CS 323E **Homework 10** **Due: April 30th 2002**

1. The MATLAB command `hilb(n)` generates an $n \times n$ Hilbert matrix, which we denote by H_n. Try $n = 3, 10, 20$ in the following problems:

   1) (4 points)
   Solve:

   $$H_n x_n = b_n$$

   for x_n, where $b_n = H_n * \texttt{ones}(n, 1)$.
   Use the MATLAB command "\" to solve the above system. (See `help mldivide`).

   b) (2 points)
   How close is x_n to the exact solution? Comment.

   c) (4 points)
   Explain the accuracy of x_n. Use the command `cond` to get the condition number of H_n.

2. (5 points)
   Does the MATLAB command "\" do pivoting? Give reasons for your answer.

3. (Use pen & paper). Let

   $$A = \begin{bmatrix} 10^{-16} & 10^{-17} \\ -10^{-16} & 10^{-17} \end{bmatrix}$$

   a) (2 points)
   Compute the determinant of A.

   b) (5 points)
   Compute $\kappa_1(A) = \|A\|_1 \cdot \|A^{-1}\|_1$.

   c) (2 points)
   Is A nearly singular? Comment.

   d) (1 point)
   Does the small magnitude of the determinant imply that A is nearly singular?

4. The MATLAB command `pascal(n)` generates an $n \times n$ Pascal matrix, which we denote by P_n. Try $n = 16$ in the following.

   a) (1 point)
   Using MATLAB, find the determinant of P_n.

   b) (1 point)
   Using MATLAB, find the condition number of P_n.

   c) (3 points)
   Is P_n close to singularity? Comment.

5. (10 points) **Numerical Integration**
   In class we have seen various rules for numerical integration: Left-point rule, Mid-point

rule, Right-point rule, Trapezoidal and Simpson's Rule. The task in this problem is to evaluate the following integral:

$$I = \int_{-1}^{+1} e^{kt}(1-t^2)^{p/2-3/2}\,dt$$

for $p = 10, 100, 1000, 10000$ and $k = p/2$. Perform the above integration by subdividing the interval [-1,1] into small subintervals and then evaluating the integral over each subinterval by all the above-mentioned methods. After obtaining the integral I above also evaluate

$$c_p = \frac{\sqrt{\pi}}{I * (p/2-1)}$$

.