ hard to parallelize
  - Many current parallel approaches
    - bulk synchronization
      $\Rightarrow$ wasted CPU power when communicating
    - complicated locking mechanism
      $\Rightarrow$ hard to scale to many machines
    - asynchronous computation using parameter server
      $\Rightarrow$ not serializable, danger of stale parameters
- Proposed NOMAD Framework
  - access graph analysis to exploit parallelism
  - asynchronous computation, non-blocking communication, and lock-free
  - serializable (or almost serializable)
  - successful applications: MF and LDA

Matrix Factorization: Recommender Systems

Rating Matrix

Items

Users

Movie 1	Movie 2								Movie 10	Movie 11
1			5			3		5		2
	2		3			5		2	5	
				3	?	5		3		
2		5			3		4		2	
			5			5				1
	5			1				5		
1			1				2			4

Matrix Factorization Approach $A \approx WH^T$

H^T

-0.07	-0.11	-0.53	-0.46	-0.06	-0.05	-0.53	-0.07	-0.35	-0.19	-0.14
0.13	-0.42	0.45	0.17	-0.25	-0.17	-0.18	0.27	-0.59	0.05	0.14
-0.21	-0.43	-0.23	0.16	0.08	0.17	0.57	-0.39	-0.37	-0.08	-0.15

W

-8.72	0.03	-1.03
-7.56	-0.79	0.62
-4.07	-3.95	2.55
-3.52	3.73	-3.32
-7.78	2.34	2.33
-2.44	-5.29	-3.92
-1.78	1.90	-1.68

1			5			3		5		2
	2		3			5		2	5	
				3		5		3		
2		5			3		4		2	
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	5			1				5		
1			1				2			4

Matrix Factorization Approach

$$\min_{\substack{W \in \mathcal{R}^{m \times k} \\ H \in \mathcal{R}^{n \times k}}} \sum_{(i,j) \in \Omega} (A_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda \left(\|W\|_F^2 + \|H\|_F^2 \right),$$

- $\Omega = \{(i,j) \mid A_{ij} \text{ is observed}\}$
- Regularized terms to avoid over-fitting

A transform maps users/items to latent feature space $\mathbb{R}^k$

- the i^{th} user $\Rightarrow$ i^{th} row of W, $\mathbf{w}_i^T$,
- the j^{th} item $\Rightarrow$ j^{th} column of H^T, $\mathbf{h}_j$.
- $\mathbf{w}_i^T \mathbf{h}_j$: measures the interaction.

SGM: Stochastic Gradient Method

SGM update: pick $(i, j) \in \Omega$

- $R_{ij} \leftarrow A_{ij} - \mathbf{w}_i^T \mathbf{h}_j$,
- $\mathbf{w}_i \leftarrow \mathbf{w}_i - \eta(\frac{\lambda}{|\Omega_i|}\mathbf{w}_i - R_{ij}\mathbf{h}_j)$,
- $\mathbf{h}_j \leftarrow \mathbf{h}_j - \eta(\frac{\lambda}{|\bar{\Omega}_j|}\mathbf{h}_j - R_{ij}\mathbf{w}_i)$,

  Ω_i: observed ratings of i-th row.

  $\bar{\Omega}_j$: observed ratings of j-th column.

$$\begin{pmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{w}_1^T \\ \mathbf{w}_2^T \\ \mathbf{w}_3^T \end{pmatrix} \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

An iteration : $|\Omega|$ updates

- Time per update: $O(k)$
- Time per iteration: $O(|\Omega|k)$,

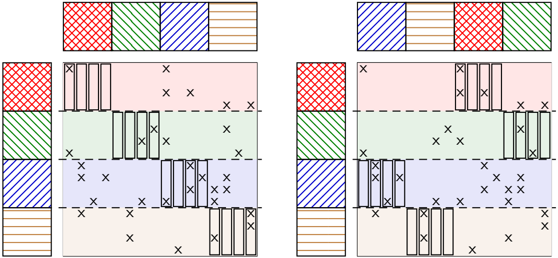
  better than $O(|\Omega|k^2)$ for ALS

Parallel Stochastic Gradient Descent for MF

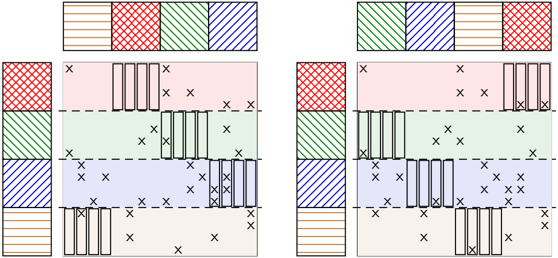
Challenge: direct parallel updates $\Rightarrow$ memory conflicts.

- Multi-core parallelization
  - Hogwild [Niu 2011]
  - Jellyfish [Recht et al, 2011]
  - FPSGD** [Zhuang et al, 2013]
- Multi-machine parallelization:
  - DSGD [Gemulla et al, 2011]
  - DSGD ++ [Teflioudi et al, 2013]

DSGD/JellyFish [Gemulla et al, 2011; Recht et al, 2011]



Synchronize and communicate



Synchronize and communicate

Proposed Asynchronous Approach: NOMAD-MF [Yun et al, 2014]

Motivation

Most existing parallel approaches require

- **Synchronization** and/or
  - E.g., ALS, DSGD/JellyFish, DSGD++, CCD++
  - Computing power is wasted:
    - Interleaved computation and communication
    - Curse of the last reducer
- **Locking** and/or
  - E.g., parallel SGD, FPSGD**
  - A standard way to avoid conflict and guarantee *serializability*
  - Complicated remote locking slows down the computation
  - Hard to implement efficient locking on a distributed system
- **Computation using stale values**
  - E.g., Hogwild, Asynchronous SGD using parameter server
  - Lack of serializability

Q: Can we avoid both *synchronization* and *locking* but keep CPU from being *idle* and guarantee *serializability*?

