CS 323E

Homework 10

Due: April 30th 2002

1. The MATLAB command hilb(n) generates an $n \times n$ Hilbert matrix, which we denote by $H_n$. Try $n = 3, 10, 20$ in the following problems:

   a) (4 points)
   Solve:

   $$H_n x_n = b_n$$

   for $x_n$, where $b_n = H_n * \text{ones}(n,1)$.
   Use the MATLAB command \"\" to solve the above system. (See help mldivide).

   b) (2 points)
   How close is $x_n$ to the exact solution? Comment.

   c) (4 points)
   Explain the accuracy of $x_n$. Use the command cond to get the condition number of $H_n$.

2. (5 points)
Does the MATLAB command \"\" do pivoting? Give reasons for your answer.

3. (Use pen & paper). Let

   $$A = \begin{bmatrix} 10^{-16} & 10^{-17} \\ -10^{-16} & 10^{-17} \end{bmatrix}$$

   a) (2 points)
   Compute the determinant of $A$.

   b) (5 points)
   Compute $\kappa_1(A) = \|A\|_1 \cdot \|A^{-1}\|_1$.

   c) (2 points)
   Is $A$ nearly singular? Comment.

   d) (1 point)
   Does the small magnitude of the determinant imply that $A$ is nearly singular?

4. The MATLAB command pascal(n) generates an $n \times n$ Pascal matrix, which we denote by $P_n$. Try $n = 16$ in the following.

   a) (1 point)
   Using MATLAB, find the determinant of $P_n$.

   b) (1 point)
   Using MATLAB, find the condition number of $P_n$.

   c) (3 points)
   Is $P_n$ close to singularity? Comment.

5. (10 points) Numerical Integration
In class we have seen various rules for numerical integration: Left-point rule, Mid-point
rule, Right-point rule, Trapezoidal and Simpson’s Rule. The task in this problem is to evaluate the following integral:

\[ I = \int_{-1}^{1} e^{kt} (1 - t^2)^{p/2 - 3/2} dt \]

for \( p = 10, 100, 1000, 10000 \) and \( k = p/2 \). Perform the above integration by subdividing the interval \([-1,1]\) into small subintervals and then evaluating the integral over each subinterval by all the above-mentioned methods. After obtaining the integral \( I \) above also evaluate

\[ c_p = \frac{\sqrt{\pi}}{I \star (p/2 - 1)} \].