

1. (10 points)

Solve a linear system of equations given by

$$\begin{aligned}2x_1 + x_2 \quad \quad + x_4 &= 4 \\2x_1 + 2x_2 + 2x_3 + 4x_4 &= 10 \\4x_1 + 2x_2 - 3x_3 + 3x_4 &= 6 \\-2x_1 + \quad x_2 + x_3 + 5x_4 &= 5\end{aligned}$$

You should first write the above system in matrix form,  $Ax = b$ . Then, use Gaussian Elimination to factor the matrix  $A = LU$ , where  $L$  is unit lower triangular and  $U$  is upper triangular. Now you have  $Ax = b \Rightarrow (LU)x = b$ . Use forward substitution to find  $y$  such that  $Ly = b$ , and as the last step use backward substitution to find the final answer  $x$  such that  $Ux = y$ .

**Note 1:** Do the above computations using pen/pencil and paper. Show all intermediate steps.

**Note 2:** Verify your answer in MATLAB using  $A \setminus b$ .

2. (10 points)

Write a program in Matlab  $[x] = \text{lsolve}(A,b)$  to do the above computations. Note that  $A$  can be any matrix and  $b$  can be any right hand side. However, assume that  $A$  is nonsingular and that no “pivoting” is required to solve the linear system. You can use ‘if’, ‘for’, ‘while’ statements and ‘+’, ‘-’, ‘\*’ and ‘/’ operations. DO NOT USE any MATLAB functions that directly solve the linear system, such as, LU `mldivide`, `\`, etc.