Our answer: NOMAD

A: Yes, NOMAD keeps CPU and network busy simultaneously

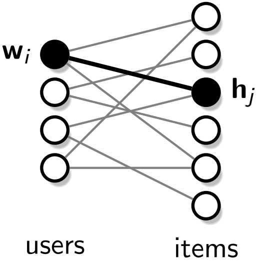
- Stochastic gradient update rule
  - only a small set of variables involved
- *Nomadic token passing*
  - widely used in telecommunication area
  - avoids conflict without explicit remote locking
  - Idea: “owner computes”
  - NOMAD: multiple “active tokens” and nomadic passing

Features:

- fully asynchronous computation
- lock-free implementation
- non-blocking communication
- serializable update sequence

Access Graph for Stochastic Gradient

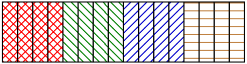
- Access graph $G = (V, E)$:
  - $V = \{\mathbf{w}_i\} \cup \{\mathbf{h}_j\}$
  - $E = \{e_{ij} : (i,j) \in \Omega\}$
- Connection to SG:
  - each e_{ij} corresponds to a SG update
  - only access to $\mathbf{w}_i$ and $\mathbf{h}_j$
- Parallelism:
  - edges without common node can be updated in parallel
  - identify "matching" in the graph
- Nomadic Token Passing:
  - mechanism s.t. active edges always form a "matching"
  - serializability guaranteed



More Details

Nomadic Tokens for $\{\mathbf{h}_j\}$:

- n tokens
- $(j, \mathbf{h}_j)$: $O(k)$ space



Worker:

- p workers
- a computing unit + a concurrent token queue
- a block of W: $O(mk/p)$
- a block row of A: $O(|\Omega|/p)$

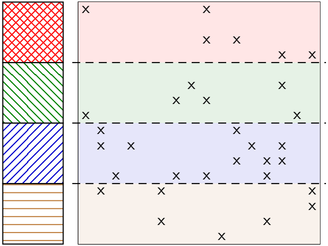
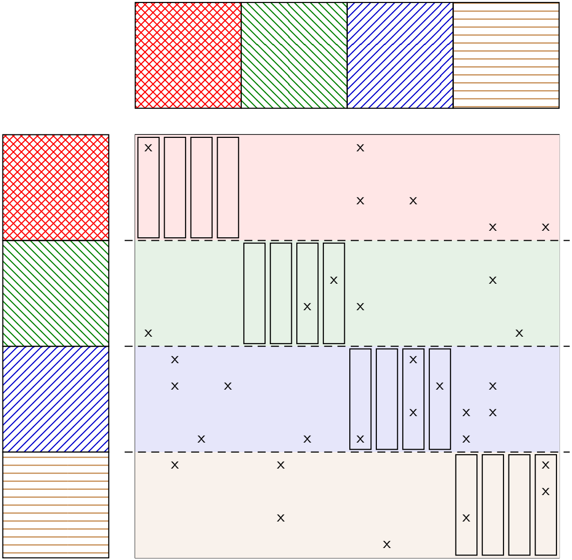


Illustration of NOMAD communication



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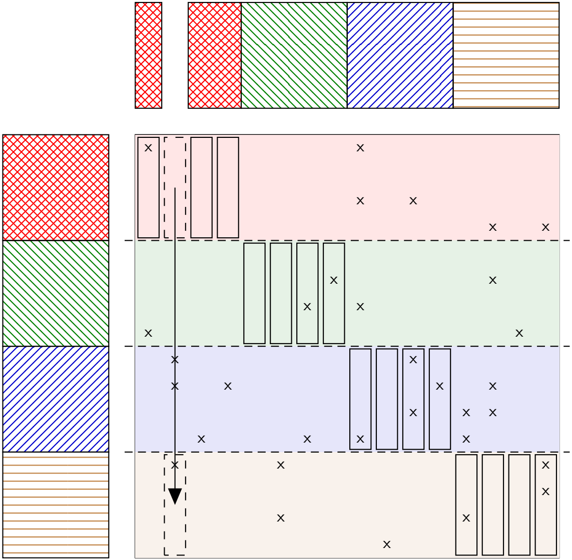

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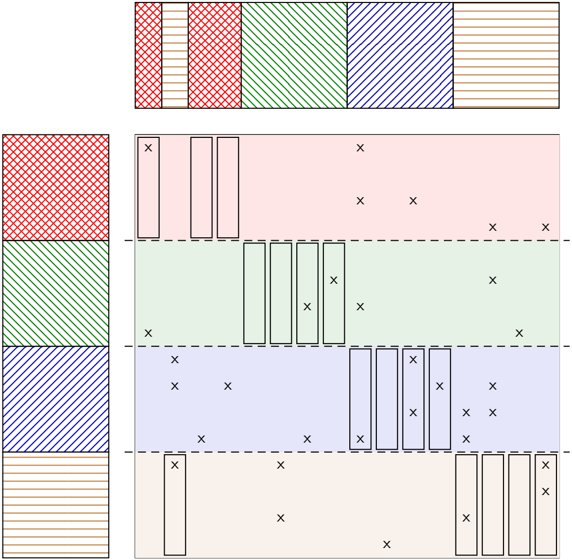

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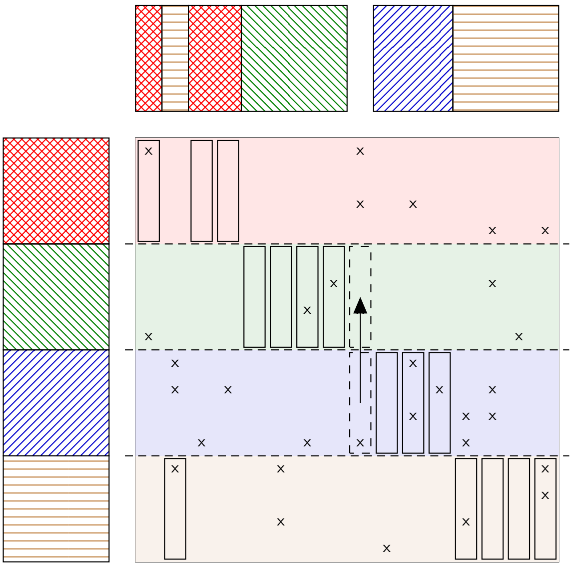

