

(a) (4 points)

Write a MATLAB function  $[L, U] = \text{MyLU}(A)$  which computes the  $LU$  factorization of the input matrix  $A$ . Write it as 3 nested loops.

(b) (2 points)

Write the function  $y = \text{UTriSol}(U, x)$  which solves for  $y$  such that  $Uy = x$  (given  $U$  and  $x$ ); write the function  $x = \text{LTriSol}(L, b)$  which solves for  $x$  such that  $Lx = b$  (given  $L$  and  $b$ ).

(c) (5 points)

Write a function  $x = \text{LUSolve}(A, b)$ . You should use  $\text{MyLU}$ ,  $\text{UTriSol}$  and  $\text{LTriSol}$ .

(d) (6 points)

Use the above to solve for  $x$  when

(i)

$$A = \begin{bmatrix} -2 & 4 & -1 & -1 & 3 \\ 4 & -9 & 0 & 5 & 3 \\ -4 & 5 & -5 & 5 & 3 \\ -8 & 8 & -23 & 20 & 3 \\ -1 & 1 & 2 & 3 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 12 \\ -32 \\ 3 \\ -13 \\ -8 \end{bmatrix} \quad (1)$$

(ii)

$$A = \begin{bmatrix} 10^{-16} & 1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (2)$$

(iii)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 6 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix} \quad (3)$$

(e) (2 points)

Comment on the accuracy of  $x$  in (1), (2) and (3).

(f) (5 points)

Incorporate partial pivoting in  $\text{MyLU}$ , i.e., write the function  $[P, L, U] = \text{MyPLU}(A)$ . Note that  $PA$  must be equal to  $LU$ .

(g) (6 points)

Use  $\text{MyPLU}$  to solve  $Ax = b$  when  $A$  and  $b$  are as in (1), (2) and (3). Comment on the solution accuracy.