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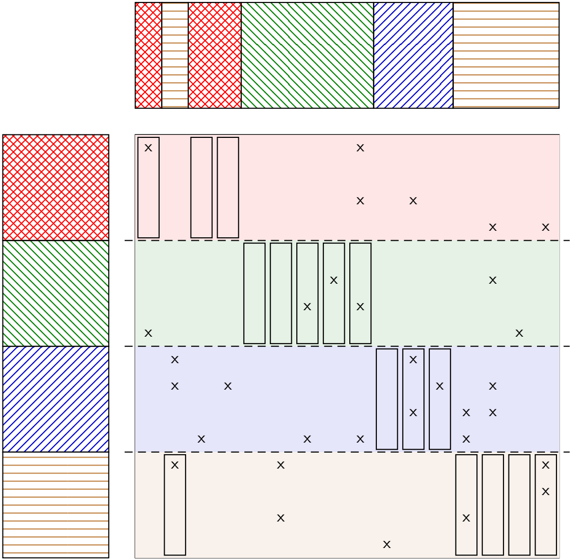

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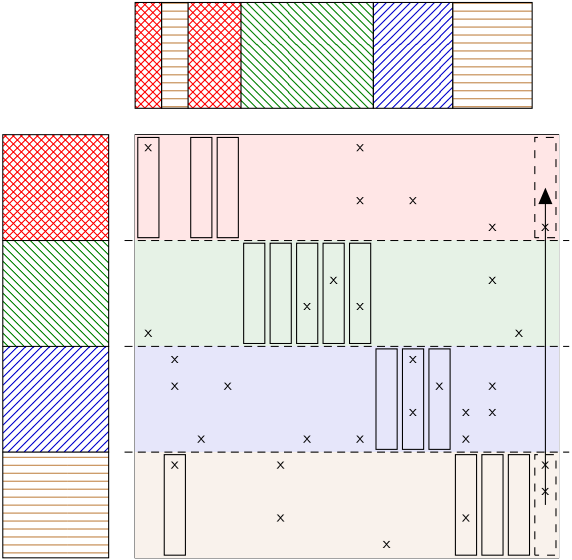

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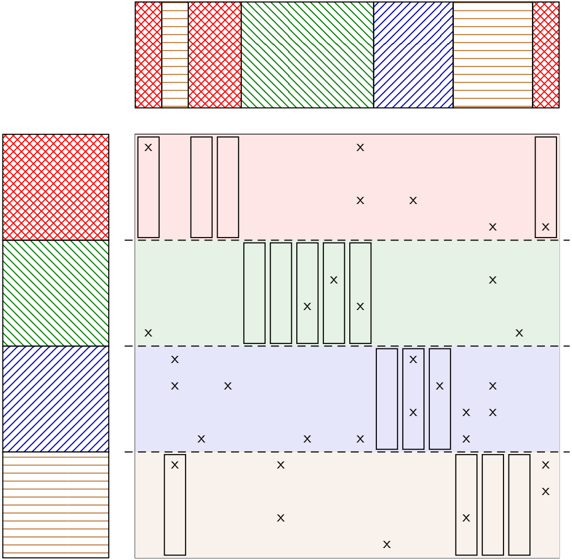

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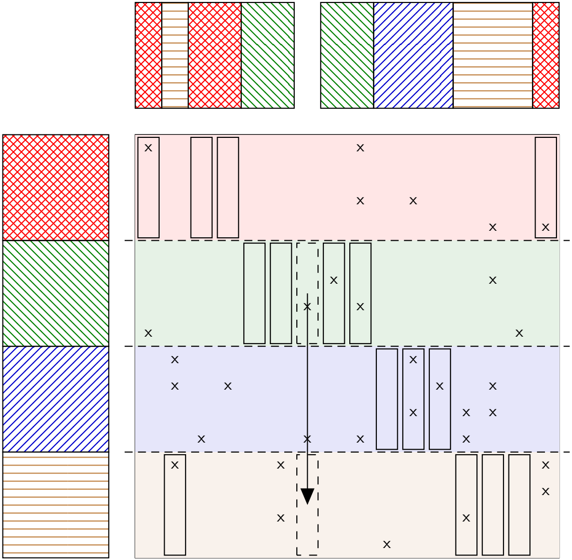

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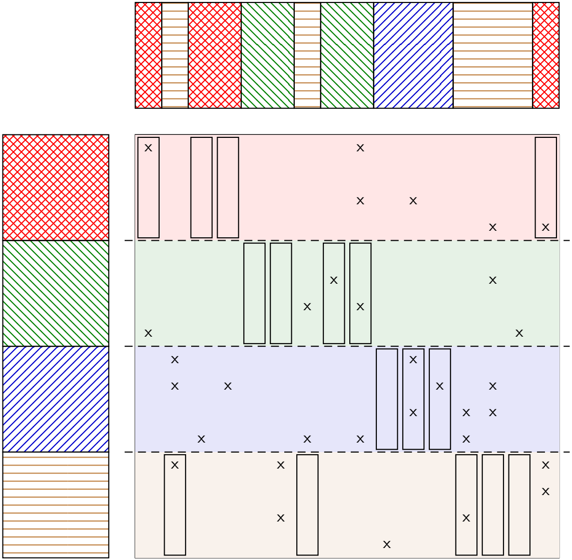

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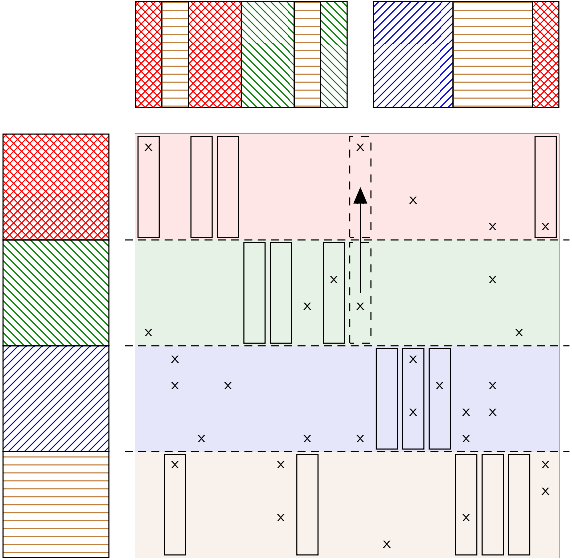

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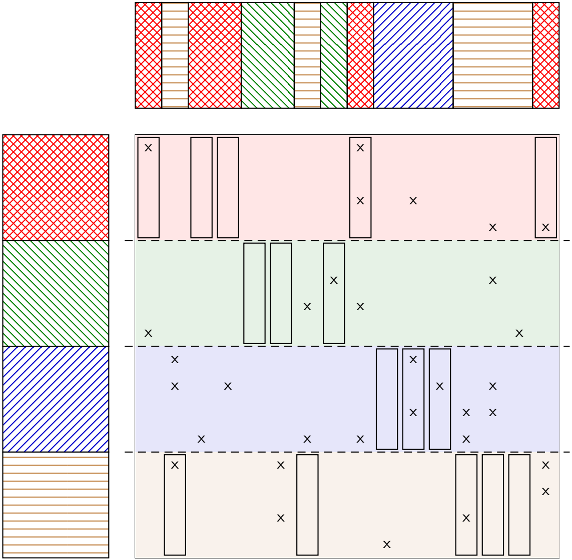
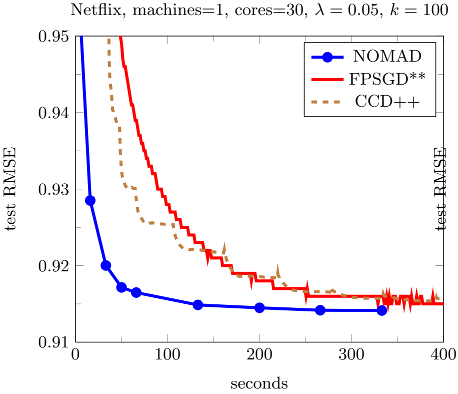


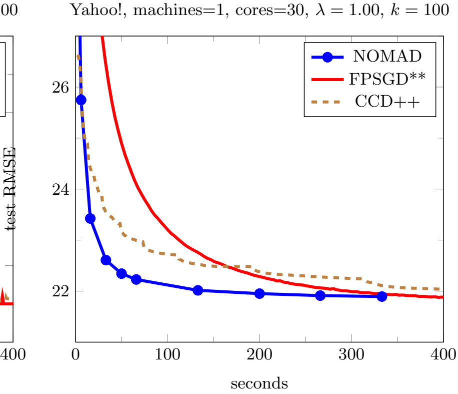
Comparison on a Multi-core System

- On a 32-core processor with enough RAM.
- Comparison: NOMAD, FPSGD**, and CCD++.

(100M ratings)



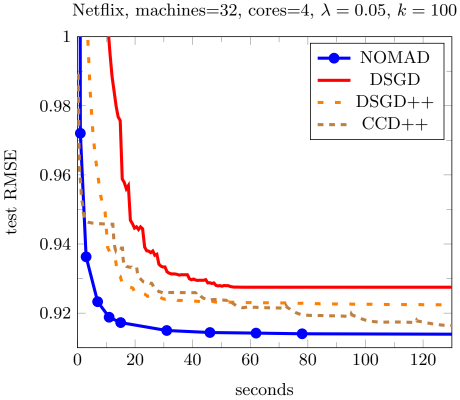
(250M ratings)



Comparison on a Distributed System

- On a distributed system with 32 machines.
- Comparison: NOMAD, DSGD, DSGD++, and CCD++.

(100M ratings)



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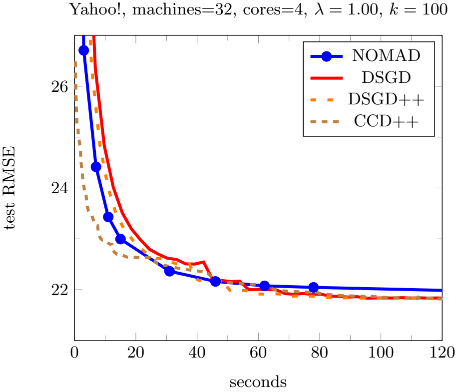
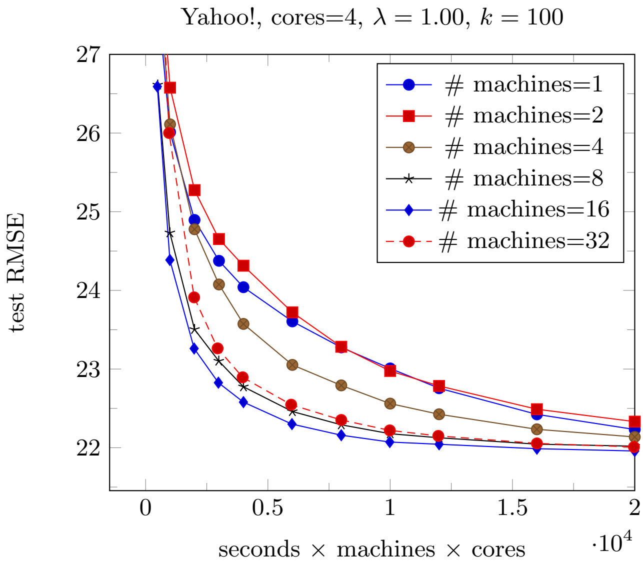
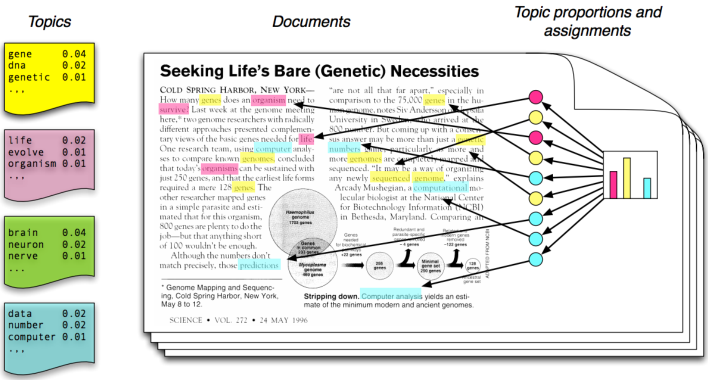


Super Linear Scaling of NOMAD-MF



Topic Modeling: Latent Dirichlet Allocation

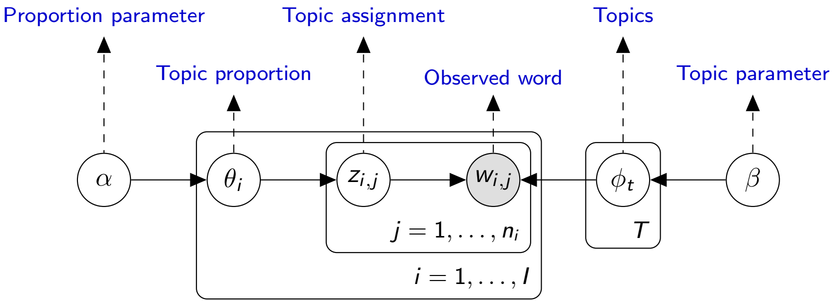
Latent Dirichlet Allocation (LDA)



- Each **topic** is a multinomial distribution over words
- Each **document** is a multinomial distribution over topics
- Each **word** is drawn from one of these topics

[1]source: http://www.cs.columbia.edu/~blei/papers/icml-2012-tutorial.pdf

Graphical Model for LDA

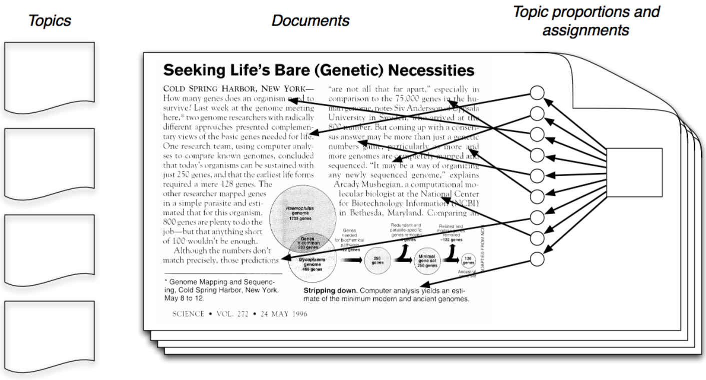


- Joint distribution

$$Pr(\cdot) = \prod_{t=1}^{T} Pr(\phi_t \mid \beta) \prod_{i=1}^{I} Pr(\theta_i \mid \alpha) \left(\prod_{j=1}^{n_i} Pr(z_{i,j} \mid \theta_i) Pr(w_{i,j} \mid \phi_{z_{i,j}}) \right)$$

- $Pr(\phi_t \mid \beta)$, $Pr(\theta_i \mid \alpha)$: Dirichlet distributions
- $Pr(w \mid \phi_t)$, $Pr(z \mid \theta_i)$: multinomial distributions

Inference for LDA



- Only documents are observed
- $\theta_t, \phi_t, z_{i,j}$ are latent
- Goal: infer these latent structures

[1] source: http://www.cs.columbia.edu/~blei/papers/icml-2012-tutorial.pdf

Posterior Inference for LDA

Task: $Pr(\theta_i, \phi_t, z_{i,j} \mid \{d_i\}, \alpha, \beta)$

- Given
  - a corpus of documents $\{d_i : i = 1, \ldots, N\}$, α, β
  - each document $d_i = \{w_{i,j} : j = 1, \ldots, n_i\}$
- Exact inference for $z_{i,j}$, θ_i, ϕ_t
  - Intractable
  - Latent variables are dependent when conditioned on data

Approximate Inference approaches:

- Variational Methods
  - See [Blei et al, 2003]
  - an optimization approach
  - runs faster
  - but generates biased results
- Gibbs Samplings
  - See [Griffiths & Steyvers, 2004]
  - an MCMC approach
  - more accurate
  - but slower with a vanilla implementation

Goal: Design a scalable Gibbs sampler for LDA

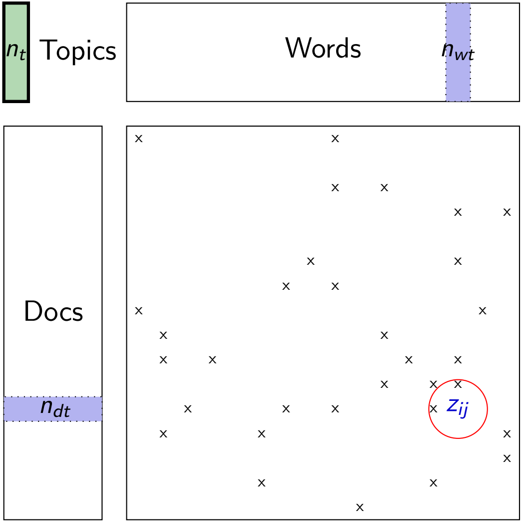
Gibbs Sampling for LDA [Griffiths & Steyvers, 2004]

- Count matrices for topic assignment $\{z_{i,j}\}$:
  - n_{dt}: # words of document d assigned to topic t
  - n_{wt}: # of times word w assigned to topic t
  - $n_t := \sum_w n_{wt} = \sum_d n_{dt}$
- Gibbs Sampling Step
  1. choose $w := w_{i,j}$ with old assignment $t_o := z_{i,j}$ of document $d := d_i$
  2. Decrease n_{dt_o}, n_{wt_o}, n_{t_o} by 1
  3. Resample a new assignment $t_n := z_{i,j}$ according to

     $$Pr(z_{i,j} = t) \propto \frac{(n_{dt} + \alpha)(n_{wt} + \beta)}{n_t + \bar{\beta}}, \quad \forall t = 1, \ldots, T.$$

  4. Increase n_{dt_n}, n_{wt_n}, n_{t_n} by 1
- Constants
  - J: vocabulary size
  - $\bar{\beta} = \beta \times J$

Access Pattern for Gibbs Sampling

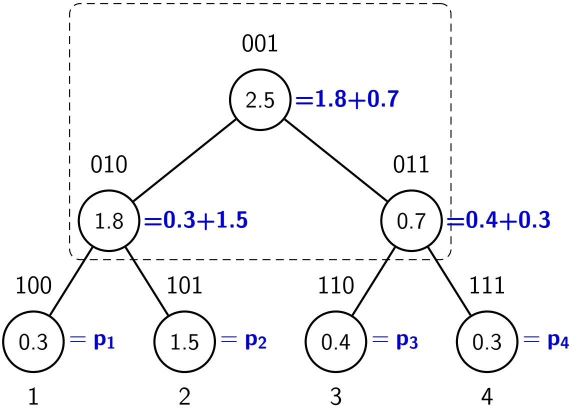


Multinomial Sampling Techniques for $\mathbf{p} \in R_+^T$

	Initialization Time	Initialization Space	Generation Time	Parameter Update Time
LSearch	$\Theta(T)$	$\Theta(1)$	$\Theta(T)$	$\Theta(1)$
BSearch	$\Theta(T)$	$\Theta(1)$	$\Theta(\log T)$	$\Theta(T)$
Alias Method	$\Theta(T)$	$\Theta(T)$	$\Theta(1)$	$\Theta(T)$
F+tree Sampling	$\Theta(T)$	$\Theta(1)$	$\Theta(\log T)$	$\Theta(\log T)$

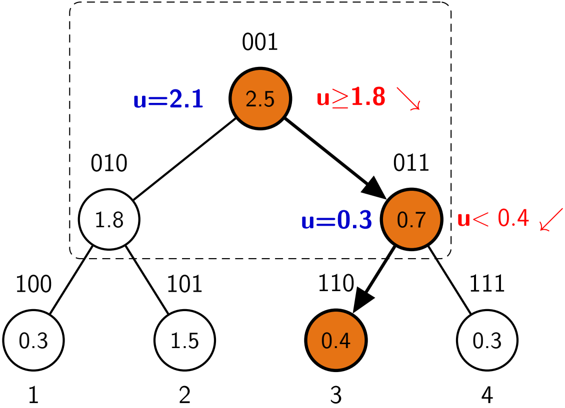
- LSearch
  - maintain $c_T = \mathbf{p}^\top \mathbf{1}$
  - linear search
  - $\Theta(1)$ update
- BSearch
  - maintain $\mathbf{c} = \texttt{cumsum}(\mathbf{p})$
  - binary search
  - no support for update
- Alias Method
  - Alias table
  - construction has some overhead
  - no support for updates
- F+tree
  - a variant of Fenwick tree
  - construction has low overhead
  - logarithmic time for sampling and update

- Construction in $\Theta(T)$ time
- $\mathbf{p} = [0.3, 1.5, 0.4, 0.3]^\top$



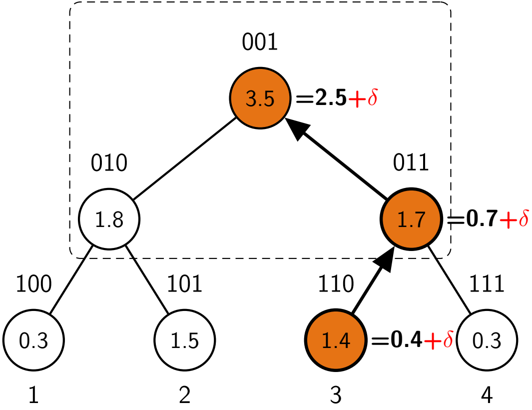
F+Tree: Sampling

- Multinomial sampling in $\Theta(\log T)$ time
- Initial u: a uniformly number drawn from $[0, F[1])$



F+Tree: Update

- Update in $\Theta(\log T)$ time
- $p_3 \leftarrow p_3 + \delta$



F+LDA = LDA with F+tree Sampling

- Decomposition of $\mathbf{p}$

$$p_t = \frac{(n_{dt}+\alpha)(n_{wt}+\beta)}{n_t+\bar{\beta}}, \quad \forall t = 1,\ldots,T.$$
$$= \underbrace{\beta\left(\frac{n_{dt}+\alpha}{n_t+\bar{\beta}}\right)}_{q_t} + \underbrace{n_{wt}\left(\frac{n_{dt}+\alpha}{n_t+\bar{\beta}}\right)}_{r_t}. \quad (1)$$

- $\mathbf{p} = \beta\mathbf{q} + \mathbf{r}$
  - two-level sampling for $\mathbf{p}$
- $\mathbf{q}$ is dense
  - only 2 entries (q_{t_o}, q_{t_n}) change for each Gibbs step in the same document
  - use F+Tree for $\mathbf{q}$
- $\mathbf{r}$ is sparse
  - nonzero entries: $T_w := \{t : n_{tw} \neq 0\}$
  - entire $\mathbf{r}$ changes for each Gibbs step
  - use BSearch for $\mathbf{r}$
- Can also work on word-by-word update sequence

F+LDA: Alternative Decomposition

- Word-by-word Gibbs sampling sequence
- Decomposition of $\mathbf{p}$

$$
\begin{aligned}
p_t &= \frac{(n_{dt}+\alpha)(n_{wt}+\beta)}{n_t+\bar{\beta}}, \quad \forall t = 1, \ldots, T. \\
&= \alpha \underbrace{\left(\frac{n_{wt}+\beta}{n_t+\bar{\beta}}\right)}_{q_t} + \underbrace{n_{dt}\left(\frac{n_{wt}+\beta}{n_t+\bar{\beta}}\right)}_{r_t}.
\end{aligned} \tag{2}
$$

- $\mathbf{p} = \alpha\mathbf{q} + \mathbf{r}$
- $\mathbf{q}$: slight changes for this sequence $\Rightarrow$ use F+Tree
- $\mathbf{r}$: $|T_d := \{t : n_{dt} \neq 0\}|$ nonzeros $\Rightarrow$ use BSearch

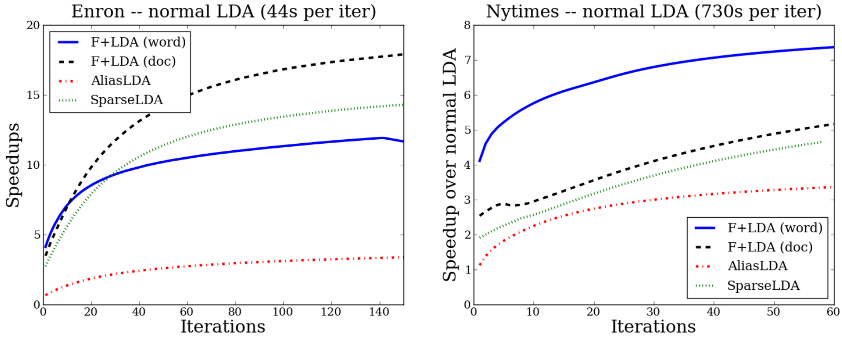
Comparison to Other LDA Sampling

	F+LDA		F+LDA		Sparse-LDA			Alias-LDA	
Sequence	Word-by-Word		Doc-by-Doc		Doc-by-Doc			Doc-by-Doc	
Exact?	Yes		Yes		Yes			No	
Decomposition	$\alpha\left(\frac{n_{wt}+\beta}{n_t+\beta}\right)$	$+n_{dt}\left(\frac{n_{wt}+\beta}{n_t+\beta}\right)$	$\beta\left(\frac{n_{dt}+\alpha}{n_t+\beta}\right)$	$+n_{wt}\left(\frac{n_{dt}+\alpha}{n_t+\beta}\right)$	$\frac{\alpha\beta}{n_t+\beta}$	$+\beta\left(\frac{n_{dt}}{n_t+\beta}\right)$	$+n_{wt}\left(\frac{n_{dt}+\alpha}{n_t+\beta}\right)$	$\alpha\left(\frac{n_{wt}+\beta}{n_t+\beta}\right)$	$+n_{dt}\left(\frac{n_{wt}+\beta}{n_t+\beta}\right)$
Structure	F+tree	BSearch	F+tree	BSearch	LSearch	LSearch	LSearch	Alias	Alias
Fresh samples	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes
Initialization	$\Theta(\log T)$	$\Theta(\lvert T_d\rvert)$	$\Theta(\log T)$	$\Theta(\lvert T_w\rvert)$	$\Theta(1)$	$\Theta(1)$	$\Theta(\lvert T_w\rvert)$	$\Theta(1)$	$\Theta(\lvert T_d\rvert)$
Sampling	$\Theta(\log T)$	$\Theta(\log\lvert T_d\rvert)$	$\Theta(\log T)$	$\Theta(\log\lvert T_w\rvert)$	$\Theta(T)$	$\Theta(\lvert T_d\rvert)$	$\Theta(\lvert T_w\rvert)$	$\Theta(\#MH)$	$\Theta(\#MH)$

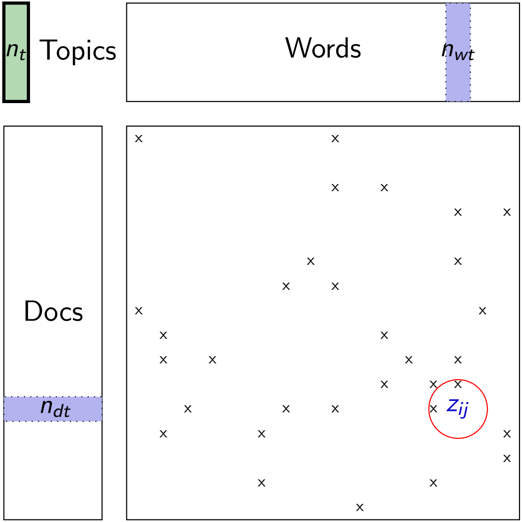
- F+LDA: word-by-word faster than doc-by-doc for large I
  - $|T_d|$ bounded by n_i, but $|T_w|$ approaches to T
  - per Gibbs step cost: $\rho_F \log T + \rho_B |T_d|$
- SparseLDA:
  - per Gibbs step cost: $\Theta(T + |T_d| + |T_w|)$
  - the first $\Theta(T)$ rarely happens but $|T_w| \to T$ for large I
- AliasLDA:
  - per Gibbs step cost: $\rho_A |T_d| + \#MH$
  - $\rho_A \approx 3 \times \rho_B$: construction overhead of Alias table
  - If $(\rho_A - \rho_B)|T_d| > \rho_F \log T \Rightarrow$ AliasLDA slower than F+LDA
  - say $|T_d| \approx 100$, F+LDA still faster for $T < 2^{50}$

Comparison of various sampling methods

- Single machine, single thread
- y-axis: speedup over normal $O(T)$ multinomial sampling
- $T = 1024$
- Enron: 38K docs with 6M tokens
- NyTimes: 0.3M docs with 100M tokens



Access Pattern for Gibbs Sampling

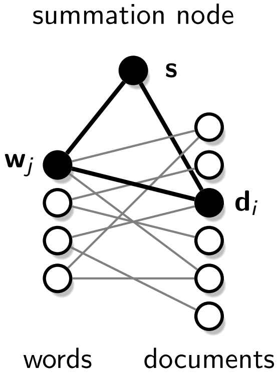


Access Graph for Gibbs Sampling

- $G = (V, E)$: a hyper graph

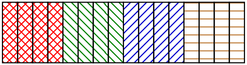
$$V = \{\mathbf{d}_i\} \cup \{\mathbf{w}_j\} \cup \{\mathbf{s}\}$$
$$E = \{e_{ij} = (\mathbf{d}_i, \mathbf{w}_j, \mathbf{s})\}$$

- Connection to Gibbs sampling
  - $(\mathbf{d}_i)_t := n_{d_i t}$, $(\mathbf{w}_j)_t := n_{w_j t}$, $(\mathbf{s})_t := n_t$
  - each e_{ij}: a Gibbs step for word w_j in d_i access to $(\mathbf{d}_i, \mathbf{w}_j, \mathbf{s})$
- Parallelism: more challenging
  - all edges incident to $\mathbf{s}$
  - all $(\mathbf{s})_t$ are large in general
    $\Rightarrow$ slightly stale $\mathbf{s}$ is fine for accuracy
  - duplicate $\mathbf{s}$ for parallelism



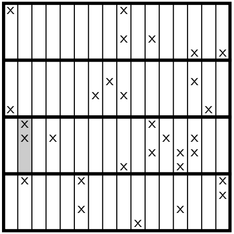
Nomadic Tokens for $\{\mathbf{w}_j : j = 1, \ldots, J\}$:

- J tokens
- $(j, \mathbf{w}_j)$: $O(T)$ space



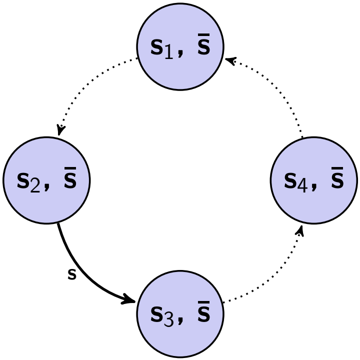
Worker:

- p workers
- a computing unit + a concurrent token queue
- a subset of $\{\mathbf{d}_i\}$: $O(IT/p)$
- "x": an occurrence of a word
- bigger rectangle: a subset of corpus
- smaller rectangle: a unit subtask



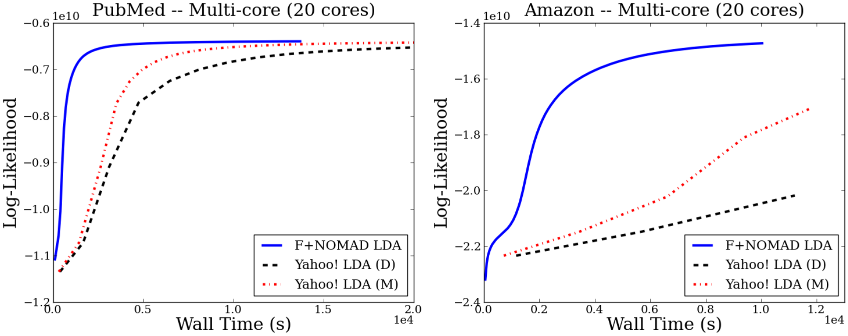
Nomadic Token for $\mathbf{s}$: Circular Delta Update

- Single global $\mathbf{s}$
  - travels among machines as a messenger
  - broadcasts local delta updates
- Every machine p: $(\mathbf{s}_p, \bar{\mathbf{s}})$
  - $\mathbf{s}_p$: local working copy
  - $\bar{\mathbf{s}}$: snapshot version of global $\mathbf{s}$



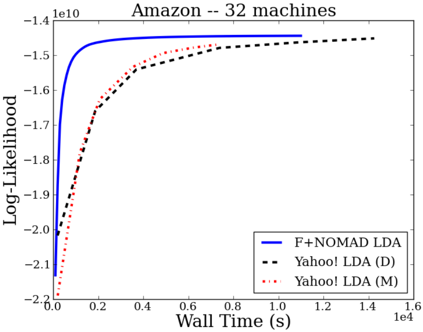
$$\mathbf{s} \leftarrow \mathbf{s} + (\mathbf{s}_3 - \bar{\mathbf{s}})$$
$$\bar{\mathbf{s}} \leftarrow \mathbf{s}$$
$$\mathbf{s}_3 \leftarrow \mathbf{s}$$

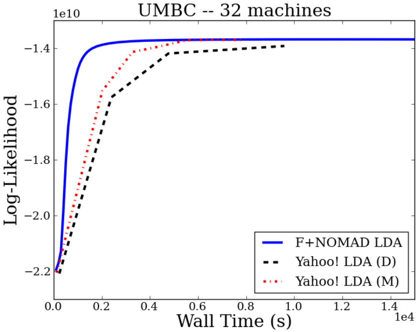
- On a machine with a 20-core processor
- Comparison: F+NOMAD LDA, Yahoo! LDA
- PubMed: 9M docs with 700M tokens
- Amazon: 30M docs with 1.5B tokens



Comparison on a Multi-machine System

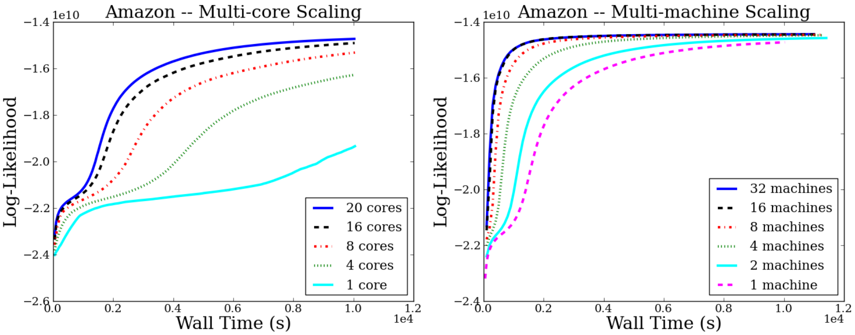
- 32 machines, each with a 20-core processor, $T = 1024$
- Comparison: F+NOMAD LDA, Yahoo! LDA
- Amazon: 30M docs with 1.5B tokens
- UMBC: 40M docs with 1.5B tokens





Scaling of NOMAD-LDA

- Each machine has 20 cores
- Multi-machine setting:
  - 18 cores for sampling
  - 2 cores for communication



Conclusions

- NOMAD framework uses nomadic tokens to provide
  - Asynchronous computation
  - Non-blocking communication
  - Lock-free implementation
  - Serializable or near Serializable
- Recommender System: Matrix factorization
  - scalable parallel stochastic gradient
  - Serializability guarantee
- Topic Modeling: Latent Dirichlet Allocation
  - Logarithmic F+tree sampling
  - Efficient Gibbs Sampling
  - Duplicated nomadic tokens for the common node
  - Outperforms Yahoo! LDA